0+ second excited sublevel with a logft value from 6.5 to 7 and a gamma transition of intensity at most 10% of that of the 0.445-Mev gamma ray, its corresponding energy would lie between 0.230 and 0.440 Mev. On the other hand, if we suppose that the gamma transition has an intensity of 1% or less of that of the 0.445-Mev gamma ray, its corresponding energy would lie between 0.530 and 0.740 Mev. These rough calculations are not in contradiction with our experimental results, since these relative intensities at the corresponding energy ranges cannot be observed.

We shall attempt to perform the external conversion of the gamma rays from I^{128} and observe the corresponding electron lines in our orange-type beta-ray spectrometer.

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Fusion Chain Reaction—Chain Reaction with Charged Particles

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It is shown that in the case of a medium which is an exoergic nuclear mixture (a mixture of nuclei which can lead to an exoenergetic reaction) and which possesses a high temperature ($\sim 10^7$ °K) or a relatively high density ($\sim 10^4$ g/cc), a fusion chain reaction can take place. This is due to the decrease in the stopping power of the medium under the conditions given above. Equations for determining the multiplication factor for a binary mixture under various physical conditions are derived. The multiplication factor is calculated for a DT mixture. It is concluded that for an exoergic nuclear mixture there exists a critical temperature or a critical density which limits the slow release of fusion nuclear energy. For an infinite medium of 50% DT mixture the critical temperature and the critical density are $\sim 10^7$ °K and $\sim 10^3$ g/cc, respectively. In a finite medium the values are higher and there exists a critical mass which limits the possibility for the development of a fusion chain reaction. This critical mass was estimated and in first approximation is $m_{\rm cr}$ (in grams) $=1/(\text{density of the medium in g/cc})^2$.

I. INTRODUCTION

SIMULTANEOUSLY with discovery of the fission chain reaction with neutrons, the possibility of obtaining a chain reaction with charged praticles was abandoned, because of the small efficiency of charged particles in nuclear reactions. In the most advantageous case, $D+T \rightarrow He^4+n$, the efficiency attained is only $\sim 5\times 10^{-3}$ reaction per 14-Mev deuteron. But no note was taken of the fact that the efficiency depends on the physical conditions and in some cases it may be greatly increased. This is especially true for nuclei of small charge, where the Coulomb barrier penetration factor is not too high. The development of a chain reaction with charged particles is, therefore, possible only for light nuclei, where the release of nuclear energy is due to the process of fusion. Only highly exoenergetic reactions of large cross sections may lead to the fusion chain reaction; these are the same reactions which are involved in thermonuclear reaction.[1]

II. FORMULATION OF THE PROBLEM

The mechanism of a fusion chain reaction, which is due to *in statu nascendi* reactions, is as follows. In an exoergic reaction $A+B$ in which weakly bound groups of nucleons of nuclei A and B form strongly bound groups of reaction products, we obtain particles of high kinetic energy. Part of their kinetic energy is transferred in elastic collisions directly to the A and B nuclei of the medium. The recoiling A and B nuclei, in the process of slowing down to thermal energy, have some probability of leading again to the reaction $A+B$. Under normal physical conditions, the dissipation of energy of charged particles in collisions with electrons is so large that their range (L) in the medium is much smaller than the mean free path (λ) with respect to nuclear processes. Therefore, only a small fraction of the recoil nuclei lead again to the $A+B$ reaction. The development of the avalanche is possible when the sum of the ranges of the recoil nuclei $\sum_i L_i$ is comparable to λ. Since $\sum_i L_i \simeq E_Q/\langle(dE/dx)\rangle_{\rm Av}$ and $\lambda \simeq 1/N\bar{\sigma}$, where E_Q= the kinetic energy released in the $A+B$ reaction, $\langle(dE/dx)\rangle_{\rm Av}$= the average energy losses of recoiling nuclei per unit path, N= the density of reacting nuclei of the medium, and $\bar{\sigma}$= the mean cross section for the $A+B$ reaction, we can write

$$E_Q\bar{\sigma}/[(1/N)\langle(dE/dx)\rangle_{\rm Av}]\sim 1. \qquad (1)$$

[1] W. B. Thompson, Proc. Phys. Soc. (London) B70, 1 (1957).

