thinnest foil ($\frac{1}{3}$-mil Al), Fig. 3. A Gaussian and the Molière prediction are fitted to the data, and it can be seen that the accuracy does not allow any conclusion to be reached regarding a possible divergence from a Gaussian.

The width of the curves were compared with the values predicted by the Molière theory using a relation derived by Hanson *et al.*[3]:

$$\theta_w = \theta_1(B-1.2)^{\frac{1}{2}},$$

$$\theta_1^2 = 0.157\frac{Z(Z+1)}{A}\frac{t}{E_0^2},$$

$$B - \ln B = \ln[(\theta_1/\theta_a)^2] - 0.154,$$

$$\left(\frac{\theta_1}{\theta_a}\right)^2 = 7800\frac{(Z+1)Z^{\frac{1}{3}}t}{A[1+3.35Z^2(e^2/\hbar c)^2]},$$

where θ_w is the scattering angle at the $1/e$ point, θ_1 the maximum scattering angle, θ_a the screening angle, A the atomic weight, t the foil thickness in g/cm^2, and E the electron energy in Mev.

A comparison of the widths of the curves with the predictions of Molière theory is given in Table I. It can be seen that they are in excellent agreement. In this there is a slight and possibly not significant disagreement with the lower energy data of Hanson *et al.*[3] They found excellent agreement in the case of gold, but their value for beryllium indicated scattering about 3–7% less in width than the Molière predictions. It was suggested that this disagreement with theory might be caused by a breakdown of the Fermi-Thomas atomic model used in the Molière calculations since this calculation would not necessarily be valid for such a light element as beryllium. However, later work by Mohr and Tassie[4] using a Hartree model gives the same result as the Fermi-Thomas calculation.

[3] Hanson, Lanzl, Lyman, and Scott, Phys. Rev. **84**, 634 (1951).

TABLE I. Comparison of the measured multiple scattering with the predictions of the Molière theory.

Foil material	Thickness in g/cm²	Half of the width at $1/e$ (in radians $\times E_0/mc^2$) Molière theory	Measured (aperture function unfolded)
Be	0.0123 (5 mil)	0.282	0.281±0.009
Al	0.00244 ($\frac{1}{3}$ mil)	0.176	0.174±0.005
Al	0.00706 (1 mil)	0.359	0.350±0.009
Al	0.0205 (3 mil)	0.699	0.685±0.017
Au	0.0103 ($\frac{1}{2}$ mil)	0.952	0.950±0.024

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[4] C. B. O. Mohr and L. J. Tassie, Australian J. Phys. **7**, 217 (1954).

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Time-Reversal Invariance and Radiative Muon Decay*

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The radiative muon decay $\mu \rightarrow e+\nu+\bar{\nu}+\gamma$ is investigated as a source of information on the time-reversal properties of the muon decay interaction. It is shown that for any muon decay process, which may include electromagnetic interactions, terms in the transition probability which violate time-reversal invariance must be pseudoscalars. Further, only ten combinations of the coupling constants of the four-fermion interaction can occur in the transition probability. These may be classified in accordance with the types of terms in which they occur; a term which violates time-reversal invariance must contain as a factor either the electron mass or the transverse electron polarization. In the radiative muon decay, if such a term is proportional to the electron mass, it will also contain as a factor the longitudinal electron polarization.

If the two-component neutrino theory is assumed, the four-fermion interaction representing muon decay is characterized by two coupling constants, g_V and g_A.[1] The transition probability for a decay process must be quadratic in these constants; the combinations which may occur are $|g_V|^2+|g_A|^2$, $|g_V|^2-|g_A|^2$, $\mathrm{Re}(g_A{}^*g_V)$, and $\mathrm{Im}(g_A{}^*g_V)$.[2] A normalized decay spectrum will be

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[1] T. Lee and C. Yang, Phys. Rev. **105**, 1671 (1957).

