PHYSICAL REVIEW VOLUME 111, NUMBER 3 AUGUST 1, 1958

Λ-Nucleon Interaction in the Hypernucleus $_\Lambda Be^9$†

KIU S. SUH
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received April 7, 1958)

A variational method has been used to calculate the strength of the Λ-nucleon interaction required to account for the observed binding of $_\Lambda Be^9$. The hypernucleus is treated as a three-body system (two alpha particles and the hyperon), in which the spin-average of the Λ-nucleon interaction is effective. Results in agreement with previous calculations of the Λ-nucleon interaction in $_\Lambda He^5$ are obtained without appreciable overlapping of the two alpha particles. This absence of overlapping is taken to be a reasonable criterion for the validity of the model.

I. INTRODUCTION

An analysis of the binding energy data for the light hypernuclei in terms of a phenomenological spin-dependent, two-body Λ-nucleon interaction potential has recently been given by Dalitz and Downs.[1] In that analysis a hypernucleus was assumed to consist of a Λ particle loosely bound to a relatively undistorted stable core nucleus. The hypernucleus $_\Lambda Be^9$ was not included in the analysis because of the absence of a stable bound state for the core nucleus Be^8. It is the purpose of the present paper to present an analysis of $_\Lambda Be^9$ to complement the analysis of Dalitz and Downs.

The three-body model of $_\Lambda Be^9$ in which two alpha particles are considered to be held together by a Λ particle is suggested by the observed mesonic decay $_\Lambda Be^9 \rightarrow \pi^- + p + Be^8$ followed by $Be^8 \rightarrow He^4 + He^4$; analysis of decays of this type indicates a value $B_\Lambda = 6.4 \pm 0.4$ Mev for the binding energy of the Λ particle.[2]

A variational calculation is made to determine the volume integral of the Λ-nucleon interaction required to account for the observed value of B_Λ for two assumed values of the range of the interaction. If the three-body model gives an adequate description of $_\Lambda Be^9$, then the volume integrals calculated here ought to agree with those calculated from the two-body model of $_\Lambda He^5$. The results do, in fact, give this agreement, indicating that the three-body model is reasonable while providing some confirmation of the previous analysis of $_\Lambda He^5$.[1]

In the course of the analysis, an effective nuclear interaction potential between two alpha particles had to be introduced. Since the nucleons inside each alpha particle form a tightly bound closed shell, the Pauli principle for individual nucleons tends to prevent appreciable overlap between two alpha particles. This tendency is incorporated in our effective alpha-alpha interaction, which we assume to be repulsive at short distances. The details of this potential are introduced in a rather arbitrary manner, but it is adjusted to give the correct energy for the Be^8 ground state (alpha-alpha resonance energy of 96 kev). It will be shown, however, that the final results are not very sensitive to the detailed shape of the potential, provided only that the assumed repulsion is sufficient to prevent any appreciable overlap of the two alpha particles.[3] For the variation calculation a two-parameter trial function is employed; one of these parameters governs the average separation of the alpha particles. The extent of the alpha-alpha overlap is measured by a comparison between the mean square alpha-alpha separation and twice the mean square radius of the alpha particle. It will be seen later that, in fact, no overlap does occur in our model.

The Λ-nucleon interaction can be expected to arise mainly from two physical mechanisms: exchange of a single K meson and/or exchange of two pions.[1,4] For simplicity in these calculations, the Λ-nucleon interaction potential is taken to be of Gaussian form.[5] Calculations are made for two intrinsic ranges of this potential, intrinsic ranges equal to those of a Yukawa potential $e^{-\kappa r}/\kappa r$ with ranges of the Compton wavelength of the K meson $(1/\kappa)_K = \hbar/M_K c$ and one-half the pion Compton wavelength $(1/\kappa)_{\pi/2} = \hbar/2M_\pi c$, respectively; these ranges correspond to the two physical mechanisms mentioned above.[6]

† Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958) this issue. See also earlier references cited there.

[2] Levi Setti, Slater, and Telegdi, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Sec. 8, p. 6., and W. Slater, University of Chicago dissertation, 1958 (unpublished).

[3] If the short-distance repulsion between the alpha particles were omitted entirely, the resultant wave function would (incorrectly) contain a large overlap between the alpha particles, and would grossly overestimate the Λ binding energy for a given volume integral of the Λ-nucleon interaction. In fact, for a (quite unrealistic) wave function for which the overlap is complete, the observed Λ binding energy of $_\Lambda Be^9$ would lead to a value for the volume integral of Λ-nucleon interaction only about half as big as that obtained previously from $_\Lambda He^5$ (or as that obtained in this paper from a more realistic treatment of $_\Lambda Be^9$).

[4] K. Nishijima, Progr. Theoret. Phys. Japan **14**, 526 (1956).