If we assume $\bar{\sigma}\sim 10^{-24}$ cm^2, $E_Q\sim 10$ Mev, we find that the atomic stopping cross section $[\bar{\sigma}(dE/dx) = (1/N)\langle(dE/dx)\rangle_{Av}]$ would be $\sim 10^{-17}$ ev atom^{-1} cm^2. Under normal physical conditions it is about a thousand times higher,[2,3] and we are far from satisfying the criterion (1).

The main idea of the problem involves the dependence of the atomic stopping cross section on the physical conditions.

Under normal physical conditions, in the moderate energy range, the most important losses of energy of heavy charged particles are due to scattering from electrons. They are about four thousand times higher than the energy losses in other processes.[3] The atomic stopping cross section due to scattering from electrons was discussed in detail by the author,[4] and according to Eq. (6) of reference 4, for a particle ξ having a velocity v_ξ and a charge $Z_\xi e$, it is

$$\sigma\left(\frac{dE}{dx}\right)_{\text{el}}^{\text{sc}} = -\frac{4\pi e^4}{m v_\xi^2} Z_\xi^2 \int_0^\infty f(v_e) G[d(v_e);\lambda(v_e)] dv_e, \quad (2)$$

where $f(v_e)$ is the momentum distribution of electrons in the medium and G is the universal stopping power function given in reference 4 by Eq. (8). In the above paper it was shown that these losses depend mainly on the velocity distribution of the electrons, especially in the case $v_\xi \lesssim v_e$. In the limiting case $v_\xi \ll v_e$, the asymptotic value of G becomes $\frac{2}{3}(v_\xi/v_e)^3$, whereupon we have

$$\sigma(dE/dx)_{\text{el}}^{\text{sc}} \sim 1/v_e^3. \quad (3)$$

Hence we see that the energy losses connected with the scattering from electrons decrease very strongly with their velocity. The electron momentum distribution can be shifted to higher velocities by a considerable rise of temperature or by increasing the density up to the strong degeneration of the electron gas. In this way we can decrease the atomic stopping cross section so that the condition (1) is fulfilled.

III. ATOMIC STOPPING CROSS SECTION AT HIGH TEMPERATURES OR HIGH DENSITY

As was mentioned above, the main energy losses of charged particles are due to scattering from electrons and thus depend on the state of the medium.

To evaluate the stopping cross section of plasma electrons,[5] we have to use in Eq. (2) the Maxwellian momentum distribution. We obtain an approximate dependence on the temperature of the plasma if we make the substitution $f(v_e) = \delta(\bar{v}_e - v_e)$, where $\bar{v}_e = (8kT/\pi m)^{\frac{1}{2}}$ is the mean thermal velocity of electrons. In the case interesting us, $v_\xi \ll v_e$, we have

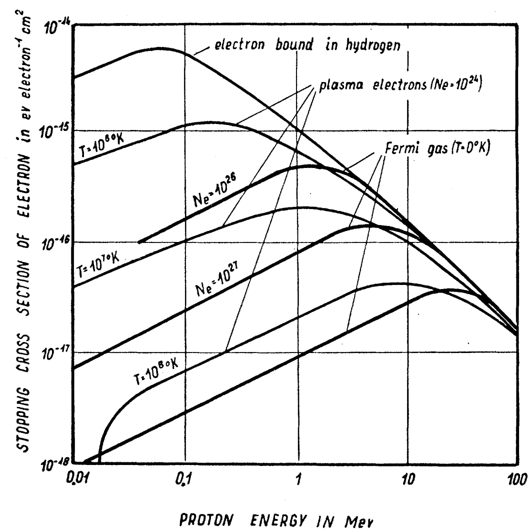

FIG. 1. Stopping cross sections of electrons in various media for protons, as functions of proton energy.

$$\sigma\left(\frac{dE}{dx}\right)_{\text{plasma elect.}}^{\text{sc}} \simeq -\frac{8\pi}{3}\frac{e^4}{m}\left(\frac{\pi m}{8kT}\right)^{\frac{3}{2}} Z_\xi^2 v_\xi. \quad (4)$$

