Magnetic Form Factor of the Neutron at 600 Mev*

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The study of electrons scattered inelastically from the deuteron has been extended to 600 Mev. Data at 500 Mev were reported in an earlier paper. Results are presented for seven scattering angles between 45° and 135°. A comparison of the deuteron's cross section at these angles with the corresponding free proton cross sections permits a determination of the form factor associated with the magnetic moment distribution of the neutron. Two different methods of analyzing the data are described, and the rms radius of the neutron for an exponential density distribution of the magnetic cloud can be shown to lie between the limits of 0.75×10^{-13} cm and 0.95×10^{-13} cm. The rms radius of the neutron may be taken to be $(0.80\pm0.10)\times10^{-13}$ cm and is very close to the value found for an exponential model of the magnetic cloud in the proton.

I. INTRODUCTION

In a previous paper[1] we have described a method of studying the internal magnetic structure of the neutron. The method involves electrons of high-energy scattered inelastically from the deuteron. The work reported there was done primarily at a bombarding energy of 500 Mev. The conclusions of that paper were that (a) the neutron's magnetic moment distribution is spread out (i.e., the neutron is not a point), (b) that within experimental error the neutron has the same magnetic form factor as does the proton, and (c) that the possible difference between the two form factors cannot be larger than can be represented by the rms sizes of 0.90×10^{-13} cm (neutron) and 0.80×10^{-13} cm (proton).[2-5] In addition to these conclusions, there were some other observations regarding the theoretical background of this problem. All of the conclusions of the earlier paper will be discussed again later in this paper.

In this paper we shall report data taken with the same apparatus at a bombarding energy of 600 Mev. We have also continued the analysis of the earlier 500-Mev data.[1] We now have a fairly complete angular distribution at 600 Mev; that is, we have investigated the electrons scattered inelastically from the deuteron (electrodisintegration) at laboratory angles from 45° to 135°.

Since this paper is essentially a continuation of our earlier one, we shall describe the experimental arrangement and general method only briefly.

In this experiment we wish to probe within the meson clouds of the neutron and proton with electrons to examine the difference, if any, in the angular distributions of the corresponding scattered electrons. Since we desire to work at large momentum transfers and at large angles, we are dealing with the magnetic clouds in both nucleons. The electron-scattering technique and related theory will be described here only briefly, since several review articles on these subjects have appeared lately.[6-8]

In order to make a comparison between neutron and proton, we use a liquid deuterium target; and this means that, although the neutron and proton in the deuteron are almost free, we shall have to take into account the fact that the nucleons are in rapid motion. For purposes of comparison we also scatter electrons from free protons.

Under conditions of large angle and large momentum transfers the neutron should scatter electrons with a cross section which depends only on its magnetic moment and on the form factor associated with its magnetic moment distribution. This is precisely true in the case that the neutron's electric form factor (F_{1n}) is identically zero. That it is small is known from the fact that the over-all charge and second moment of the charge of the neutron are zero.[6,7,9] In any event the conditions of the experiment are such that magnetic scattering is dominant. Since the magnetic structure of the proton is known quite accurately,[2-5] we can make the comparison fairly easily, provided we can take into account the fact that in the deuteron the nucleons move with a certain momentum distribution.

Because of the above considerations, the momentum distribution of the scattered electrons will appear as a continuum with a maximum lying near the position of the sharp scattering peak observed from a free proton. The coherent scattering from the whole deuteron (elastic scattering) is usually too small to be observable under these conditions. In the entire experiment we have looked for it only once, namely, at 600 Mev and 60° (Fig. 2).

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[1] M. R. Yearian and R. Hofstadter, Phys. Rev. **110**, 552 (1958).
[2] R. Hofstadter and R. W. McAllister, Phys. Rev. **98**, 217 (1955).
[3] R. W. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956).
[4] E. E. Chambers and R. Hofstadter, Phys. Rev. **105**, 1454 (1956).
[5] F. Bumiller and R. Hofstadter, Bull. Am. Phys. Soc. Ser. II, **2**, 390 (1957); also Bull. Am. Phys. Soc. Ser. II, **3**, 50 (1958).
[6] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).
[7] R. Hofstadter, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 231.
[8] Bumiller, Hofstadter, and Yearian, Revs. Modern Phys. (to be published).
[9] Yennie, Lévy, and Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

