PHYSICAL REVIEW VOLUME 111, NUMBER 3 AUGUST 1, 1958

Effective Parameters in Ferrimagnetic Resonance

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(Received April 10, 1958)

The steady-state solution for the susceptibility tensor of a two-sublattice system has been found by using sublattice equations of motion which include complete Landau-Lifschitz relaxation terms with individual relaxation parameters and which describe relaxation toward the instantaneous total field acting on the sublattice. It is shown that one can define effective parameters describing the behavior of the system as a whole which remain finite throughout the compensation region and, in particular, insure that the absorption coefficient will remain positive. It is also found that the effective gyromagnetic ratios characterizing the absorption and Faraday effect are different in principle, and a new term is found in the expression for the off-diagonal element which is a consequence both of the sublattice structure and total field relaxation. In the case of small damping, many of these distinctions disappear and some parameters reduce to those previously obtained. It is shown that the inclusion of total field relaxation is necessary to obtain results which are unambiguous and correct in principle; the magnetization line width product is also shown to be continuous, but vanishing at the compensation point for angular momentum.

INTRODUCTION

If the electronic magnetic moments in a ferromagnetic material are subjected to a magnetic field having a constant component in the z direction and a small oscillating component in the xy plane, then the gyroscopic behavior of the spins leads to expressions for the transverse components of the magnetization of the form

$$M_x=\chi_x H_x - i\phi H_y,\quad M_y = i\phi H_x+\chi_y H_y. \tag{1}$$

The components of the susceptibility tensor (χ_x,χ_y,ϕ) can be calculated once the proper equations of motion of the magnetization components are known.

An equation which has been extensively used for this purpose is the Landau-Lifschitz equation, which we shall write in the form

$$d\mathbf{M}/dt=\gamma\mathbf{M}\times\mathbf{H}-\alpha\mathbf{M}\times(\mathbf{M}\times\mathbf{H}). \tag{2}$$

The form of this equation is a direct consequence of the simple requirement that $|\mathbf{M}|$ remain constant—a reasonable expectation for the strongly coupled ferromagnetic moments. For, if $|\mathbf{M}|$ is constant, $d\mathbf{M}/dt$ must lie in the plane perpendicular to $\mathbf{M}$, and can therefore be written as a linear combination of two orthogonal vectors in this plane, namely $\mathbf{M}\times\mathbf{H}$ and $\mathbf{M}\times(\mathbf{M}\times\mathbf{H})$. This immediately yields (2). The coefficient α has the dimensions of the reciprocal of angular momentum and must be positive in order that the last term of (2) actually describe a relaxation toward the direction of $\mathbf{H}$ as it is meant to do. It is common practice to assume α to be a constant, in particular, to be independent of the components of the magnetization; for simplicity, we shall also adopt this assumption.

Let us neglect anisotropy and demagnetizing fields so that $\mathbf{H}$ in (2) is just the applied field. Then, if we write $H_z=H$, assume that H_x and H_y are proportional to $e^{i\omega t}$ and are both small compared to H, and neglect second-order terms in (2), we can easily find the steady-state solution of (2) in the form given by (1). The result is that $\chi_x=\chi_y=\chi$, and

$$D_l\chi=\gamma^2[1+(\alpha S)^2]HM+i\alpha\omega M^2, \tag{3}$$

$$D_l\phi=\gamma\omega M, \tag{4}$$

where

$$D_l=\gamma^2[1+(\alpha S)^2]H^2-\omega^2+2i\alpha\omega HM, \tag{5}$$

$M_z=M\simeq$const, and $S=M/\gamma$. We shall have occasion to refer to these specific results later.

It is also common practice to use these formulas (or their equivalent in other notations) to analyze results of experiments performed on multiple sublattice systems, i.e., ferrimagnetic systems. One obtains in this way values of γ and α which characterize the material studied. We shall refer to these quantities as "effective parameters" and denote them by γ_e and α_e. Previous experience with the effective gyromagnetic ratio[1] shows that they can be regarded as suitable averages of the separate parameters of the individual sublattices.