[5] Dalitz and Downs (reference 1) have shown that the volume integrals of the Λ-nucleon potential required to account for the observed hypernuclear binding energies are insensitive to the details of the potential shape and also to possible exchange character of the interaction. Although the exchange of a K meson implies an exchange force, only ordinary forces are used here; the two ranges used are just intended to span the reasonable ranges.

[6] H. Wilhelmsson and P. Zielinski [Nuclear Phys. **6**, 219 (1958)], have recently reported an attempt to establish both the strength and range of the Λ-alpha interaction by an analysis of $_\Lambda He^5$ and $_\Lambda Be^9$. Their analysis of $_\Lambda Be^9$ is similar to that presented here except that the alpha-alpha nuclear interaction is omitted.

II. DISCUSSION OF THE MODEL

In the three-body model, $_\Lambda Be^9$ is assumed to consist of two undistorted alpha particles held together by a Λ particle in an S state with respect to the center of mass of the two alpha particles. Since the spin-saturated alpha particle is such a tightly bound structure, the effect of possible deformation of the individual alpha particles is expected to be unimportant provided that this model does not imply appreciable overlapping of the two alpha particles.[7] The variational principle for the three-body model of $_\Lambda Be^9$ can be expressed as

$$\int\int\Psi\left\{-\frac{\hbar^2}{2\mu_\alpha}\nabla_l^2-\frac{\hbar^2}{2\mu_\Lambda}\nabla_\rho^2+V_{\Lambda\alpha}(|\boldsymbol{\rho}-\tfrac{1}{2}\mathbf{l}|)\right.$$
$$\left.+V_{\Lambda\alpha}(|\boldsymbol{\rho}+\tfrac{1}{2}\mathbf{l}|)+V_{\alpha\alpha}(l)+C_{\alpha\alpha}(l)-E\right\}\psi d\mathbf{l}d\boldsymbol{\rho}\geqslant 0, \quad (1)$$

where $\mathbf{l}$ is the alpha-alpha separation vector and $\boldsymbol{\rho}$ is the position vector of the Λ particle from the center of mass of the two alpha particles. Equation (1) is used as a variation principle for the strength of the Λ-nucleon interaction. The reduced masses are $\mu_\alpha=M_\alpha/2$ and $\mu_\Lambda=2M_\Lambda M_\alpha/(M_\Lambda+2M_\alpha)$, where M_α and M_Λ are the masses of the alpha and Λ particle, respectively. The total energy of the system is $E=(0.096-B_\Lambda)$ Mev, the alpha-alpha resonance energy being 96 kev. Calculation has shown that an uncertainty in E is not important for this analysis, i.e., an uncertainty of 10% in E introduces an uncertainty of about 2% in the volume integral of the Λ-nucleon interaction.

The nucleon distribution in the alpha particle is taken to be the same as that for the proton distribution deduced from the results of electron scattering experiments.[8] This Gaussian distribution, normalized to unity, is

$$P(\eta)=(\alpha^2/\pi)^{\frac{3}{2}}\exp(-\alpha^2\eta^2), \quad (2)$$

the rms radius of the distribution being 1.44 ± 0.07 fermi [1 fermi (f)$\equiv10^{-13}$ cm] with $\langle\eta^2\rangle=3/2\alpha^2$. The Gaussian potential taken for the Λ-nucleon interaction is

$$V_{\Lambda n}(r')=-\left(\frac{\beta^2}{\pi}\right)^{\frac{3}{2}}\Omega\exp(-\beta^2r'^2), \quad (3)$$

r' being the distance between the Λ particle and any particular nucleon. The volume integral of the Λ-nucleon interaction Ω is defined by

$$\Omega=-\int V_{\Lambda n}(r')d\mathbf{r}', \quad (4)$$

and β is related to the intrinsic range b of the potential by $\beta^2=2.0604/b^2$ which is, in turn, related to the range parameter κ by $b=2.1196/\kappa$. Integration of (3) over the four nucleons in the alpha particle distributed according to (2) yields the Λ-alpha interaction potential

$$V_{\Lambda\alpha}(r)=-4\Omega(q^2/\pi)^{\frac{3}{2}}\exp(-q^2r^2), \quad (5)$$

where $q=\alpha\beta/(\alpha^2+\beta^2)^{\frac{1}{2}}$ and $r=|\boldsymbol{\rho}\pm\frac{1}{2}\mathbf{l}|$ in our coordinate system. The volume integral Ω is the average volume integral per nucleon. For spin $\frac{1}{2}$ for the Λ particle, Ω corresponds to $(3\Omega_p+\Omega_a)/4$, where Ω_p is the volume integral of the triplet interaction and Ω_a is that for the singlet interaction.[9]

