Production of Strange Particles by π-Nucleon and Photonucleon Interactions near Threshold

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An investigation of the behavior of the π-nucleon reaction cross sections near the strange particle threshold shows that the variation with energy of the Λ^0-K cross section near the $\Sigma-K$ threshold depends strongly upon the relative parities of the Σ and the Λ^0 hyperons. If the elements of the Wigner R matrix are nearly stationary near the $\Sigma-K$ threshold, and if the decay of the compound system is nearly independent of its mode of formation, the variation of the Λ^0 and Σ cross sections is almost uniquely established from cross sections measured at one energy. In particular, the Λ^0 cross section exhibits strong cusps at the $\Sigma-K$ thresholds. Angular distributions and polarizations are discussed from the viewpoint of these assumptions. The photonucleon production of strange particles is shown to be closely related to the π-nucleon production, essentially because the same final-state interactions are important. The strength and characteristics of these interactions lead to the conclusion that the matrix elements for photonucleon production of strange particles will, in general, be strongly affected in magnitude by the final-state interactions and they will not be real, even at threshold. The analysis of cusps, which are predicted in the $\gamma+p\to\pi+$nucleon cross section at the Λ^0+K threshold and the $\Sigma+K$ threshold, and in the $\gamma+p\to\Lambda^0+K$ reaction at the $\Sigma+K$ threshold, should provide a means of determining the relative parities of the strange particles, in a manner almost independent of the dynamics of the reaction.

INTRODUCTION

IN the study of processes involving strong short-range interactions, it is often convenient to divide the problem into two parts. Particularly over a small energy range, the short-range interaction can usually be expressed by some simple approximation while the behavior in the external region can be treated exactly. The Gamow-Condon description of α-particle decay, the effective range theory of nucleon-nucleon scattering, and Fermi's statistical theory of high-energy processes, are diverse examples of phenomena in which the complexity of the process may be understood primarily in terms of the properties of particles in an external region, defined as a region in which they do not interact or interact only through Coulomb forces.

In particular, the behavior of cross sections near threshold is determined largely by the properties of the wave equation in the external region.[1,2] Since the form of the wave equation will depend upon the spins of the particles and upon their relative parities, an investigation of the behavior of the strange-particle production cross sections near threshold might be expected to provide information concerning these intrinsic properties of the particles as well as values of the matrix elements determined by the properties of the internal region.

The threshold for the production of Λ^0 hyperons by π^-+p interactions occurs at a π-meson bombarding energy of about 760 Mev while the $\Sigma-K$ thresholds are at about 900 Mev. A measurement of the strange particle production cross section by the interaction of 960-Mev π^--mesons with protons has been used[3] to obtain cross sections of 0.58 ± 0.11 millibarn for Λ^0-K^0 production, 0.34 ± 0.09 millibarn for Σ^0-K^0 production, and 0.09 ± 0.04 millibarn for Σ^--K^+ production. Furthermore, it seems likely that the production intensity is primarily the result of S-wave production in a state of isotopic spin, T, equal to $\frac{1}{2}$. In the center-of-mass system this measurement takes place about 100 Mev above the Λ^0-K^0 threshold and only about 30 Mev above the $\Sigma-K$ threshold. Since these energies are small compared to the natural energies that one might expect to be important in this process, such as the rest energy of the K meson, it might be expected that an approximation, based on the assumption that the effect of the internal region does not change much over the energy interval of interest, would adequately describe the variation of cross section from the thresholds up to, and past, 960 Mev. Calculations made on this basis can then be considered extrapolations from the measurements at 960 Mev downward in energy to the Λ^0 and Σ thresholds and upwards to a limited extent. Since the radiative capture process, and hence photonucleon production processes are intimately related to the $\pi-p$ interaction, the study of the $\pi-p$ interactions provides information concerning the photoproduction of strange particles.