Similarly, taking into account the momentum distribution of a Fermi gas, we obtain the stopping cross section of Fermi-gas electrons (Gryziński,[4] Eq. (18); see also Fermi and Teller[6]):

$$\sigma\left(\frac{dE}{dx}\right)_{\text{Fermi elect.}}^{\text{sc}} = -\frac{1}{N_e}\left(\frac{4}{3\pi}\right)\left(\frac{2e^4m^2}{\hbar^3}\right) Z_\xi^2 v_\xi \times \ln\left(\frac{v_{\max}\hbar}{e^2}\right), \quad (5)$$

where $v_{\max} = (3\pi^2)^{\frac{1}{3}}(\hbar/m)N_e^{\frac{1}{3}}$, and N_e = the number of electrons per cc.

The results of exact computations, where for the maximum impact parameter we have put $D_{\max} = N_e^{-\frac{1}{3}}$ (see Appendix), are plotted in Fig. 1 for various temperatures and densities.

A decrease in the electron scattering losses increases the role of energy losses connected with the interaction with the nuclei of a medium.

The contribution to the atomic stopping cross section due to elastic scattering from nuclei of mass m_A and charge $Z_A e$ is

$$\sigma\left(\frac{dE}{dx}\right)_{\text{nuc}}^{\text{sc. elas.}} = -\frac{4\pi e^4}{m_A v_\xi^2}(Z_\xi Z_A)^2 \times \ln\left(\frac{\mu_{\xi A} v_\xi^2}{Z_\xi Z_A e^2 N^{\frac{1}{3}}}\right) - \frac{1}{2}K_{\xi A} E_\xi \sigma_{\xi A}{}^{\text{sc}}(E_\xi), \quad (6)$$

[2] P. K. Weyl, Phys. Rev. **91**, 289 (1953).
[3] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 779 (1953).
[4] M. Gryziński, Phys. Rev. **107**, 1471 (1957).
[5] E. N. Parker, Phys. Rev. **107**, 830 (1957).
[6] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

where $\mu_{\xi A}$ is the reduced mass, $K_{\xi A}=4m_\xi m_A/(m_\xi+m_A)^2$, $\sigma_{\xi A}{}^{sc}$= the elastic nuclear scattering cross section of particle ξ from the nucleus A, and N= the number of nuclei per cc. The first term in Eq. (6) represents the Coulomb scattering, and the second the nuclear scattering, which we have assumed isotropic in the center-of-mass system.

The stopping cross section due to inelastic collisions with nuclei is

$$\sigma\left(\frac{dE}{dx}\right)_{\text{nuc}}^{\text{sc. incl.}}=-\sum_i \Delta E_i \sigma_{\xi A}{}^{\text{sc. incl.}}(E_\xi). \qquad (7)$$

The sum is taken over all channels with the exciting energy ΔE_i and the cross section $\sigma_{\xi A}{}^{\text{sc. incl.}}(E_\xi)$.

The energy losses connected with the bremsstrahlung[7] of heavy charged particles are very low in comparison with the losses given above, and therefore can be safely neglected.

Finally, the stopping cross section of nucleus A and its Z_A electrons is

$$\sigma\left(\frac{dE}{dx}\right)=Z_A\sigma\left(\frac{dE}{dx}\right)_{\text{elect.}}^{\text{sc}}+\sigma\left(\frac{dE}{dx}\right)_{\text{nuc}}^{\text{sc. elas.}}+\sigma\left(\frac{dE}{dx}\right)_{\text{nuc}}^{\text{sc. incl.}}. \qquad (8)$$

The total atomic stopping cross sections of hydrogen plasma[8] for protons and the relative contribution of their components in various conditions are plotted in Fig. 2.

IV. EVALUATION OF THE MULTIPLICATION FACTOR

To determine the exact conditions for the development of an avalanche, we shall examine an infinite homogeneous medium formed by a mixture of two kinds of nuclei A and B which can initiate the exoergic reaction. We denote by N_A and N_B the densities of the reacting particles, and by $\sigma_{AB}{}^{\xi}$ the laboratory cross section for the reaction $A+B$ (the bombarding particle is denoted by the first lower index) with the emission of the particle ξ. The particles of high kinetic energy, obtained from this reaction, produce a certain number of recoil nuclei.