II. THEORY

At present there is no complete, relativistic treatment of incoherent scattering from the deuteron. However, certain good approximations have been developed by Jankus[10] and Blankenbecler.[11] Their results may be given in the following form:

$$(d\sigma_d/d\Omega)^{\text{in}}=(1+\Delta)[d\sigma_p/d\Omega+d\sigma_n/d\Omega], \quad (1)$$

where the two cross sections on the right are for finite-size nucleons. Δ is a very small correction to the simple sum rule that was derived by Blankenbecler,[11] and this correction includes the effects of nucleon size as well as the final state interaction. A more complete treatment of Jankus' and Blankenbecler's work as well as representative values of Δ will be found in references 1 and 7. For the present we shall ignore the small Blankenbecler correction (i.e., we set $\Delta=0$). It can be shown from Rosenbluth's work[6,12] that

$$R\equiv\frac{d\sigma_n/d\Omega}{d\sigma_p/d\Omega}=\frac{F_{2n}^2}{F_p^2}\times\frac{(\kappa_n^2\hbar^2q^2/4M^2c^2)[2\tan^2(\theta/2)+1]}{1+(\hbar^2q^2/4M^2c^2)[2(1+\kappa_p)^2\tan^2(\theta/2)+\kappa_p^2]}, \quad (2)$$

where $\kappa_n=\mu_n$, $1+\kappa_p=\mu_p$, and where we have assumed $F_{1n}=0$. Here F_{2n} is the magnetic form factor of the neutron, θ is the scattering angle, M is the nucleon mass, the μ's are the magnetic moments, and q is the magnitude of the four-vector energy-momentum transfer given by

$$q=\frac{2p_0}{\hbar}\sin(\theta/2)\bigg/\left(1+\frac{2E_0}{Mc^2}\sin^2(\theta/2)\right)^{\frac{1}{2}}$$
$$=(2/\lambda\!\!\!^{-})\sin(\theta/2)\bigg/\left(1+\frac{2E_0}{Mc^2}\sin^2(\theta/2)\right)^{\frac{1}{2}}. \quad (3)$$

From Eq. (1), with $\Delta=0$, it is obvious that a simple subtraction, involving the area under the free proton peak and the area under the deuteron inelastic continuum, will yield the neutron's cross section. We can then form the ratio R and from Eq. (2) extract values of F_{2n}^2, since F_p^2 is known.[2–5] Thus we obtain R experimentally for different angles, plot F_{2n}^2 as a function of q^2, and from this plot we are able to determine the rms radius of the magnetic moment distribution by the usual methods.[1–8] As a refinement, we can calculate the Blankenbecler correction ($\Delta\neq0$), which gives a slightly different value for R.

The above procedure, which we shall designate the "area method," suffers from the fact that there is one correction which cannot be taken into account. That is the meson exchange effect, where (for example) the electron makes a meson on one of the nucleons and the meson is absorbed by the other nucleon. We believe this contribution is small; however, the "area method" may be in error because of this effect by as much as 10%.

However, there is a second method of analyzing the data which is semi-independent of the first. We shall call this the "differential method." Jankus[10] has given an expression for the shape of the deuteron inelastic spectrum which, when modified to a certain extent (see reference 1), is valid even in the region of large momentum transfers. Jankus' Eqs. (9)–(11) are replaced by

$$\frac{d^2\sigma_d^{\text{in}}}{d\Omega dp}=\frac{1}{4\pi}\frac{e^4}{p_0^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{2\alpha}{(1-\alpha r_t)}\frac{O^2}{q^2k}, \quad (4)$$