Recently, an investigation was made of the possible existence and properties of the effective relaxation parameter α_e.[2] For the purposes of this initial study, sublattice relaxation terms of the Bloch type were used, since the line shapes predicted by Bloch and Landau-Lifschitz terms are indistinguishable at low signal levels. It was found that one could indeed define an effective relaxation parameter by comparison with the results for the analogous ferromagnetic case. However, it was shown that, in the compensation region, these equations predicted negative values for the absorption coefficient for appropriate relative values of the sublattice parameters. It was concluded from this that the set of sublattice equations of motion which had been used could not serve as completely adequate descriptions throughout the whole range of possible ferrimagnetic behavior. It was further pointed out that the equations were inconsistent in the sense that they did not describe relaxation toward the instantaneous total field acting on each sublattice, and it was suggested that if this defect were remedied, one would obtain consistent and correct results in the compensation region as well as in the region of apparent ferromagnetic behavior.

Our purpose in this paper is to carry out this suggested program and to see whether the inclusion of

[1] R. K. Wangsness, Phys. Rev. **91**, 1085 (1953); **93**, 68 (1954); **95**, 339 (1954); Am. J. Phys. **24**, 60 (1956).
[2] R. K. Wangsness, Bull. Am. Phys. Soc. Ser. II, **3**, 43 (1958).

instantaneous field relaxation terms for the sublattices will still enable one to define effective parameters in a consistent and precise manner and whether the previous difficulties in the compensation region will be satisfactorily alleviated. In order to avoid excessive complications, we shall only consider the case of two sublattices. Further, we shall include only the external and molecular fields in the total field acting on a sublattice, thereby neglecting anisotropy and demagnetizing fields. Although these fields can also be included in our general equations in a straightforward way, the results become so much more cumbersome as to almost entirely obscure the basic points of interest.

COMPLETE SOLUTION FOR TWO SUBLATTICES

Since each sublattice represents a strongly coupled system of electronic moments, we adopt an equation of motion for the ith sublattice of the Landau-Lifschitz form (2), i.e., we write

$$d\mathbf{M}_i/dt=\gamma_i\mathbf{M}_i\times\mathbf{H}_i-\alpha_i\mathbf{M}_i\times(\mathbf{M}_i\times\mathbf{H}_i). \quad (6)$$

In (6), γ_i and α_i are the gyromagnetic ratio and relaxation parameter of the ith sublattice, while $\mathbf{H}_i$, the total field acting on it, is given by

$$\mathbf{H}_1=\mathbf{H}+\lambda\mathbf{M}_2,\quad \mathbf{H}_2=\mathbf{H}+\lambda\mathbf{M}_1, \quad (7)$$

where $\mathbf{H}$ is the external field and λ the molecular field coefficient.

We assume the external field has a constant component $H_z=H$, and two small transverse components, H_x and H_y, each proportional to $e^{i\omega t}$. Inserting (7) into (6), assuming that M_x and M_y are also proportional to $e^{i\omega t}$, and neglecting second-order terms, one finds that

$$M_{iz}=M_i\simeq\text{const}, \quad (8)$$

and that the remaining four components of the equations of motion can be put in the form

$$(i\omega+A)M_x-CM_y+J\Delta_x-F\Delta_y=\alpha_+H_x-\gamma_+H_y, \quad (9a)$$

$$CM_x+(i\omega+A)M_y+F\Delta_x+J\Delta_y=\gamma_+H_x+\alpha_+H_y, \quad (9b)$$

$$EM_x-DM_y+(i\omega+B)\Delta_x-G\Delta_y=\alpha_-H_x-\gamma_-H_y, \quad (9c)$$

$$DM_x+EM_y+G\Delta_x+(i\omega+B)\Delta_y=\gamma_-H_x+\alpha_-H_y, \quad (9d)$$

where (with $j=x, y$):