SCATTERING MATRIX

The following calculations were made by using the R-matrix formalism of Wigner and others.[4–6] In particular, the forms used in discussing threshold reactions are quite general. If the interaction between pairs of particles, which are the products of a nuclear reaction, is sufficiently singular, a surface in configuration spase will exist which will divide the total configuration space into an internal region of strong interaction and an

[1] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).
[2] G. Breit, Phys. Rev. **107**, 1612 (1957).
[3] L. B. Leipuner and R. K. Adair, Phys. Rev. **109**, 1358 (1958).
[4] E. P. Wigner, Phys. Rev. **70**, 606 (1946).
[5] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[6] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

TABLE I. Variation of quantities δ_l, P_l, and Δ_l with k and l. The quantity X is equal to ka, where a is the channel radius and $k=\hbar|(E^2-M^2)|^{\frac{1}{2}}$. The total energy of the system is E, and M is the mass of the particles in the channel. The phases of the quantities P_l and Δ_l are adjusted so that this definition of k is to be used in Eqs. (2) and (4).

l	$\delta_l(E>E)$	$\Delta_l(E>M)$	$P_l(E>M)$	$\Delta_l(E<M)$	$P_l(E<M)$
0	$-X$	0	1	1	0
1	$-X+\frac{1}{2}\pi-\cot^{-1}X$	$\frac{1}{X}\left(-1+\frac{X^2}{(1+X^2)}\right)$	$\frac{X^2}{1+X^2}$	$\frac{1}{X}\left(-1-\frac{(X^3+X^2)}{1-X^2}\right)$	0
2	$-X+\pi-\cot^{-1}\left(\frac{X^2-3}{3X}\right)$	$\frac{1}{X}\left(-2+\frac{3X^2+2X^4}{9+3X^2+X^4}\right)$	$\frac{X^4}{9+3X^2+X^4}$	$\frac{1}{X}\left(-2-\frac{(3X^2-2X^4-X^5)}{9-3X^2+X^4}\right)$	0
3	$-X+\frac{3}{2}\pi-\cot^{-1}\left(\frac{X^3-15X}{6X^2-15}\right)$	$\frac{1}{X}\left(-3+\frac{45X^2+12X^4+3X^6}{225+45X^2+6X^4+X^6}\right)$	$\frac{X^6}{225+45X^2+6X^4+X^5}$	$\frac{1}{X}\left(-3-\frac{45X^2-12X^4+3X^6+X^7}{225-45X^2+6X^4-X^6}\right)$	0

external region of no interaction. Complete sets of external standing wave functions can then be constructed, a set, D, which has a finite normal derivative but zero value on the surface, and a set V which has a finite value but zero derivative on the surface. At any particular energy the value function V_{sl} for the pair of particles s with relative angular momentum l, and a definite total angular momentum and parity, can be expanded in terms of the derivative functions $D_{s'l'}$, where s' runs over all sets of particles with positive kinetic energy in any part of the energy region of interest. The set of equations $V_{sl}=\Sigma R_{sls'l'}D_{s'l'}$, written for all s, forms the matrix equation $V=RD$, where the elements of the real symmetric R matrix contain all the information in the scattering process. The scattering matrix, U, can be expressed in terms of the R matrix:

$$U=e^{-1}(q-R)^{-1}(q^*-R)e^*, \tag{1}$$

where

$$e_{sl}=\tfrac{1}{2}k_s^{\frac{1}{2}}(G_{sl}'+iF_{sl}'),$$

and

$$q_{sl}=2k_s^{-1}(G_{sl}+iF_{sl})(G_{sl}'+iF_{sl}'^{-1})^{-1}$$

are diagonal matrices, G_{sl} and F_{sl} are the irregular and regular free-particle wave functions whose asymptotic form is $\cos(k_sr-\frac{1}{2}l\pi)$ and $\sin(k_sr-\frac{1}{2}l\pi)$, and k_s is the wave number in channel s. The prime represents the derivative with respect to kr and the functions are evaluated on the surface of radius a_s. The dimension of the R matrix is then equal to the number of open channels. In order to study the behavior of cross sections near the threshold[1,2] for a process, i, it is then desirable to use the same R matrix above and below the threshold, which requires the continuation of the quantities e_i and q_i below the threshold of channel i.