[2] D. Candlin, Nuovo cimento **6**, 390 (1957). A more complete expression for the transition probability has been given by R. Sharp and G. Bach, Can. J. Phys. **35**, 1199 (1957). Similar remarks hold for the general four-component theory with its ten coupling constants; this has been treated by T. Kinoshita and A. Sirlin, Phys. Rev. **108**, 844 (1957).

linear in the parameters 1, ξ, ζ, η, where

$$\xi=2\,\mathrm{Re}\frac{g_A{}^*g_V}{|g_V|^2+|g_A|^2},\quad \zeta=\frac{|g_V|^2-|g_A|^2}{|g_V|^2+|g_A|^2},\quad \eta=2\,\mathrm{Im}\frac{g_A{}^*g_V}{|g_V|^2+|g_A|^2}. \tag{1}$$

The particular significance of the parameter η is that it vanishes unless g_V and g_A differ in phase; that is, unless there is a failure of time-reversal invariance. Measurements of the angular and energy distribution of electrons from polarized muons permit a direct determination of ξ and ζ, but not η. From recent experiments one may conclude that $\xi=-0.91\pm0.14$[3]; the determination of ζ will require measurement of the low-energy electron spectrum.[2] The three parameters are connected by the relation $\xi^2+\zeta^2+\eta^2=1$, so that, in principle, η could be determined from ξ and ζ; in practice, however, even accurate measurements of ξ and ζ would allow only a very rough determination of η. In the transition probability for nonradiative muon decay, η appears as a coefficient only for a term proportional to $\boldsymbol{\sigma}_\mu\cdot\boldsymbol{\sigma}_e\times\mathbf{p}$, where $\boldsymbol{\sigma}_\mu$ and $\boldsymbol{\sigma}_e$ are muon and electron spins, $\mathbf{p}$ the electron momentum; this term clearly changes sign under time reversal and is present only when η is nonzero. Determination of η by observation of this term requires measurement of the component of electron polarization perpendicular to the plane of the muon spin and the electron momentum. However, the polarization detectors presently available for high-energy electrons are rather insensitive to transverse polarization.

Hence it is of interest to consider whether the time-reversal properties of the muon decay interaction could be more easily established from the radiative muon decay $\mu\rightarrow e+\nu+\bar{\nu}+\gamma$.[4] The process is rare, but its transition probability might contain terms proportional to η which would not depend on the transverse electron polarization. Of particular interest would be a term proportional to $\eta\boldsymbol{\sigma}_\mu\cdot\mathbf{p}\times\mathbf{l}$, where $\mathbf{l}$ is the photon momentum. It will be shown, however, that any term of the decay which violates time-reversal invariance depends on the electron polarization. Further, any such term which does not depend on the transverse polarization of the electron is proportional to the electron mass m_e and so must be small except possibly for low-energy electrons.

These conclusions may be obtained from a consideration of the more general muon decay interaction

$$F=\textstyle\sum_r (\bar{\psi}_e\Gamma_r\psi_\mu)[\bar{\psi}_\nu\Gamma_r(g_r+g_r{}'\gamma_5)\psi_\nu], \tag{2}$$

where the sum r is over the usual coupling forms S, V, T, A, P. The transition probability P for a decay process is quadratic in the coupling constants. It will be shown that for any muon decay process, including arbitrary radiative effects, only ten combinations of the coupling constants can appear in P. This limitation results from the zero neutrino mass, the lack of other neutrino interactions, and the fact that neutrinos are not directly detected in the decay. Further, the ten combinations can be classified according to the kinds of terms in P with which they occur; due to the smallness of the electron mass, this is a useful procedure.[5]

The matrix element for any muon decay process [including the decay interaction (1) only to lowest order but allowing arbitrary electromagnetic interactions] will have the form

$$M=\textstyle\sum_r R_r[\bar{u}_k\Gamma_r(g_r+g_r{}'\gamma_5)v_{k'}], \tag{3}$$

where k and k' are the neutrino momenta. No neutrino variables appear in the factors R_r of the matrix element. Since the neutrino is massless, the transition probability P must be invariant under the substitution $v_{k'}\rightarrow\gamma_5 v_{k'}$ in M and thus invariant under the interchange of primed and unprimed coupling constants. Hence the combinations of coupling constants which appear in P are symmetric in primed and unprimed constants.