If $f_\xi(E_\xi{}^0)$ is the energy distribution of the ξ particles obtained from each reaction $A+B$, then the number of ξ particles in the energy interval $E_\xi{}^0$ to $E_\xi{}^0+dE_\xi{}^0$ is $f_\xi(E_\xi{}^0)dE_\xi{}^0$. Since the major part of the reaction $A+B$ in the avalanche occurs in the moderate energy range (100–500 kev), and since the reaction $A+B$ is strongly exoenergetic, we have assumed that this distribution is independent of the energy of the entrance channel. If in the result of reaction $A+B$ we obtain two particles, the function $f_\xi(E_\xi{}^0)$ is the $\delta(E_\xi{}^0-E_\xi)$ function. Owing to the destruction of particles ξ on interaction with the A and B nuclei, the initial number $f_\xi(E_\xi{}^0)dE_\xi{}^0$ of particles along the path x drops to the value $q(E_\xi{}^0,x)f_\xi(E_\xi{}^0)dE_\xi{}^0$, where

$$q(E_\xi{}^0,x)=\exp\left[-\int_0^x (N_A\sigma_{\xi A}+N_B\sigma_{\xi B})dx\right], \qquad (9)$$

and $\sigma_{\xi A}$ ($\sigma_{\xi B}$) is the total reaction cross section of the particle ξ with the nucleus A (B). Taking into account that the energy of particle ξ on the path x drops, due to energy losses, from $E_\xi{}^0$ to E_ξ, we can write the last expression in terms of E_ξ:

$$q(E_\xi{}^0,E_\xi)=\exp\left[-\int_{E_\xi{}^0}^{E_\xi}\frac{N_A\sigma_{\xi A}+N_B\sigma_{\xi B}}{(dE_\xi/dx)}dx\right], \qquad (10)$$

where (dE_ξ/dx) is the loss of energy of particle ξ on the unit path. Upon introducing $\sigma_{\xi A}{}^{sc}(E_\xi,E_A)dE_A$, the cross section for the production of recoil nuclei A of energy E_A to E_A+dE_A by the particle ξ with energy E_ξ, the number of the recoil nuclei A with energy interval E_A to E_A+dE_A produced by the particles ξ from the

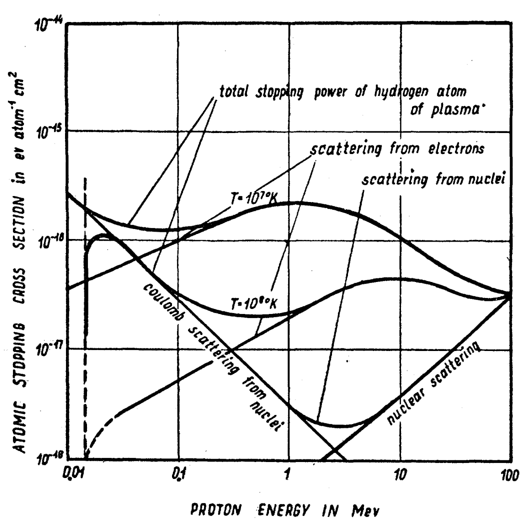

FIG. 2. Total stopping cross section of hydrogen plasma for protons, as a function of proton energy. The contributions of the various relevant processes are shown separately, as well as the total.

[7] L. Landau and L. Lifshitz, *The Theory of Fields* (Moscow and Leningrad, 1948), second edition, pp. 208, 219 [translation: *The Classical Theory of Fields* (Addison-Wesley Press, Inc., Cambridge, 1951), Chap. 9].