Although the explicit form of Eq. (1) for reactions with a large number of channels is unwieldy, it appears desirable to write out the results for 3 channels to illustrate the relationships between the behavior of the $\pi-p$, the Λ^0-K, and the $\Sigma-K$ cross sections. The subscripts a, b, and c represent the three channels, where the subscripts implicitly include the orbital angular momentum and spin of the channel. We can write for an interaction of definite total angular momentum and parity:

$$U_{ab}=\omega_a(\delta_{ab}-N_{ab}/D)\omega_b,$$

where

$$\begin{aligned}
N_{ab}&=-2ik_a^{\frac{1}{2}}P_a^{\frac{1}{2}}k_b^{\frac{1}{2}}P_b^{\frac{1}{2}}[R_{ab}-k_cQ_c(R_{cc}R_{ab}-R_{ac}R_{bc})],\\
N_{aa}&=-2ik_aP_a[R_{aa}-k_cQ_c(R_{aa}R_{cc}-R_{ca}R_{ac})\\
&\quad-k_bQ_b(R_{bb}R_{aa}-R_{ab}R_{ba})+k_bQ_bk_cQ_c(R_{aa}R_{bb}R_{cc}\\
&\quad+R_{ac}R_{cb}R_{ba}+R_{ab}R_{bc}R_{ca}\\
&\quad-R_{cc}R_{ab}R_{ba}-R_{bb}R_{ac}R_{ca})],\\
D&=1-k_aQ_aR_{aa}-k_bQ_bR_{bb}-k_cQ_cR_{cc}\\
&\quad+k_aQ_ak_bQ_b(R_{aa}R_{bb}-R_{ab}R_{ba})\\
&\quad+k_aQ_ak_cQ_c(R_{aa}R_{cc}-R_{ac}R_{ca})\\
&\quad+k_bQ_bk_cQ_c(R_{bb}R_{cc}-R_{bc}R_{cb})\\
&\quad-k_aQ_ak_bQ_bk_cQ_c(R_{aa}R_{bb}R_{cc}\\
&\quad+2R_{ab}R_{cb}R_{ac}-R_{aa}R_{bc}R_{cb}\\
&\quad-R_{bb}R_{ac}R_{ca}-R_{cc}R_{ab}R_{ba}).
\end{aligned} \tag{2}$$

In these equations $P_a=(G_aF_a'-F_aG_a')(F_a^2+G_a^2)^{-1}$, and $Q_a=\Delta_a+iP_a$, where $\Delta_a=d\ \ln(F_a^2+G_a^2)^{\frac{1}{2}}/d\ \ln(k_aa_a)$, $\omega_a=\exp(i\delta_a)$, and $\delta_a=-\tan^{-1}(F_a/G_a)$. Values of the quantities P, Δ, and δ are given in Table I for small values of l. Partial reaction cross sections σ_{ab} for states of a definite parity, and angular momentum j, then take the form

$$\sigma_{ab}=\frac{2j+1}{(2I+1)(2S+1)}\frac{\pi}{k_a^2}|\delta_{ab}-u_{ab}|^2, \tag{3}$$

where I and S are the intrinsic spins of the particles of channel a.

Equations (2) are greatly simplified if relationships of the type $(R_{aa}R_{bb}=R_{ab}R_{ba})$ hold. Then the quantities in the parentheses, ( ), vanish and

$$\begin{aligned}
N_{aa}&=-2ik_aP_aR_{aa},\\
N_{ab}&=-2ik_a^{\frac{1}{2}}P_a^{\frac{1}{2}}R_{aa}^{\frac{1}{2}}R_{bb}^{\frac{1}{2}}k_b^{\frac{1}{2}}P_b^{\frac{1}{2}},\\
D&=1-k_aQ_aR_{aa}-k_bQ_bR_{bb}-k_cQ_cR_{cc}.
\end{aligned} \tag{4}$$

Since the elements R and the quantities P are real, $U_{ab}{}^2$ can be written

$$|U_{ab}|^2=[2k_aP_aR_{aa}/|D|][2k_bP_bR_{bb}/|D|].$$

The production intensity is then the simple product of a formation factor and a disintegration factor. This special condition, that the decay of the system is independent of its mode of formation, is implicit in statistical theories of high-energy processes and is obtained for certain other situations, as when one intermediate state dominates the reaction.