$$M_j=M_{1j}+M_{2j},\quad \Delta_j=M_{1j}-M_{2j}, \quad (10)$$

$$\alpha_\pm=\alpha_1M_1^2\pm\alpha_2M_2^2, \quad (11)$$

$$\gamma_\pm=\gamma_1M_1\pm\gamma_2M_2. \quad (12)$$

The various coefficients are given by

$$A=\tfrac{1}{2}(H\Gamma_+-\lambda\Gamma_-\Delta), \quad (13)$$

$$B=\tfrac{1}{2}\Gamma_+(H+\lambda M), \quad (14)$$

$$C=\gamma H-\lambda\delta\Delta, \quad (15)$$

$$D=\delta H-\lambda\gamma\Delta, \quad (16)$$

$$E=\tfrac{1}{2}(H\Gamma_--\lambda\Gamma_+\Delta), \quad (17)$$

$$F=\delta(H+\lambda M), \quad (18)$$

$$G=\gamma(H+\lambda M), \quad (19)$$

$$J=\tfrac{1}{2}\Gamma_-(H+\lambda M), \quad (20)$$

where

$$M=M_1+M_2,\quad \Delta=M_1-M_2, \quad (21)$$

$$\gamma=\tfrac{1}{2}(\gamma_1+\gamma_2),\quad \delta=\tfrac{1}{2}(\gamma_1-\gamma_2), \quad (22)$$

$$\Gamma_\pm=\alpha_1M_1\pm\alpha_2M_2. \quad (23)$$

Equations (9) can now be solved for M_x and M_y. The solutions are found to have the form (1) with $\chi_x=\chi_y=\chi$. The results are

$$\mathfrak{D}\chi=\alpha_+\mathfrak{K}+\gamma_+\mathfrak{L}+\alpha_-\mathfrak{M}+\gamma_-\mathfrak{N}, \quad (24)$$

$$\mathfrak{D}\phi=-i(\gamma_+\mathfrak{K}-\alpha_+\mathfrak{L}+\gamma_-\mathfrak{M}-\alpha_-\mathfrak{N}), \quad (25)$$

where

$$\mathfrak{K}=A(B^2+G^2)+B(DF-EJ)-G(DJ+EF)-\omega^2(A+2B)+i\omega(B^2+G^2+2AB+DF-EJ-\omega^2), \quad (26)$$

$$\mathfrak{L}=C(B^2+G^2)-G(DF-EJ)-B(DJ+EF)-\omega^2C+i\omega[2BC-(DJ+EF)], \quad (27)$$

$$\mathfrak{M}=E(F^2+J^2)+J(CG-AB)-F(AG+BC)+\omega^2J-i\omega[J(A+B)+F(C+G)], \quad (28)$$

$$\mathfrak{N}=D(F^2+J^2)-F(CG-AB)-J(AG+BC)-\omega^2F-i\omega[J(C+G)-F(A+B)], \quad (29)$$

and where $\mathfrak{D}$, the determinant of the coefficients of (9) is given by

$$\mathfrak{D}=\{(A^2+C^2)(B^2+G^2)+(D^2+E^2)(F^2+J^2)+2(AB-CG)(DF-EJ)-2(AG+BC)(DJ+EF)-\omega^2[A^2+C^2+B^2+G^2+4AB+2(DF-EJ)]+\omega^4\}+2i\omega[A(B^2+G^2)+B(A^2+C^2)+(A+B)(DF-EJ)-(C+G)(DJ+EF)-\omega^2(A+B)]. \quad (30)$$

EFFECTIVE PARAMETERS

Our basic general results contained in (24)–(30) are obviously much too complicated to be easily interpreted. As usual,[1,2] however, we can expand our results in powers of the molecular field coefficient λ and keep only the terms in the highest power of λ. One finds from (13)–(23) and (26)–(30) that the coefficients of λ^4 in $\mathfrak{D}$ and of λ^3 in $\mathfrak{D}$, $\mathfrak{K}$, $\mathfrak{L}$, $\mathfrak{M}$, and $\mathfrak{N}$ are zero, so that the only terms of these expressions that we need now consider are those of order λ^2.