In order to represent the nuclear interaction between the two alpha particles, the following potential is introduced to provide a repulsion at short distances and maximum attraction at a distance of about twice the rms radius of the alpha particle:

$$V_{\alpha\alpha}(l)=A\exp(-\delta^2l^2)-B\exp(-\epsilon^2l^2). \quad (6)$$

The four parameters A, B, δ^2, and ϵ^2 of (6) are fixed by the arbitrary conditions $V(2\text{f})=0$, $(dV/dl)=0$ for $l=3$f, $|V(4\text{f})|\simeq\frac{1}{2}|V(3\text{f})|$, and the fourth condition that (6) provide the correct nuclear interaction energy between two alpha particles. In order to estimate this nuclear interaction energy, the Coulomb energy must be subtracted from the known alpha-alpha resonance energy of 96 kev.[10] The Coulomb energy is estimated to be $4e^2/\langle l\rangle\simeq1.8$ Mev where the appropriate alpha-alpha separation is taken to be $\langle l\rangle\simeq3$f. The potential (6) is then required to account for an energy -1.7 Mev between two alpha particles. The fourth condition on the parameters of (6) results from a variation calculation for a system of two alpha particles without Coulomb interaction with the trial function $l\exp(-a^2l^2)$. The four parameters thus determined are $\delta^2=0.406$ f^{-2}, $\epsilon^2=0.116$ f^{-2}, $A=96$ Mev, and $B=30$ Mev which imply that $V_{\alpha\alpha}(0)=66$ Mev and $V(3\text{f})=-10$ Mev. Because the parameters were determined by a variational calculation, the correct potential must be less strongly attractive than (or at most the same as) the calculated potential (6). Calculation has shown, however, that the total contribution of $V_{\alpha\alpha}$ to the calculated value of Ω is about 3%, so that errors in $V_{\alpha\alpha}$ are to be considered relatively unimportant.

The trial function for these calculations is then taken to be

$$\psi=l\exp(-a^2l^2)\exp(-b^2\rho^2), \quad (7)$$

where a and b are the two variation parameters. The first factor of (7) was suggested by the fact that the repulsion of the alpha-alpha nuclear interaction (6) at small distances prevents very close approach of the two alpha particles; the second by the assumed s-wave motion of the Λ particle.

[7] Dalitz and Downs (reference 1) have shown that consideration of radial compression for the alpha-particle core of $_\Lambda He^5$ allows an improvement of at most 3% in the volume integral of the Λ-nucleon interaction.

[8] R. Hofstadter, Revs. Modern Phys. 28, 214 (1956).

[9] Both triplet and singlet potentials are assumed to have the same form (3); the Λ-neutron and Λ-proton interactions have been taken to be the same.

[10] R. H. Brown and D. R. Inglis, Phys. Rev. 55, 1182 (1939).

Finally, the Coulomb energy of the two alpha particles is obtained by taking the charge distribution in the alpha particle to be given by (2) with the rms radius 1.61 ± 0.05 f of the charge distribution[8] rather than that of the nucleon distribution. With the trial function (7), the expectation value of the coulomb energy is then

$$\langle C_{\alpha\alpha}(l)\rangle = \left\langle \psi(l,\rho)\left| 4e^2\int\int \frac{P(\eta_1)P(\eta_2)}{|\mathbf{l}+\boldsymbol{\eta}_1-\boldsymbol{\eta}_2|} \times d\boldsymbol{\eta}_1 d\boldsymbol{\eta}_2 \right|\psi(l,\rho)\right\rangle \Big/ \langle\psi(l,\rho)|\psi(l,\rho)\rangle$$
$$= 4e^2\left(\frac{\alpha^2}{\pi}\right)^3 \left\langle \psi(l,\rho)\left| \int\int \frac{\exp[-\alpha^2(\eta_1^2+\eta_2^2)]}{|\mathbf{l}+\boldsymbol{\eta}_1-\boldsymbol{\eta}_2|} \times d\boldsymbol{\eta}_1 d\boldsymbol{\eta}_2 \right|\psi(l,\rho)\right\rangle \Big/ \langle\psi(l,\rho)|\psi(l,\rho)\rangle$$
$$= \frac{2^5e^2\alpha a}{3(2\pi)^{\frac{1}{2}}}\left(\frac{6a^2+\alpha^2}{[(2a)^2+\alpha^2]^{\frac{3}{2}}}\right), \quad (8)$$

where $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are the internal radius vectors of the two alpha particles and $\mathbf{l}$ is the alpha-alpha separation vector. The contribution to Ω from $\langle C_{\alpha\alpha}(l)\rangle$ turns out to be small but not negligible, i.e., omission of this term would reduce Ω by about 5%.