Formally any value may be chosen for the channel radius a_s as long as the wave function can be closely approximated by the free-particle wave function for $r_s > a_s$. In Eq. (1), e and q are functions of the choice of a, while the scattering matrix U is not. The dependence of R on a contained in Eq. (1) is complex and depends sharply upon the orbital angular momentum in the various channels. While, in general, the most useful values of a are those which enclose the strongly interacting region most closely, the effect of the particular choices used can be seen most clearly in connection with explicit results. Sachs[7] has pointed out that the use of a channel radius smaller than the Compton wavelength of the channel particles is questionable as the particle cannot be localized in a region smaller than this.[8]

When interaction strengths are very weak, the quantities kQR will be small compared to one and the cross section σ_{ab} reduces to $4\pi k_b k_a{}^{-1}|R_{ab}|^2$, where spin statistical weights are neglected and for simplicity S-wave interactions are assumed. A comparison with the usual perturbation-theory representation shows that R_{ab} is proportional to M_{ab}, where M_{ab} is the usual real matrix element of perturbation theory.

For strong interactions no such precise equivalent exists.

PI-NUCLEON INTERACTIONS

There are four important products or channels of the π^-+p interaction at 960 Mev: (1) π^-+p, which we shall label as p; (2) Λ^0+K^0, labeled Λ; (3) $\Sigma+K$, labeled as Σ; and (4) nucleon $+2\pi$, labeled as n. The development of the scattering matrix in terms of the derivative matrix refers only to channels for two-body interactions and is not immediately applicable to situations in which the production of several particles occurs, such as $\pi^-+p\rightarrow(p+\pi^-)+\pi^0$. If we look at that particular reaction, we would expect to find a set of discrete two-body reactions where the $(p+\pi^-)$ system is bound in a Coulomb state. Each of these would be described by a row and column of the R matrix, and a row and column of the U matrix. This density of such $(p+\pi^-)$ states increases as the energy of the $(p+\pi^-)$ system approached the Coulomb binding energy, until it is infinite at and above zero energy. Since it seems unlikely that the relative probability of the filling of such states would be very different if the products were the result of a π^-+p interaction, a $\Sigma+K$ interaction, or a Λ^0+K interaction, the use of an n row and n column of the R matrix with diagonal element R_{nn}, instead of an infinite number of rows and columns, to represent the multiple meson production, should not effect results significantly. On physical grounds, k_nP_n should be essentially proportional to the phase space available for three-body reactions and Δ_n can be neglected.

Using the simplifying assumption, $(R_{ab})^2=R_{aa}R_{bb}$, discussed above, we can then write the scattering matrix element for the production of Λ^0-K^0 particles in an S state,

$$U_{p\Lambda}=e^{-i\delta_p}\times\left[\frac{-2ik_p{}^{\frac{1}{2}}P_p{}^{\frac{1}{2}}R_{pp}{}^{\frac{1}{2}}k_\Lambda{}^{\frac{1}{2}}R_{\Lambda\Lambda}{}^{\frac{1}{2}}}{1-k_pQ_pR_{pp}-k_\Sigma Q_\Sigma R_{\Sigma\Sigma}-k_nQ_nR_{nn}-ik_\Lambda R_{\Lambda\Lambda}}\right]\times\exp(-ik_\Lambda a_\Lambda). \quad (5)$$

Here the Λ^0 is assigned positive parity by definition. The quantities δ_p, P_p, and Q_p depend upon the parity of the K meson, as the $\pi-p$ orbital angular momentum is zero if the K is pseudoscalar, and one if the K is scalar. The quantity Q_Σ depends similarly on the parity of the Σ.

If the total S-wave production cross section for Λ^0-K^0, $\pi k_p{}^{-2}|U_{p\Lambda}|^2$, is set equal to 0.45 millibarn at an energy of 960 Mev, and the $\Sigma-K$ S-wave cross section is set at 0.43 millibarn,[3] the sum of the two cross sections is then almost 40% of the maximum permitted by the conservation laws, $\frac{2}{3}\pi k_p{}^{-2}$. The extrapolation of these cross sections is then not sensitive to the values chosen for the elements of the R matrix, and is strongly influenced only by the relative parity of the Σ and Λ^0. It is desirable to discuss the behavior of the cross sections for various combinations of the assignments of parities.

Since the Λ^0 production must take place through the isotopic spin $\frac{1}{2}$ state, and since the $\Sigma-K$ intensity is almost entirely due to the $T=\frac{1}{2}$ state also, it is necessary to consider only the $T=\frac{1}{2}$ state carefully.