One finds that, to this approximation,

$$\mathfrak{D}/(\lambda\gamma_1\gamma_2)^2=H^2M^2(1+\alpha_1^2S_1^2)(1+\alpha_2^2S_2^2)-\omega^2[S^2+(\alpha_1+\alpha_2)^2S_1^2S_2^2]+2i\omega HM[\alpha_1S_1^2+\alpha_2S_2^2+\alpha_1\alpha_2(\alpha_1+\alpha_2)S_1^2S_2^2], \quad (31)$$

$$\mathfrak{K}/\lambda^2=HM^2\{\tfrac{1}{2}\Gamma_+[\gamma^2+\delta^2+\tfrac{1}{4}(\Gamma_+^2-\Gamma_-^2)]-\gamma\delta\Gamma_-\}+i\omega M[M(\gamma^2+\tfrac{1}{4}\Gamma_+^2)-\Delta(\gamma\delta+\tfrac{1}{4}\Gamma_+\Gamma_-)], \quad (32)$$

$$\mathcal{L}/\lambda^2=HM^2\{\gamma[\gamma^2-\delta^2+\tfrac{1}{4}(\Gamma_+^2+\Gamma_-^2)]-\tfrac{1}{2}\delta\Gamma_+\Gamma_-\} +i\omega M[\tfrac{1}{2}\Delta(\gamma\Gamma_- -\delta\Gamma_+)], \quad (33)$$

$$\mathfrak{M}/\lambda^2=HM^2\{\tfrac{1}{2}\Gamma_-[\gamma^2+\delta^2-\tfrac{1}{4}(\Gamma_+^2-\Gamma_-^2)]-\gamma\delta\Gamma_+\} +i\omega M[-M(\gamma\delta+\tfrac{1}{4}\Gamma_+\Gamma_-) +\Delta(\delta^2+\tfrac{1}{4}\Gamma_-^2)], \quad (34)$$

$$\mathfrak{N}/\lambda^2=HM^2\{-\delta[\gamma^2-\delta^2-\tfrac{1}{4}(\Gamma_+^2+\Gamma_-^2)]-\tfrac{1}{2}\gamma\Gamma_+\Gamma_-\} +i\omega M[-\tfrac{1}{2}M(\gamma\Gamma_- -\delta\Gamma_+)], \quad (35)$$

where $S_i=M_i/\gamma_i$ is the angular momentum of the ith sublattice and $S=S_1+S_2$ is the total angular momentum.

Inserting these results into (24) and (25), one finds that χ and ϕ can be written in the form

$$\mathfrak{D}\chi=\gamma_e^2[1+(\alpha_eS)^2]HM+i\alpha_e\omega M^2, \quad (36)$$

$$\mathfrak{D}\phi=\gamma_e'\omega M+i\beta_eHM^2, \quad (37)$$

where

$$\mathfrak{D}=\gamma_e^2[1+(\alpha_2S)^2]H^2-\omega^2+2i\alpha_e\omega HM, \quad (38)$$

$$\gamma_e^2[1+(\alpha_eS)^2]=[M^2(1+\alpha_1^2S_1^2)(1+\alpha_2^2S_2^2)]/\Sigma, \quad (39)$$

$$\alpha_e=[\alpha_1S_1^2+\alpha_2S_2^2 +\alpha_1\alpha_2(\alpha_1+\alpha_2)S_1^2S_2^2]/\Sigma, \quad (40)$$

$$\gamma_e'=\{M[S+(\alpha_1^2S_1+\alpha_2^2S_2)S_1S_2]\}/\Sigma, \quad (41)$$

$$\beta_e=\tfrac{1}{2}(\gamma_1-\gamma_2)\{(\alpha_1-\alpha_2)\alpha_1\alpha_2S_1^2S_2^2 -[(\alpha_1^3M_1^4-\alpha_2^3M_2^4)/(\gamma_1\gamma_2)^2]\}/\Sigma, \quad (42)$$

$$\Sigma=S^2+[(\alpha_1+\alpha_2)S_1S_2]^2. \quad (43)$$

The quantities denoted by the subscript e are the ones we designate as the effective parameters, because, by comparing (36)–(38) with the ferromagnetic result given by (3)–(5), we see that the susceptibilities for the two-sublattice system have essentially the same dependence on frequency, field, and net magnetization as do those for the ferromagnetic system.