where

$$O^2=F_p^2\left\{\frac{1}{Z^2-1}+\frac{1}{Z_1^2-1}-\frac{2}{Z_1-Z}[Q_0(Z)-Q_0(Z_1)]\right\}$$
$$\times\left\{1+\left[k^2+2\left(\frac{q}{2}\right)^2\left(\mu_p^2+\frac{F_{2n}^2}{F_p^2}\mu_n^2\right)\right]\tfrac{1}{2}[2\tan^2(\theta/2)\right.$$
$$\left.+1-Z^2k^2]-\left(\frac{q}{2}\right)^2(\mu_p^2-\kappa_p^2)\right\}$$
$$-F_p^2\left\{\frac{Z}{Z^2-1}+\frac{Z_1}{Z_1^2-1}-\frac{Z_1+Z}{Z_1-Z}[Q_0(Z)-Q_0(Z_1)]\right\}2Zk^2$$
$$-F_p^2\left\{2+\frac{1}{Z^2-1}+\frac{1}{Z_1^2-1}-\frac{2ZZ_1}{Z_1-Z}[Q_0(Z)-Q_0(Z_1)]\right\}k^2$$
$$\times\{\tfrac{1}{2}[2\tan^2(\theta/2)+1-Z^2k^2]-Z^2k^2\}$$
$$-\frac{(Z_1-Z)^2}{ZZ_1(Z_1+Z)}[Q_0(Z)+Q_0(Z_1)](-\tfrac{2}{3}\mu_p\mu_nF_pF_{2n})$$
$$\times\left(\frac{q}{2}\right)^2[2\tan^2(\theta/2)+1-Z^2k^2], \quad (5)$$

and where

$$Q_0(Z)=\text{arccoth}(Z)=\tfrac{1}{2}\ln\left(\frac{Z+1}{Z-1}\right),$$
$$Z=[\alpha^2+(q/2)^2+k^2]/qk, \quad (6)$$
$$Z_1=[\gamma^2+(q/2)^2+k^2]/qk.$$

Here p_0 is the incoming electron momentum and p is the outgoing (scattered) momentum. The vector $\mathbf{k}$ is the final momentum of the proton with respect to the center-of-mass of the recoiling deuteron. The ground-state wave function of the deuteron is the Hulthén wave function $e^{-\alpha r}-e^{-\gamma r}$, where α^2 is the binding energy and γ is fixed by the choice of the triplet effective range r_t. q is the four-momentum transfer. For convenience, the formulas are expressed in dimensionless form ($\hbar=c=M=1$).

[10] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956); also Ph.D. thesis, Stanford University, 1956 (unpublished).
[11] R. Blankenbecler, Bull. Am. Phys. Soc. Ser. II, **2**, 389 (1958).
[12] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).

TABLE I. Total cross sections for the deuteron at 500 Mev and 600 Mev at various scattering angles.

θ	500 Mev $d\sigma_d/d\Omega$(cm²/sterad)	θ	600 Mev $d\sigma_d/d\Omega$(cm²/sterad)
45°	...	45°	$(2.84\pm0.29)\times10^{-31}$
60°	...	60°	$(7.63\pm0.77)\times10^{-32}$
75°	$(4.60\pm0.49)\times10^{-32}$ and $(4.55\pm0.50)\times10^{-32}$	75°	$(3.31\pm0.34)\times10^{-32}$
90°	$(2.36\pm0.40)\times10^{-32}$	90°	$(1.08\pm0.19)\times10^{-32}$
105°	$(1.50\pm0.30)\times10^{-32}$	105°	$(9.58\pm2.40)\times10^{-33}$ and $(6.56\pm0.88)\times10^{-33}$
120°	$(9.06\pm1.98)\times10^{-33}$	120°	$(4.79\pm1.15)\times10^{-33}$
135°	$(7.58\pm1.23)\times10^{-33}$	135°	$(3.09\pm0.60)\times10^{-33}$ and $(2.92\pm0.19)\times10^{-33}$

It was pointed out by Drell[13] that at the peak of the inelastic spectrum the effects of meson exchange, final state interaction, and bremsstrahlung are all minimized. So, from the modified Jankus theory, we can calculate $d^2\sigma_d/d\Omega dE$ at the peak and compare this value with experiment. The value of the cross section at the peak depends on the choice of F_{2n}^2, and therefore we can find the correct values of the neutron's form factor. We believe this method to be quite accurate. Preliminary results of Drell and Blankenbecler,[14] who are still working on the problem, indicate that the contribution of mesonic effects at the peak is likely to be less than 10%.