Summing over all neutrino variables, the transition probability will have the form

$$P\propto\textstyle\sum_{rs} R_rR_s{}^*[(g_rg_s{}^*+g_r{}'g_s{}'^*)J_{rs} + (g_rg_s{}'^*+g_r{}'g_s{}^*)K_{rs}]. \tag{4}$$

In Eq. (4) the first term is a scalar and the second (containing cross terms between primed and unprimed coupling constants) is a pseudoscalar. The quantities J_{rs} and K_{rs} which result from the neutrino variables are given by

$$J_{rs}=\mathrm{Tr}[\gamma^\mu\Gamma_r\gamma^\rho\Gamma_s]I_{\mu\rho},\quad K_{rs}=\mathrm{Tr}[\gamma^\mu\Gamma_r\gamma_5\gamma^\rho\Gamma_s]I_{\mu\rho}, \tag{5}$$

where

$$I_{\mu\rho}=\int\int d^3kd^3k'\,\delta(G-k-k')k_\mu k_\rho{}'/k_0k_0{}' = (\pi/6)[2G_\mu G_\rho+g_{\mu\rho}G^2], \tag{6}$$

and G is the appropriate combination of other 4-momenta in the process. The function J_{rs} vanishes for $r\neq s$ when $I_{\mu\rho}$ is symmetric in μ and ρ, as it is in Eq. (6). Thus in the scalar part of P the only combinations of the coupling constants which occur are $(|g_r|^2+|g_r{}'|^2)$. Consequently any term in P which violates time-reversal invariance must be a pseudoscalar. The properties of K_{rs} follow from those of J_{rs}; the possible coupling constant combinations of the pseudoscalar part are $(g_Sg_P{}'^*+g_S{}'g_P{}^*)$, $(g_Vg_A{}'^*+g_V{}'g_A{}^*)$ and their complex conjugates, and $(g_Tg_T{}'^*+g_T{}'g_T{}^*)$. There are thus ten

[3] R. Swanson (private communication).

[4] The spectrum for this decay was obtained on the assumption that each of T, C, and P is conserved, by A. Lenard, Phys. Rev. **90**, 968 (1953) and Behrends, Finkelstein, and Sirlin, Phys. Rev. **101**, 866 (1956).

[5] A portion of these conclusions have been derived by R. Gatto and G. Lüders [Nuovo cimento **7**, 806 (1958) from the Pauli-Pursey theory of invariants] [see, for example, W. Pauli, Nuovo cimento **6**, 204 (1957)].

combinations of the coupling constants which may appear in the transition probability for a muon decay process.

It may be noted that the discussion of the possible coupling constant combinations depended on the essentially symmetrical fashion in which the neutrinos appear in the four-fermion muon decay interaction, for from this followed the symmetry of the integrand of $I_{\mu\rho}$ in neutrino momenta and hence the symmetry of $I_{\mu\rho}$ in μ and ρ. To give an example, modification of the integrand of $I_{\mu\rho}$ by the factor[6] $[1-(p+k')^2/2M^2]$ leads to the expression

$$I_{\mu\rho}(6/\pi)=[1-2p\cdot(p+G)/M^2][2G_\mu G_\rho+g_{\mu\rho}G^2] -2(p_\mu G_\rho-p_\rho G_\mu)(G^2/M^2). \quad (7)$$

This contains an antisymmetric part and allows scalar cross terms in Eq. (4).

A further classification of the ten coupling constant combinations is obtained by considering the transformation of electron wave functions

$$\psi_e\rightarrow\gamma_5\psi_e;\quad \bar{\psi}_e\rightarrow-\bar{\psi}_e\gamma_5, \quad (8)$$

which leaves the electromagnetic interactions $e\bar{\psi}_e\gamma_\mu\psi_e$ and $e\bar{\psi}_\mu\gamma_\mu\psi_\mu$ invariant. Its effect on the transition probability for any muon decay process is simply to replace the electron mass m_e everywhere by $-m_e$ and to reverse the sign of the transverse component $\boldsymbol{\sigma}_e^T$ of the electron polarization. Thus terms in (1 or $m_e\boldsymbol{\sigma}_e^T$) are invariant under the transformation; terms in (m_e or $\boldsymbol{\sigma}_e^T$) change sign.

The transformation (8) may be viewed in another way. Its effect on the muon decay interaction is equivalent to the interchange of V and A, and of S and P, leaving T unchanged. (Since Γ_T does not appear in terms with Γ_r for $r\neq T$, we may replace $\Gamma_T\gamma_5$ by Γ_T.) Thus terms in (T) and terms which are symmetric in (VA) and (ST) are invariant under the transformation; terms which are antisymmetric in (VA) and (ST) change sign.