If we write $\chi=\chi'-i\chi''$, $\phi=\phi'-i\phi''$, and set

$$\Gamma^2=\gamma_e^2[1+(\alpha_eS)^2], \quad (44)$$

we easily find that

$$\tilde{D}\chi'=HM[\Gamma^2(\Gamma^2H^2-\omega^2)+2(\alpha_e\omega M)^2], \quad (45)$$

$$\tilde{D}\chi''=\alpha_e\omega M^2(\omega^2+\Gamma^2H^2), \quad (46)$$

$$\tilde{D}\phi'=\omega M[\gamma_e'(\Gamma^2H^2-\omega^2)+2\alpha_e\beta_eH^2M^2], \quad (47)$$

$$\tilde{D}\phi''=HM^2[2\alpha_e\gamma_e'\omega^2-\beta_e(\Gamma^2H^2-\omega^2)], \quad (48)$$

where

$$\tilde{D}=|\mathfrak{D}|^2=(\Gamma^2H^2-\omega^2)^2+(2\alpha_e\omega HM)^2 =\Gamma^{-4}\{[\Gamma^4H^2-(\Gamma^2-2\alpha_e^2M^2)\omega^2]^2 +(\Gamma^2-\alpha_e^2M^2)(2\alpha_e\omega^2M)^2\}. \quad (49)$$

As a simple check on our work, we see that if we set $\alpha_i=0$, then $\alpha_e=\beta_e=0$, $\gamma_e^2=(M/S)^2$, $\gamma_e'=M/S$, and the components of χ and ϕ reduce to well-known previous results.[1] We can obtain the ferromagnetic case by setting $\gamma_i=\gamma$, $\alpha_i=\alpha$, $S_i=\tfrac{1}{2}S$; we find that $\alpha_e=\tfrac{1}{2}\alpha$, $\beta_e=0$, and $\gamma_e=\gamma_e'=M/S=\gamma$, and the values of χ and ϕ, of course, reduce to those given by (3)–(5).

There are several remarks to be made, however about the general results. The effective relaxation parameter α_e is always positive since the $\alpha_i>0$. Therefore, the absorptive component χ'' given by (46) will be always positive; thus these expressions no longer lead to the previous unacceptable situation of negative absorption.[2] We also see that since, in general, $\gamma_e\neq\gamma_e'$, the effective gyromagnetic ratio which basically determines the Faraday effect, γ_e', is different in principle from that obtained from the study of the resonance absorption. An interesting property of γ_e' is that for appropriate relative values of α_i and S_i, γ_e' can have a sign *opposite* to that of M/S since the product S_1S_2 is negative in the ferrimagnetic case. Finally, the term in ϕ proportional to β_e is a completely new term, entirely absent in the ferromagnetic case as we can see from (4). This new term is a direct consequence of the sublattice structure since it is proportional to $(\gamma_1-\gamma_2)$ from (42) and hence vanishes for the case of equivalent sublattices; we also note that it is of third order in the relaxation parameters α_i.

All of these distinctions disappear, however, if the damping is sufficiently small. If we keep only terms linear in the α_i, we find from (39)–(43) that $\beta_e=0$,

$$\alpha_e=(\alpha_1S_1^2+\alpha_2S_2^2)/S^2, \quad (50)$$

$$\gamma_e^2=(M/S)^2, \quad \gamma_e'=M/S, \quad (51)$$

and we are back to our familiar results with a simpler expression for the effective relaxation parameter which is still always positive even in this limiting case of small damping. For many cases, of course, (50) and (51) along with (45)–(49) will be sufficient to use in discussing experimental results.