III. RESULTS

The variation principle (1) has been used with the trial function (7) to determine the volume integral Ω of the Λ-nucleon potential appearing in (5). The volume integral was minimized with respect to the two variational parameters a and b. The results are given in the first four columns of Table I.

In columns (v) and (vi) are listed the corresponding values of $U_4/4$ obtained by Dalitz and Downs[1] from the analysis of $_{\Lambda}He^5$ with an undistorted alpha-particle core; U_4 is the volume integral of the total Λ-nucleon interaction in $_{\Lambda}He^5$ so that our Ω corresponds to $U_4/4$. The volume integrals listed in column (v) were obtained from an exact solution of the Schrödinger equation for the two-body model of $_{\Lambda}He^5$. Our results differ from these by 10–15%. This discrepancy is largely due to the fact that our trial function (7) is not the exact solution of the Schrödinger equation for the three-body problem of $_{\Lambda}Be^9$. Our results should rather be compared with the values of $U_4/4$ obtained by a variational calculation with a Gaussian trial function for the Λ motion in $_{\Lambda}He^5$. Such a calculation was also done by Dalitz and Downs, whose results are given in column (vi) of Table I. These values of the volume integral are much closer to our results, the discrepancies being $\lesssim 5\%$.

The values of $(\langle l^2\rangle)^{\frac{1}{2}}$ given in Table I indicate the extent of the overlapping of the two alpha particles in our model of $_{\Lambda}Be^9$. These average separations are larger than the square root of twice the mean square radius of the alpha particle (~2 f) but smaller than twice the rms radius of the alpha particle (2.88 f).

TABLE I. Values of the volume integral per nucleon required to account for the observed binding of $_{\Lambda}Be^9$ for two ranges of the Λ-nucleon interaction are listed in column (i). The optimum variational parameters a and b and $(\langle l^2\rangle)^{\frac{1}{2}}$, the rms of α-α separation distance, are listed in columns (ii), (iii), and (iv), respectively. In columns (v) and (vi), the corresponding volume integrals obtained by Dalitz and Downs[1] from the two-body analysis of $_{\Lambda}He^5$ are listed.

Range parameter	(i) Ω (Mev f^3)	(ii) a(f^{-1})	(iii) b(f^{-1})	(iv) $\langle l^2\rangle^{\frac{1}{2}}$(f)	(v) $U_4/4$ (Mev f^3)	(vi) $U_4/4$ (Mev f^3)
$(1/\kappa)_{\pi/2}=\hbar/2M_\pi c$	254	0.41	0.50	2.74	231	251
$(1/\kappa)_K=\hbar/M_K c$	208	0.43	0.57	2.60	180	198

In addition to the principal results reported here, a preliminary calculation of the same kind was made in which a Gaussian alpha-alpha nuclear potential was used with a trial function of the form

$$\exp(-a^2l^2)\exp(-b^2\rho^2).$$

This calculation resulted in an optimum alpha-alpha separation of $(\langle l^2\rangle)^{\frac{1}{2}}\simeq 1.7$ f for both interaction ranges, indicating appreciable overlapping of the two alpha particles. Also the volume integrals obtained there are appreciably smaller than those listed in Table I. A cutoff was made in this variational calculation to provide $(\langle l^2\rangle)\simeq 2.9$ f; this resulted in the volume integral values $\Omega=253$ Mev f^3 and 193 Mev f^3 for the longer and shorter interaction range, respectively.

The analysis of the binding energies of $_{\Lambda}Be^9$ and $_{\Lambda}He^5$ thus indicate the same Λ-nucleon interaction in both hypernuclei provided that the alpha-alpha nuclear interaction in $_{\Lambda}Be^9$ is such as to prevent appreciable overlapping of the two alpha particles. The present analysis of $_{\Lambda}Be^9$, therefore, supports the conclusion by Dalitz and Downs[1] that, if the hypernuclear binding energies are ascribed to a two-body Λ-nucleon interaction, then that interaction must be spin-dependent. Our results disagree with the conclusions reached by Brown and Peshkin[11] on the basis of a two-body analysis of $_{\Lambda}Be^9$ in which the nucleon core was taken to have a rms radius equal to that of normal Be^9.[8]

ACKNOWLEDGMENTS

I wish to thank Dr. B. W. Downs for having suggested this problem and for the considerable assistance and encouragement afforded me throughout the work. I also thank Professor E. E. Salpeter for his kind assistance and discussion.

[11] L. M. Brown and M. Peshkin [Phys. Rev. **107**, 272 (1957)] find that the volume integral per nucleon Ω required to account for the binding of $_{\Lambda}Be^9$ is appreciably larger than that required to account for the binding of $_{\Lambda}He^5$. It seems to us that this discrepancy is due directly to their assumption of an unreasonably large nucleon core for $_{\Lambda}Be^9$.