Since these two classifications under the transformation (8) must coincide, we may pair the terms of the transition probability for a muon decay process (including any radiative effects) and the coupling constant combinations with which they occur[5]:

(A) Terms with both m_e and $\boldsymbol{\sigma}_e^T$ or neither as factors. These can occur with the coupling constant combinations

$$\begin{aligned}&|g_S|^2+|g_S'|^2+|g_P|^2+|g_P'|^2,\\ &|g_V|^2+|g_V'|^2+|g_A|^2+|g_A'|^2,\\ &|g_T|^2+|g_T'|^2,\\ &\mathrm{Re}(g_Sg_P'^*+g_S'g_P^*),\\ &\mathrm{Re}(g_Vg_A'^*+g_V'g_A^*),\\ &\mathrm{Re}(g_Tg_T'^*).\end{aligned} \quad (9)$$

(B) Terms with either m_e or $\boldsymbol{\sigma}_e^T$ as factors. These can occur with the coupling constant combinations

$$\begin{aligned}&|g_S|^2+|g_S'|^2-|g_P|^2-|g_P'|^2,\\ &|g_V|^2+|g_V'|^2-|g_A|^2-|g_A'|^2,\\ &\mathrm{Im}(g_Sg_P'^*+g_S'g_P^*),\\ &\mathrm{Im}(g_Vg_A'^*+g_V'g_A^*).\end{aligned} \quad (10)$$

The usefulness of this classification results from the fact that terms containing the electron mass as a factor will in general be small. All ten combinations of the coupling constants do occur in the transition probabilities calculated for various muon decay processes; the pairing of terms and combinations may also be verified.[2,4] The last two combinations can occur only with a failure of time-reversal invariance in the muon decay interaction. They will occur as coefficients for pseudoscalar terms; these must contain as a factor either the electron mass m_e (and so be small except perhaps for very low energy electrons) or the transverse electron polarization $\boldsymbol{\sigma}_e^T$ (which is very difficult to measure).

These conclusions may now be specialized to the radiative muon decay $\mu\rightarrow e+\nu+\bar{\nu}+\gamma$. Since terms of the transition probability which violate time-reversal invariance are pseudoscalars, the term $\boldsymbol{\sigma}_\mu\cdot\mathbf{p}\times\mathbf{l}$ is excluded; such terms which do occur will contain either m_e or $\boldsymbol{\sigma}_e^T$ as a factor. An experiment to test time-reversal invariance by measurement of a transverse electron polarization is already possible for nonradiative muon decay, so only the m_e terms need be investigated. Of these, only a term $\boldsymbol{\varepsilon}\cdot\mathbf{p}\times\mathbf{l}\ \boldsymbol{\varepsilon}\cdot\mathbf{p}$ (multiplied by scalar functions of the momenta, $\boldsymbol{\varepsilon}$ being the photon polarization vector) does not depend on the longitudinal electron polarization $\boldsymbol{\sigma}_e^L$, but a detailed examination of the matrix element shows that it appears with coefficient zero. The simplest term depending on the longitudinal electron polarization $\boldsymbol{\sigma}_e^L$ would be proportional to $m_e\boldsymbol{\sigma}_\mu\cdot\boldsymbol{\sigma}_e^L\times\mathbf{l}$. To observe such a term would require measurement of the difference in longitudinal polarization, with photons in coincidence, according as the photons are observed above or below the plane of the muon spin and electron direction. Since the radiative process is rare and the time-reversal effect depends on the factor m_e, such an experiment is unlikely to provide a more sensitive test of time-reversal invariance than the difficult experiment which is possible with nonradiative muon decay.

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[6] This modification is suggested by a representation of the $(\mu\bar{\nu})(e\bar{\nu})$ form of the decay interaction as due to an intermediate boson coupled to $(\mu\bar{\nu})$ and $(e\bar{\nu})$. This clearly removes the symmetry of the two neutrinos. Of course, a theory involving only one intermediate boson contains essentially only one coupling constant and satisfies time-reversal invariance. See T. Lee and C. Yang, Phys. Rev. **108**, 1611 (1957).