[8] As shown above, the stopping power of hydrogen plasma under the conditions existing in the sun, 2×10^7 °K, is about one hundred times lower than the stopping power of hydrogen under normal physical conditions. Therefore, Bethe's calculations [H. Bethe, Phys. Rev. **55**, 434 (1938)] of the efficiency of reactions *in statu nascendi* in the sun (with the assumption that the energy losses are approximately the same in the both cases) are not valid.

reaction $A+B$ along their paths is

$$g_{\xi A}(E_A)dE_A=\int f_\xi(E_\xi^0)dE_\xi^0\int_{E_\xi^0}^{E_A/K_{\xi A}} N_A\sigma_{\xi A}{}^{sc}(E_\xi,E_A)\times q(E_\xi^0,E_\xi)\frac{dE_\xi}{(dE_\xi/dx)}dE_A, \quad (11)$$

$\sigma_{\xi A}{}^{sc}(E_\xi,E_A)$ is given by the differential scattering cross section $\sigma_{\xi A}{}^{sc}(E_\xi,\theta)$ and the relation between the angle of scattering and loss of energy in the collision.[9]

Summing over all the products of the $A+B$ reaction, we obtain

$$g_A{}^{(1)}(E_A)=\sum_\xi g_{\xi A}(E_A). \quad (12)$$

We can write a similar expression for the energy distribution of recoiling nuclei B. As a result of the elastic scattering of the first generation of A and B nuclei, we obtain the second generation of recoil nuclei of the medium. With the help of the above considerations we can write the energy distribution for the nth generation of recoil nuclei A:

$$g_A{}^{(n)}(E_A)=\int g_A{}^{(n-1)}(E_A')dE_A'\int_{E_A'}^{E_A} N_A\sigma_{AA}{}^{sc}(E_A'',E_A)\times(q(E_A',E_A'')\frac{dE_A''}{(dE_A''/dx)}+\int g_B{}^{(n-1)}(E_A')dE_B'\int_{E_B'}^{E_A/K_{AB}} N_B\sigma_{BA}{}^{sc}(E_B'',E_A)\times q(E_B',E_B'')\frac{dE_B''}{(dE_B''/dx)}. \quad (13)$$

If we add the energy distributions of all generations, we obtain the energy distribution of the whole cascade initiated by the particles from the reaction $A+B$:

$$G_A(E_A)=\sum_n g_A{}^{(n)}(E_A). \quad (14)$$

Having obtained the distributions $G_A(E_A)$ and, in a similar way, $G_B(E_B)$, we can give the number of $A+B$ reactions in the slowing-down process of the cascade initiated by the particles from the one reaction $A+B$:

$$k=\int G_A(E_A)dE_A\int_{E_A}^{0} N_B\sigma_{AB}(E_A')\times q(E_A,E_A')\frac{dE_A'}{(dE_A'/dx)}+\int G_B(E_B)dE_B\times\int_{E_B}^{0} N_A\sigma_{BA}(E_B')q(E_B,E_B')\frac{dE_B'}{(dE_B'/dx)}. \quad (15)$$

[9] E. Segrè, *Experimental Nuclear Physics* (John Wiley and Sons, Inc., New York, 1953), first edition, Vol. II, pp. 9, 14.

If the reaction $A+B$ has only one exoergic channel, the number k is the multiplication factor for the given medium. The condition for the development of the avalanche, therefore, is $k>1$.

In the numerical calculations, as long as the slowing-down process of products of the reaction $A+B$ and recoil nuclei is due to scattering from electrons, we can take into consideration only the first generation of recoil nuclei. Then from Eqs. (4), (5), and (15), we have: (a) In the case of charged products of the reaction $A+B$,

$$k\propto 1\bigg/\left[\sigma\left(\frac{dE}{dx}\right)^{sc}_{\text{elect.}}\right]^2 \propto\begin{cases}T^3 & \text{for plasma}\\ N_e^2 & \text{for degenerate medium.}\end{cases} \quad (16)$$

(b) In the case of neutrons,

$$k\propto 1\bigg/\left[\sigma\left(\frac{dE}{dx}\right)^{sc}_{\text{elect.}}\right] \propto\begin{cases}T^{312} & \text{for plasma}\\ N_e & \text{for degenerate medium.}\end{cases} \quad (17)$$

V. CRITICAL MASS

All our present considerations concern the conditions for the development of fusion chain reactions in infinite media. In a finite medium the conditions are different, and then a critical mass exists as in a fission chain reaction.