The values at the compensation points $M=0$ and $S=0$ are of particular interest. Since α_e is always positive, there will not be a compensation point for effective relaxation parameter, in contrast to previous simpler results.[2] At the compensation point for magnetization ($M=0$), $\chi=\phi=0$ and γ_e^2 and γ_e' also vanish; we return to this case below. At the compensation point for angular momentum ($S=0$), $S_2=-S_1$, $M=\gamma_1S_1+\gamma_2S_2=(\gamma_1-\gamma_2)S_1$, and we easily find from (39)–(43) that

$$\alpha_e=(1+\alpha_1\alpha_2S_1^2)/[(\alpha_1+\alpha_2)S_1^2], \quad (52)$$

$$\gamma_e^2=[(\gamma_1-\gamma_2)/(\alpha_1+\alpha_2)S_1]^2(1+\alpha_1^2S_1^2)(1+\alpha_2^2S_1^2), \quad (53)$$

$$\gamma_e'=-(\gamma_1-\gamma_2)(\alpha_1-\alpha_2)/(\alpha_1+\alpha_2), \quad (54)$$

$$\beta_e=\tfrac{1}{2}(\gamma_1-\gamma_2)(\alpha_1+\alpha_2)^{-2} \times\{(\alpha_1-\alpha_2)\alpha_1\alpha_2-(\gamma_1\gamma_2)^{-6}(\gamma_2^4\alpha_1^3-\gamma_1^4\alpha_2^3)\}. \quad (55)$$

Equation (54) shows quite clearly that γ_e' can in principle be of either sign, and upon further comparison with (53), we see that $\gamma_e^2\neq(\gamma_e')^2$. It is easily shown that, for this case,

$$\gamma_e^2-(\gamma_e')^2=[(\gamma_1-\gamma_2)\alpha_eS_1]^2.$$

We have seen that our expansion to order λ^2 has given us finite, nonvanishing results except for the single point $M=0$. In order to investigate the situation more closely here, we can calculate the terms of order λ in (24)–(29). Since (30) does not vanish for $M=0$, we need not consider it; our work can be further simplified since we need only find the terms of order λ when $M=0$, since, for $M\neq 0$, our expansion to order λ^2 suffices. The procedure is quite straightforward, and we find that $\mathfrak{D}=-\lambda^2(\omega M_1)^2[(\gamma_1-\gamma_2)^2+(\alpha_1+\alpha_2)^2M_1^2]$, $\mathfrak{D}\chi=-\mathfrak{D}/\lambda$, and $\mathfrak{D}\phi=0$. Therefore, at $M=0$, we can say that $\chi=(-1/\lambda)+$terms of order (λ^{-2}) and that ϕ is of order (λ^{-2}). Therefore, the absorption at $M=0$ is, at most, of order λ^{-2}, and can usually be taken as zero. If anisotropy is included, then χ'' turns out to be of order $(1/\lambda)$ at $M=0$.

NECESSITY OF TOTAL FIELD RELAXATION

Since the Eqs. (6) lead to results which are reasonable and avoid the difficulty of a negative absorption throughout the whole range of magnetization, including the compensation region, the question then naturally arises as to whether this is really due to the inclusion of relaxation toward the instantaneous total field on the sublattice or whether it is simply a result of using the complete Landau-Lifschitz relaxation term in (6). The simplest way to answer this question is to look at the consequences of dropping the terms involving λ in the relaxation term of (6). One easily sees that this means that all terms involving a product $\lambda\alpha_i$, or, equivalently, $\lambda\Gamma_\pm$, will not appear in (13)–(20). One can now proceed exactly as before but with the omission of these terms. One finds, first of all, that the coefficient of $-\omega^2$ in (31) is simply S^2. The final results still can be written in the form (36)–(38), but now the values of the effective parameters, indicated by dashed symbols, are found to be simply