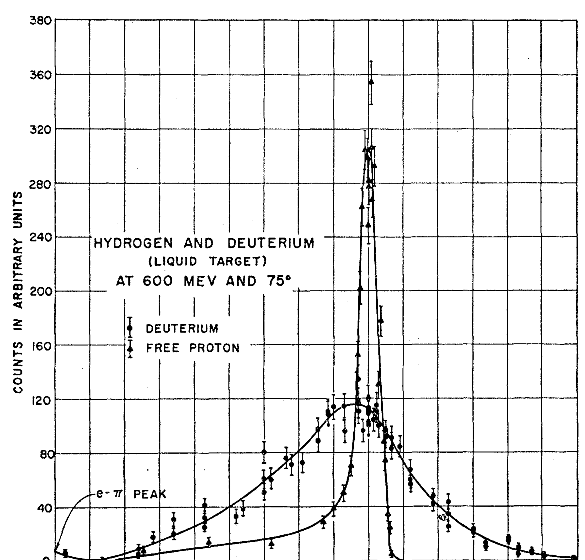

FIG. 1. The inelastic electron-deuteron scattering peak observed at 600 Mev and a laboratory scattering angle of 75°. The comparison electron-proton peak is also shown. The background due to π^- mesons has been subtracted. At this energy and angle, the e-π peak is seen to be unimportant (see text for discussion). The deuterium curve has to be multiplied by 0.870 to take into account the different atomic densities of liquid hydrogen and liquid deuterium.

[13] S. Drell (private communication).

[14] S. Drell and R. Blankenbecler (private communication).

III. EXPERIMENTAL RESULTS

In addition to the 500-Mev data reported previously,[1] we have taken new data at 600 Mev at seven angles—45°, 60°, 75°, 90°, 105°, 120°, and 135°. The free-proton peaks used for comparison were observed at 45°, 75°, 105°, and 135°; $d\sigma_p/d\Omega$ for the other angles was found by interpolating from the well-known proton cross section.[2–5] The data at all these angles have been analyzed by both the "area method" and the "differential method." The combined results of all the data, 500 Mev and 600 Mev, are shown in Figs. 4–6 and in Table I.

There are a number of small corrections that must be made to the data. These were discussed in some detail in reference 1.

(a) The liquid target acts as an extended source, so that, unless precautions are taken to accept very small

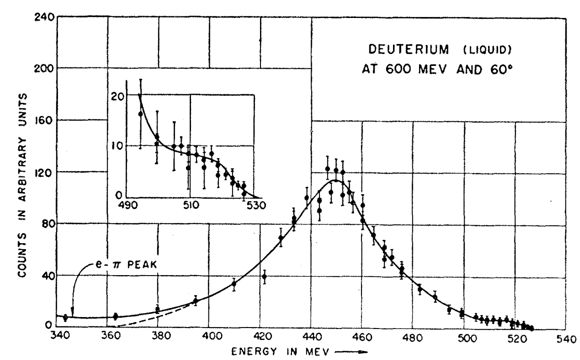

FIG. 2. The experimental inelastic continuum at 600 Mev and 60°. In the inset and at the high-energy end of the continuum, there is a small plateau. This is at the correct position and has the correct area to be the electron-deuteron *elastic* peak. There is very little area left over to attribute to the final-state interaction at this energy and angle.

Čerenkov counter pulses, the cross sections fall off too steeply with angle, due to a net loss of electrons from the scattered beam (via multiple scattering, etc.) at scattering angles different from 90°. We have checked that our explanation of this effect is correct by placing an additional radiator in the scattered beam. The effect is not important here, since we always take the *ratio* of deuterium to hydrogen.

(b) The background due to π^- mesons coming through the spectrometer must be subtracted out. (See reference 1 for details.)

(c) Both the proton and the deuteron peaks have to have radiative corrections applied.

(d) The contribution of the "e-π peak" must be subtracted out. (See below.)

(e) The data must be corrected for the relative densities of liquid deuterium and liquid hydrogen.

After all the corrections [except (d) and (e)] have been made, the data are plotted as shown in Figs. 1