In first approximation, the critical mass can be estimated very easily if we consider that the mean free path (λ) with respect to the elastic scattering of the particles taking part in the reaction has to be comparable with the dimension (L) of the system. If we denote by N the number of nuclei of the medium in a unit volume, m the mass of a nucleus in grams, and σ^{sc} the cross section for elasting scattering, we obtain $m_{cr}\sim m_1N\lambda^3=m_1/(\sigma^{sc})^3N^2$. Taking into account that $\sigma^{sc}\sim 10^{-24}$ cm^2 and $m_1\sim 10^{-24}$ g, the critical mass in grams is

$$m_{cr}\sim 10^{48}/N^2. \quad (18)$$

We see that the critical mass is very strongly dependent on the density of the medium. For densities $N=10^{28}$, 10^{24}, and 10^{20} nuclei/cc the critical masses are 10^{-8}, 10^0, and 10^8 grams, respectively.

VI. NUMERICAL CALCULATIONS

Now, to illustrate the theory given above we shall determine the conditions for the development of a fusion chain reaction in a DT mixture.

As a result of the reaction D+T, we obtain alphas and neutrons with energies ~3.5 Mev and ~14.1 Mev, respectively. Because of the much greater initial energy and much lower energy losses, most of the recoil

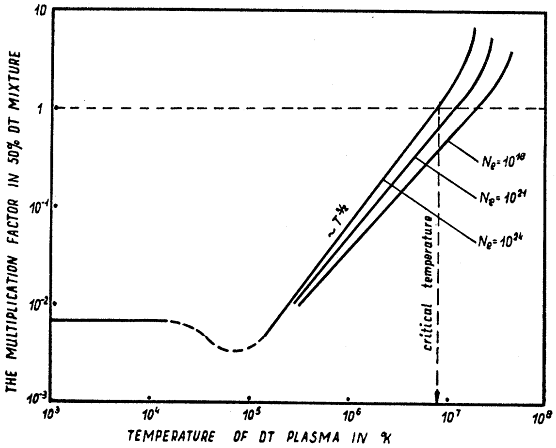

FIG. 3. Multiplication factor k for a 50% deuterium-tritium mixture as a function of temperature, for three values of the electron N_e.

nuclei D and T result from the scattering of neutrons; therefore, according to Eq. (12), $g_{\mathrm{D}}^{(1)}(E_{\mathrm{D}})\simeq g_{n\mathrm{D}}(E_{\mathrm{D}})$ and $g_{\mathrm{T}}^{(1)}(E_{\mathrm{T}})\simeq g_{n\mathrm{T}}(E_{\mathrm{T}})$. Taking into account the fact that the absorption of fast and intermediate neutrons in the DT medium is negligibly small, we have $q_n\simeq 1$. Assuming the scattering of neutrons from D and T nuclei to be isotropic in the center-of-mass system, we obtain

$$\sigma_{n\mathrm{D}}^{\mathrm{sc}}(E_n,E_{\mathrm{D}})\simeq\frac{\sigma_{n\mathrm{D}}^{\mathrm{sc}}(E_n)}{K_{n\mathrm{D}}E_n}$$
$$\sigma_{n\mathrm{T}}^{\mathrm{sc}}(E_n,E_{\mathrm{T}})\simeq\frac{\sigma_{n\mathrm{T}}^{\mathrm{sc}}(E_n)}{K_{n\mathrm{T}}E_n}, \tag{19}$$

