Application to Electron Scattering of Center-of-Mass Effects in the Nuclear Shell Model

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The usual shell-model treatment of the nuclear scattering of high-energy electrons neglects the motion of the center of mass of the nucleus. The correction due to this is calculated for the case of an oscillator potential well and gives a simple additional factor multiplying the nuclear form factor.

SCHIFF'S calculations[1] of the cross section for scattering of high-energy electrons from nuclei have been extended in two respects. Firstly nonrelativistic recoil of the nucleus is included, and secondly the charge and current densities are treated as operators connecting states of the nucleus described by the shell model, for which an oscillator potential well is assumed. Other approximations used by Schiff are used here, such as Born approximation and use of the Møller fields, and exchange currents are neglected. The calculations are particularly relevant for light nuclei.

In the laboratory system, the interaction energy between electron and nucleus is described by

$$H'=\int d\mathbf{r}[(i/\omega)\mathbf{j}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r})-\mathbf{M}(\mathbf{r})\cdot\mathbf{H}(\mathbf{r})],$$

where $\hbar\omega$ is the loss of energy of the electron, which differs from the excitation energy of the nucleus. We follow Schiff in breaking up H' into electric and magnetic multipole terms of all orders, with the expansion performed about the center of mass of the nucleus, in which case a recoil-current term can be separated out in H'. For electric multipole transitions with no change in the z component of the nuclear spin, which are the only ones to contribute to elastic scattering from a nucleus with zero spin, one finds, with Schiff's notation,[1]

$$\sum_l H'_{l,0}{}^{(E)}=-(4\pi e a_0/q^2)\int d\mathbf{r}'\, e^{i\mathbf{q}\cdot\mathbf{r}'}\rho'(\mathbf{r}').$$

Here $\hbar\mathbf{q}$ is the momentum change of the electron, and for point nucleons, the charge density is

$$\rho'(\mathbf{r}')=\int\cdots\int d\mathbf{r}_1'\cdots d\mathbf{r}_{A-1}'\,\phi_b^*\sum_{k=1}^{A}e_k\delta(\mathbf{r}'-\mathbf{r}_k')\phi_a,$$

where $\phi_a(\mathbf{r}_1'\cdots\mathbf{r}_{A-1}')$, $\phi_b(\mathbf{r}_1'\cdots\mathbf{r}_{A-1}')$ are initial and final internal nuclear wave functions, and $\mathbf{r}_k'$ is measured from the center of mass of the nucleus

$$(\mathbf{r}_A'=-\textstyle\sum_{k=1}{}^{A-1}\mathbf{r}_k').$$

Thus the nuclear form factor is

$$F=(Ze)^{-1}\int\cdots\int d\mathbf{r}_1'\cdots d\mathbf{r}_{A-1}'\,\phi_b^*\sum_{k=1}^{A}e_k e^{i\mathbf{q}\cdot\mathbf{r}_k'}\phi_a.$$

In the usual shell-model treatments,[2] one has calculated

$$F^{\mathrm{SM}}=(Ze)^{-1}\int\cdots\int d\mathbf{r}_1''\cdots d\mathbf{r}_A''\,\psi_b^*\sum_{k=1}^{A}e_k e^{i\mathbf{q}\cdot\mathbf{r}_k''}\psi_a,$$

where $\psi_a(\mathbf{r}_1''\cdots\mathbf{r}_A'')$, $\psi_b(\mathbf{r}_1''\cdots\mathbf{r}_A'')$ are shell-model wave functions and $\mathbf{r}_k''$ is measured from the center of the potential well. For an harmonic oscillator well, Elliott and Skyrme[3] have shown that, for nonspurious states,

$$\psi(\mathbf{r}_1''\cdots\mathbf{r}_A'')=(A/\pi^3a^6)^{\frac{1}{4}}\times\exp(-AR^2/2a^2)\phi(\mathbf{r}_1'\cdots\mathbf{r}_{A-1}'),$$

where $\mathbf{r}_k'=r_k''-\mathbf{R}$, $\mathbf{R}=A^{-1}\sum_{k=1}{}^{A}\mathbf{r}_k''$, and a is the length parameter for the oscillator well. After integrating over the coordinate $\mathbf{R}$, one obtains

$$F^{\mathrm{SM}}=\exp(-q^2a^2/4A)F.$$

Thus the usual shell-model form factors should be multiplied by the factor $\exp(q^2a^2/4A)$. (It can be shown that the same is true for electric and magnetic transitions in which the z component of spin changes.)

As an example, we note that for elastic scattering of electrons from C^{12}, the square of the form factor at the secondary maximum following the diffraction minimum is increased by a factor 1.7 over the previous shell-model value. A change in the opposite direction is produced by the finite proton size; if a Gaussian model is chosen for the proton, the form factor is multiplied by $\exp(-q^2a_p^2/6)$ where a_p is the root-mean-square radius of the proton.[4] This reduces the secondary maximum by a factor 0.43.

To perform phase-shift calculations of the elastic electron scattering, it is necessary to know the nuclear charge distribution $\rho'(\mathbf{r}')$. This can be obtained as the Fourier transform of F, and for unoriented nuclei of the $1p$ shell ($Z=2$ to 8), with a Gaussian model for the proton and allowing for the motion of the well center, the charge distribution is[5]

$$\rho'(\mathbf{r})=(2e/\pi^{\frac{3}{2}}a_m^3)[1+\tfrac{3}{2}\alpha(1-a^2/a_m^2)+\alpha a^2r^2/a_m^4]\exp(-r^2/a_m^2),$$

where

$$a_m^2=[(A-1)/A]a^2+\tfrac{2}{3}a_p^2 \quad\text{and}\quad \alpha=\tfrac{1}{3}(Z-2).$$

[1] L. I. Schiff, Phys. Rev. **96**, 765 (1954).

[2] R. Hofstadter, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 231.

[3] J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232**, 561 (1955).

[4] See reference 2, Eq. (132).

[5] Compare reference 2, Eq. (127).