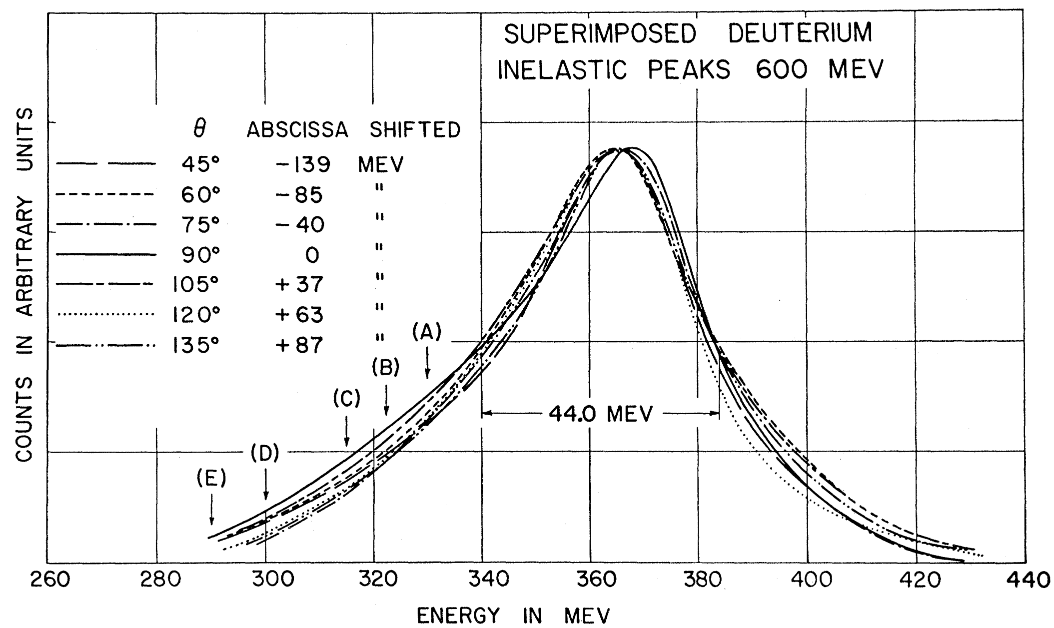

FIG. 3. The electron-deuteron inelastic peaks at 600 Mev are superimposed on one another. All have similar shapes, and the width at half-maximum is about 44 Mev. This similarity of shape is predicted by the modified Jankus theory. The vertical arrow marks the point on the energy scale below which the curve (identified by the letter above the arrow) is slightly uncertain due to contributions from the e-π peak. (A) 45°, 120°, 135°; (B) 105°, (C) 90°, (D) 60°, and (E) 75°. In general, as θ decreases, the importance of the e-π peak becomes smaller and smaller. The point of uncertainty of the 45° curve is abnormally high because the π^- mesons were not subtracted at this angle.

and 2. Figure 1 shows the data at 600 Mev and 75°. Both the free-proton peak and the deuteron inelastic peak are shown. Figure 2 shows the deuterium peak at 60°; in addition, we show the only case in which we have looked for the elastic scattering, and thus one can see how small this peak is. The elastic peak is, of course, much smaller than this at the larger angles and is difficult to see in the inelastic continuum.

At the low-energy end of Figs. 1 and 2, one can see the beginnings of another peak due to electrons which have made pions and then scattered (e-π peak). This peak contributes to the background at the larger angles.

This background can be subtracted out on the basis of the Jankus theory. The theory predicts that all of the inelastic peaks (in the region of q covered by the experiment) should have similar shapes. For example, the width at half-maximum should be approximately constant. Figure 3 shows that this prediction is experimentally verified. In this figure we have superimposed all the 600-Mev peaks; the ordinates have been normalized to the same value at the maximum, but the curves all have about the same width (44 Mev at half-maximum). The same behavior was observed at 500 Mev. Thus the Jankus theory, coupled with the experimental shapes at the smaller angles (where the e-π peak is unimportant), enables us to subtract out the contribution of the e-π peak. In the worst cases, we estimate that the maximum residual effect is at most 5% (see Fig. 12 of reference 1).

The (corrected) deuteron cross sections are shown in Table I. We can then find the corresponding values of F_{2n}^2 by using Eqs. (1) and (2). The results of the "area method" for all the data are shown in Fig. 4. There we reproduce the experimental values of F_{2n}^2 as a function of q^2. The triangles are 600-Mev points and the circles