Because, in the first approximation, the slowing down of neutrons is due to elastic scattering from D and T nuclei,

$$(dE_n/dx)\simeq\tfrac{1}{2}K_{n\mathrm{D}}E_n\sigma_{n\mathrm{D}}^{\mathrm{sc}}N_{\mathrm{D}}+\tfrac{1}{2}K_{n\mathrm{T}}E_n\sigma_{n\mathrm{T}}^{\mathrm{sc}}N_{\mathrm{T}}, \tag{20}$$

the energy distributions of the first generation of recoiling nuclei are, respectively,

$$g_{n\mathrm{D}}(E_{\mathrm{D}})\simeq\int_{14.1}^{E_{\mathrm{D}}/k_{n\mathrm{D}}}\frac{dE_n}{(K_{n\mathrm{D}}E_n)^2}\times\frac{1}{1+\sigma_{n\mathrm{T}}^{\mathrm{sc}}K_{n\mathrm{T}}N_{\mathrm{T}}/\sigma_{n\mathrm{D}}^{\mathrm{sc}}K_{n\mathrm{D}}N_{\mathrm{D}}}, \tag{21a}$$

$$g_{n\mathrm{T}}(E_{\mathrm{T}})\simeq\int_{14.1}^{E_{\mathrm{T}}/k_{n\mathrm{T}}}\frac{dE_n}{(K_{n\mathrm{T}}E_n)^2}\times\frac{1}{1+\sigma_{n\mathrm{D}}^{\mathrm{sc}}K_{n\mathrm{D}}N_{\mathrm{D}}/\sigma_{n\mathrm{T}}^{\mathrm{sc}}K_{n\mathrm{T}}N_{\mathrm{T}}}. \tag{21b}$$

Since the energy losses of deuterons and tritons up to 10^7 °K in the case of plasma, and up to 10^3 g/cc in the case of degenerate medium, are due to scattering from electrons, we can write

$$k\simeq\frac{N_{\mathrm{D}}}{N_{\mathrm{D}}+N_{\mathrm{T}}}\int g_{n\mathrm{D}}(E_{\mathrm{D}})dE_{\mathrm{D}}\times\int_{E_{\mathrm{D}}}^{0}\frac{\sigma_{\mathrm{DT}}}{\sigma(dE_{\mathrm{D}}'/dx)_{\mathrm{elect.}}^{\mathrm{sc}}}dE_{\mathrm{D}}'+\frac{N_{\mathrm{T}}}{N_{\mathrm{D}}+N_{\mathrm{T}}}\int g_{n\mathrm{T}}(E_{\mathrm{T}})dE_{\mathrm{T}}\times\int_{E_{\mathrm{T}}}^{0}\frac{\sigma_{\mathrm{TD}}}{\sigma(dE_{\mathrm{T}}'/dx)_{\mathrm{elect.}}^{\mathrm{sc}}}dE_{\mathrm{T}}', \tag{22}$$

where $\sigma(dE_{\mathrm{D}}/dx)_{\mathrm{elect.}}^{\mathrm{sc}}$ and $\sigma(dE_{\mathrm{T}}/dx)_{\mathrm{elect.}}^{\mathrm{sc}}$ are given by Eq. (2).

The value of the multiplication factor obtained by the numerical calculations for a 50% DT mixture under various conditions are plotted in Fig. 3 and Fig. 4.

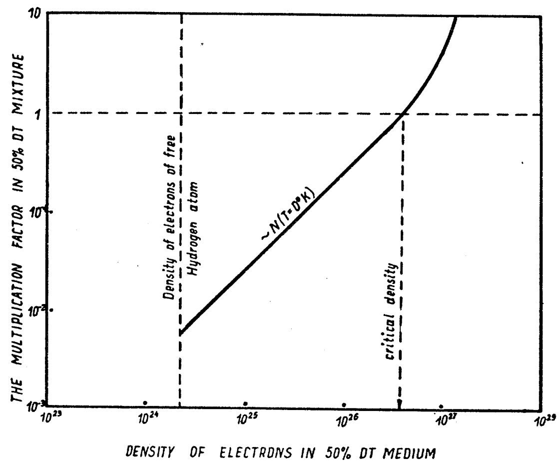

FIG. 4. Multiplication factor k for a 50% deuterium-tritium mixture at 0°K, as a function of electron density N_e.