$$\bar{\gamma}_e=\bar{\gamma}_e'=M/S, \tag{56}$$

$$\bar{\alpha}_e=(\alpha_1M_1S_1+\alpha_2M_2S_2)/MS, \tag{57}$$

$$\bar{\beta}_e=0. \tag{58}$$

Although it is still possible to define effective parameters, they are immediately seen to have the undesirable properties that were eliminated by (39)–(43), even for the case linear in α_i. In particular, the effective gyromagnetic ratios and relaxation parameters have infinite discontinuities at the compensation points. In addition, (57) can be written, with the help of (56), in the form

$$\bar{\alpha}_e=-(1/\bar{\gamma}_e)\{|\gamma_1|\alpha_1S_1^2+|\gamma_2|\alpha_2S_2^2\}/S^2, \tag{59}$$

and the sign is thus determined as the negative of the sign of $\bar{\gamma}_e$. Since $\bar{\gamma}_e$ is positive between the compensation points, $\bar{\alpha}_e$ will be negative in this region, thus once again leading to the possibility of an unacceptable negative absorption. These results thus show quite conclusively that an adequate macroscopic representation of relaxation in multiple sublattice systems must describe relaxation toward the instantaneous value of the total field acting on the individual sublattice.

In this connection, we see from (56) and (58) that the distinction between γ_e and γ_e', as well as the new term in (37) proportional to β_e, is also a direct consequence of total field relaxation and in principle offers a means of experimentally checking these results.

Although we shall not discuss it in detail here, one can apply the same approximate methods previously used for an arbitrary number of sublattices[1,2] to Eqs. (6) and (7). One easily finds that the effective parameters obtained in this way are exactly those given in (56)–(58). Thus, this method, which has been previously quite useful, fails to give the correct result for an arbitrary number of sublattices *even though* the total fields are used in the initial set of sublattice equations of motion.

A brief mention of the effect of assigning different sublattice parameters has been made by Calhoun, Smith, and Overmeyer.[3] Although they do not discuss their derivation of their result, their expression for the effective relaxation parameter in their notation is only linear in the α_i and is of the same general class as that given in (50), i.e., will also not remain finite in the compensation region.

LINE WIDTH BEHAVIOR

We can get some idea of the expected behavior of the line width by looking at the conditions on the external field for the minimum value and twice the minimum value of $\tilde{D}$ in (49) since these are almost the same as the conditions for the maximum and half-maximum value of χ''. If the value of the field which makes $\tilde{D}=\tilde{D}_{\min}$ be designated by H_0, and that which makes $\tilde{D}=2\tilde{D}_{\min}$ by $H^*=H_0+\Delta H$, we easily find from (49) that

$$2H_0\Delta H+(\Delta H)^2=2\alpha_e\omega^2M(\Gamma^2-\alpha_e^2M^2)^{\frac{1}{2}}/\Gamma^4. \tag{60}$$

If, for simplicity, we assume that the α_i are small enough so that we need only consider the linear approximation, we have $\Gamma\simeq\gamma_e\simeq M/S$, and $H_0\simeq\omega/|\gamma_e|$, so that (60) becomes

$$(M\Delta H)+[(M\Delta H)^2/2\omega|S|]=\omega(\alpha_eS^2)$$
$$=\omega(\alpha_1S_1^2+\alpha_2S_2^2)=\omega a, \tag{61}$$

with the use of (50). We note that the right-hand side of (61) is always finite and positive. Solving (61) for $M\Delta H$, we get

$$M\Delta H=\omega|S|\{[1+(2a/|S|)]^{\frac{1}{2}}-1\}. \tag{62}$$

If $a/|S|\ll 1$, we get $M\Delta H\simeq\omega a$, while if $|S|\to 0$, $M\Delta H\simeq\omega(2a|S|)^{\frac{1}{2}}$ and vanishes at $S=0$. Thus, we see that the product $M\Delta H$ varies continuously through the compensation region, in agreement with previous results.[2,3] This result does not mean that ΔH itself vanishes at $S=0$, but is a consequence of keeping only the linear terms.

[3] Calhoun, Overmeyer, and Smith, IBM Research Memo. RC 79, November 1957 (unpublished); Calhoun, Smith, and Overmeyer, J. Appl. Phys. 29, 427 (1958).