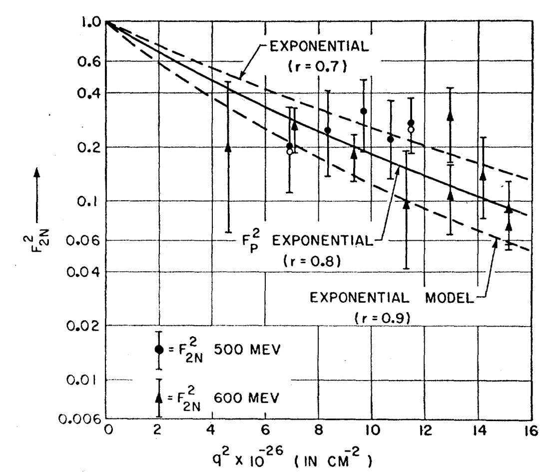

FIG. 4. This figure shows values of F_{2n}^2 (as obtained by the "area method") plotted as a function of q^2 for the data taken at 500 Mev and 600 Mev. The curves are theoretical curves given by the exponential model of the magnetic moment density of the neutron for three different choices of the rms radius. The solid curve ($r=0.8\times10^{-13}$ cm) corresponds to the proton F_p^2. (See note added in proof.)

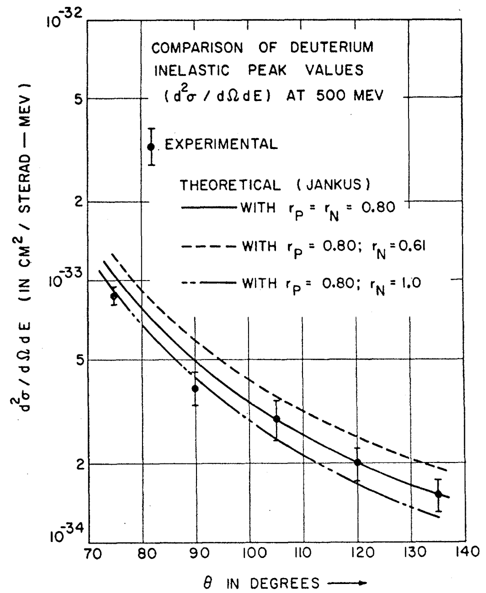

FIG. 5. The results of the "differential method" for the 500-Mev data. The differential cross section at the peak of the deuteron's inelastic continuum is plotted as a function of θ. The theoretical curves are calculated from the modified Jankus theory for three different choices of the rms radius of the neutron's magnetic moment distribution (exponential model). In all three curves the proton radii are taken to be the accepted value of 0.8×10^{-13} cm. It is seen that the experimental points lie between $r_n=0.8\times10^{-13}$ cm and 1.0×10^{-13} cm. Thus the "differential method" indicates that the neutron's magnetic rms radius is slightly larger than the proton's but not definitely outside experimental error. (See note added in proof.)

are the earlier 500-Mev points. For two of the earlier points, we have shown the effect of the Blankenbecler corrections to the simple sum rule—the solid circles are shifted to the positions indicated by the open circles—the corrections are extremely small. The solid curve represents the values F_{2n}^2 would have if the rms radius of the magnetic moment distribution were the same as that of the proton; the upper dashed curve represents the form factors if the rms size were 0.70×10^{-13} cm. This "area method" seems to indicate a radius of about 0.78×10^{-13} cm as being the best fit and is in agreement with the data in our earlier paper. The radii quoted above are all based on the exponential model.

Thus the "area method" indicates that the neutron's magnetic form factor is essentially the same as the proton's. However, the "area method" cannot distinguish the effects of meson exchange, whose inclusion would tend to decrease the best fit radius.

Figure 14 of reference 1 shows a comparison of the experimental shape of the deuterium inelastic spectrum (at 500 Mev and 135°) with the theoretical shape as calculated from the modified Jankus theory [our Eqs. (4)–(6)]. The difference between the experimental and theoretical curves is taken to be a measure of the amount of contribution of meson exchange (and also of interaction in the final state, which is covered by the Blankenbecler correction). It is upon this basis that we think that mesonic effects might possibly contribute up to 10% to the cross section.