We have put the cross sections $\sigma_{n\mathrm{D}}^{\mathrm{sc}}$ and $\sigma_{n\mathrm{T}}^{\mathrm{sc}}$ equal to the geometrical cross section, and σ_{DT} is taken from Bame and Perry.[10]

VII. CONCLUSIONS

The role of *in statu nascendi* reactions in the release of nuclear energy depends on the physical conditions, and in the case of high temperature or high density they are decisive. If we denote by E_{th} the energy released in a unit volume in the thermonuclear process, then the energy released in a unit volume with inclusion of *in statu nascendi* reactions is

$$E_{\mathrm{tot}}=E_{\mathrm{th}}/[1-k(\mathrm{T,N})]$$

where k is the multiplication factor for the given

[10] S. J. Bame and J. Perry, Phys. Rev. **107**, 1616 (1957).

medium; the factor k depends on the temperature of the medium or, more accurately, on the temperature of its electrons, and on its density. As the multiplication factor approaches unity the process of energy release has an avalanche character, and the entire nuclear energy of an exoergic mixture is released instantaneously. The stationary state for a slow release of energy does not exist above the critical temperature or above the critical density; even at a temperature of absolute zero the exoergic mixture is explosive.

A plot of E_{th}[11] and E_{tot} as functions of temperature for 50% DT mixture is given on Fig. 5.

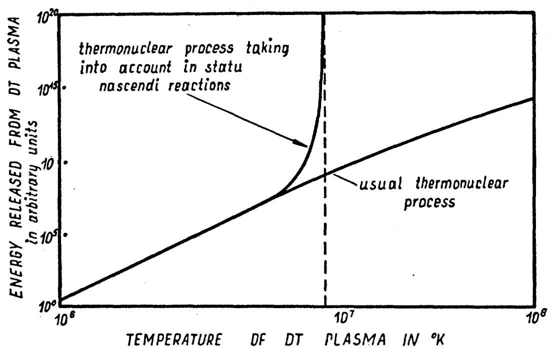

FIG. 5. Energy release per unit volume from a thermonuclear process in a 50% deuterium-tritium mixture with and without inclusion of *in statu nascendi* processes, as functions of plasma temperature.

APPENDIX

As was pointed out previously,[4] in general, the maximum impact parameter is a function of the velocities of interacting particles, their masses, and their charges, as well as of the external fields. In each problem this parameter must be determined separately.

In the case of electrons bound in atoms or Fermi-gas electrons the determination of the maximum impact parameter does not present any difficulty, but in the case of plasma electrons it is the subject of many discussions. According to Cowling,[12] Chandrasekhar,[13] and others, it is suitable to put the maximum impact parameter equal to the mean distance between the ions, but according to Landau,[14] Cohen, Spitzer, and Routly,[15] and others it must equal the Debye radius.

From Eq. (2) it follows at once that in the limiting case, $v_\xi \ll v_e$, the atomic stopping cross section is independent of the assumed value of $D_{\max}$.

In the second limiting case, to determine the maximum impact parameter we must take into account the fact that the charged particles of the plasma are interacting with each other. Consider two particles with charges $+Ze$ and $-Ze$ separated by a distance r. The Coulomb force between them is $(Ze/r)^2$. The transfer of momentum to such a binary system from particle ξ is neglegibly small when the force of interaction between the particle ξ and each particle of the system is less than the force of internal interaction, or $Z_\xi Z/D^2 < ZZ/r^2$. Taking into account the mean value of the distance between charged particles in the plasma, we finally obtain $D_{\max} \sim N^{-\frac{1}{3}}$.

The assumption that the maximum impact parameter is equal to the Debye radius will change the numerical results only slightly owing to the logarithmic dependence on $D_{\max}$ of $\sigma(dE/dx)^x{}_{\text{plasma elect}}$ in the $v_\xi \ll v_e$ region.

ACKNOWLEDGMENTS

I am grateful to Professor A. Soltan and Professor Y. Werle for helpful discussions.

[11] R. F. Post, Revs. Modern Phys. **28**, 338 (1956).

[12] T. G. Cowling, Proc. Roy. Soc. (London) **A183**, 453 (1945).

[13] S. Chandrasekhar, Astrophys. J. **97**, 255, 263 (1943).

[14] L. D. Landau, Physik. Z. Sowjetunion **10**, 154 (1936).

[15] Cohen, Spitzer, and Routly, Phys. Rev. **80**, 230 (1950).