We have also applied the "differential method" to all the data. We have calculated the absolute value of $d^2\sigma_d/d\Omega dE$ at the peak in each case and compared with experiment. The value of this cross section depends on the choice of F_{2n}^2. In Figs. 5 and 6 we show $d^2\sigma_d/d\Omega dE$ at the peak *vs* θ for 500 Mev and 600 Mev, respectively. The smooth curves are the values of the differential cross section at the peak, when the neutron rms radius is chosen to be the same as the proton's (0.8×10^{-13} cm). The dashed curves are for choices corresponding to other radii. It is easy to see that the experimental cross sections indicate that the neutron's rms radius lies between 0.8×10^{-13} cm and 1.0×10^{-13} cm. We feel that the "differential method" is the more accurate of the two methods. Moreover, the direction of any corrections (such as bremsstrahlung, etc.) is to *decrease* the radius of the best fit in the "differential method," while the opposite is true in the "area method." Thus the two methods give radii which are close to upper and lower limits of the correct radius. It must be mentioned that our method of plotting the data suffers from the fact that the values of the differential cross section are more sensitive to small radii than large radii for the neutron. That is to say, in Fig. 6, if we had plotted the curve for $r_n=2.0\times10^{-13}$ cm, it would not fall down much faster with θ than does the curve with $r_n=1.0\times10^{-13}$ cm. However, the method of plotting the data is sufficiently sensitive to be used with the present accuracy of the

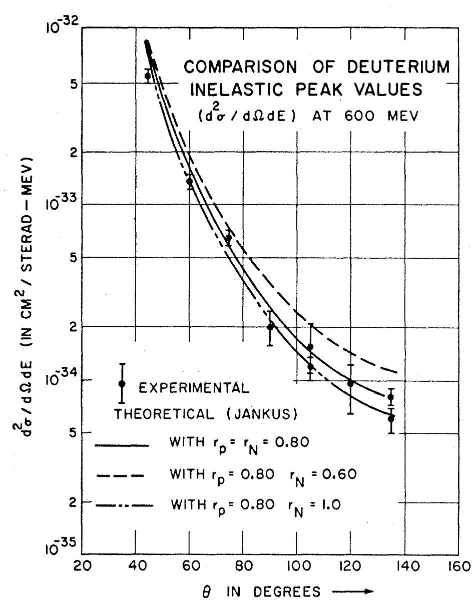

FIG. 6. The results of the "differential method" for 600 Mev. The curves are calculated from the modified Jankus theory with various choices of rms radius of the neutron (exponential model). The proton's radii are taken to be 0.8×10^{-13} cm. Again a neutron radius of 0.9×10^{-13} cm seems to fit the data fairly well. (See note added in proof.)

experiment. By making careful measurements, the "differential method" may yield greatly increased sensitivity.

IV. CONCLUSIONS

Most of our conclusions are little modified from those of the earlier paper.[1] We shall repeat only those conclusions which the higher energy data now make more precise.

(A) The neutron's magnetic cloud is not a point.

(B) The neutron's magnetic form factor is similar to the proton's.

(C) The "differential method" indicates that the two nucleons have slightly different magnetic form factors and sizes; however, the differences are small.

(D) The Jankus theory of electrodisintegration of the deuteron appears to be valid beyond the limits within which it was originally developed. However, the theory has to be modified as outlined in reference 1.

(E) Since in Fig. 2 the deuteron elastic peak has approximately the right area under it to entirely fill in the "plateau" (see insert to figure), and since this is the region where the final state interaction should appear if it is large enough to be detected,[15] the conclusion is that the final state interaction is probably quite small at these large momentum transfers.

(F) In view of the uncertainty in deciding between the "area method" and the "differential method," the neutron's rms size may be given as $(0.80\pm0.1)\times10^{-13}$ cm. This value is consistent with all the above conclusions within experimental error.

[15] See Fig. 15 of reference 7.

Conclusions VI, VII, and VIII, of reference 1, are unaltered by the work reported here.‡

ACKNOWLEDGMENTS

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‡ *Note added in proof.*—Recent calculations of radiative corrections, applying only to the differential method of evaluating the deuteron peak, indicate a larger effect than was estimated originally. The experimental points in Figs. 5 and 6 should be multiplied by approximately 1.26 at 135°, 1.32 at 75°, and 1.42 at 45°. Intermediate points should be corrected in proportion. These corrections will be discussed in detail in a subsequent paper. Application of these results to the differential method gives the neutron "best-fit" magnetic radius, for an exponential model, the value $(0.77\pm0.10)\times10^{-13}$ cm. In Fig. 4 an error has been made in the point at $q^2=9.33$. The proper value should be $F_{2N}^2=0.356\pm0.1$. Further, in reference 1, the last entries of Table II at 600 Mev and 135° should read 1.34 ± 0.10 and 0.34 ± 0.10, respectively.