If the Σ parity is even and the K is pseudoscalar, we can immediately calculate the ratios, $k_\Lambda R_{\Lambda\Lambda}:k_\Sigma R_{\Sigma\Sigma}:D/(k_pR_{pp})$ for S-wave production from the measured cross sections. Since the input strange-particle production cross section is so large, the sum $|U_{p\Lambda}|+|U_{p\Sigma}|$ is nearly equal to one. This requires that the terms $k_\Lambda R_{\Lambda\Lambda}$ and k_pR_{pp} dominate the denominator D, and therefore that the ratios of these terms, and hence the ratios of three elements, $R_{\Lambda\Lambda}$, $R_{\Sigma\Sigma}$, and R_{pp} determined by the two measured cross sections, essentially determine the cross section as a function of energy. This behavior with energy is not significantly changed if the K meson is scalar, though the $\pi-p$ channel orbital angular momentum is then one. This is the case because the penetration factor, $P=(k_pa_p)^2(1+k_p{}^2a_p{}^2)^{-1}$ is close to one for reasonable values of a_p, where $a_p\geqslant\hbar/M_Kc$ and in any

[7] R. G. Sachs, Phys. Rev. **95**, 1065 (1954).
[8] T. D. Newton and E. P. Wigner, Revs. Modern Phys. **21**, 400 (1949).

case does not vary strongly over the small energy region which is of interest.

The solid lines in Fig. 1 show the strange-particle production cross sections calculated using Eq. (5) on the assumption that the Σ parity is even and the K is pseudoscalar. Values for the R-matrix elements were chosen so that $R_{\Lambda\Lambda}=1.0$, $R_{\Sigma\Sigma}=1.5$, and $R_{pp}=0.25$, in units of the K-meson Compton wavelength. The term $k_nQ_nR_{nn}$ was chosen arbitrarily to be equal to $k_pQ_pR_{pp}$. Since the Σ^0-K^0 and Σ^--K^+ channels have different thresholds, the term $\frac{1}{3}k_{\Sigma 0}Q_{\Sigma 0}+\frac{2}{3}k_{\Sigma +}Q_{\Sigma +}$ was used to substitute for $k_\Sigma Q_\Sigma$ in the denominator. The numerical factors represent the squares of Clebsch-Gordan coefficients appropriate for the resolution of the $T_{\frac{1}{2}}$ isotopic spin state into the two possible $\Sigma-K$ states, while $Q_{\Sigma 0}$ and $Q_{\Sigma +}$ represents the function Q calculated using the momenta of the Σ^0-K^0 and Σ^--K^+ system, respectively. Though the $T_{\frac{3}{2}}$ amplitude does not contribute much to the $\pi^-+p\rightarrow\Sigma+K$ cross section, it affects the ratio of Σ^+ to Σ^0 strongly. The $T_{\frac{3}{2}}$ cross section is also largely determined near threshold once the value is known for one energy. Equation (2) was also used to calculate this amplitude with $R_{\Sigma\Sigma}=0.15$ and $R_{pp}=0.4$ in units of $\hbar/M_Kc$ and $k_nQ_nR_{nn}=k_pR_{pp}$. These values were chosen in accord with the considerations of reference 3, which are based on that work and on the Michigan measurements of the $\pi^++p\rightarrow\Sigma^++K^+$ cross section.[9] Values of δ, and hence a, are needed when the relative phases of the amplitudes are necessary to calculate interference effects. In this case, only differences between the radii chosen for $T=\frac{3}{2}$ channels and $T=\frac{1}{2}$ channels are at all pertinent and these have been taken as zero. The Σ^0 and Σ^- cross sections, calculated separately using the above parameters, are also represented on Fig. 1. It is desirable to emphasize again that while all of the values of the parameters for both $T_{\frac{3}{2}}$ and $T_{\frac{1}{2}}$ amplitudes used in the calculations are presented in order to be definite and for orientation purposes, the cross sections calculated as a function of energy are very nearly independent of the particular choice of values used when this choice is constrained to fit the experimental cross sections at 960 Mev.

There are important qualitative features evident in the Λ^0 cross section shown by the solid line of Fig. 1. The sharp double cusp at the Σ^0 and Σ^- thresholds results from the competition above threshold with real $\Sigma-K$ production and below threshold with virtual $\Sigma-K$ production. Formally this behavior results from the rapid increase in the magnitude of $k_\Sigma Q_\Sigma R_{\Sigma\Sigma}$ in the denominator as the reaction energy varies upwards or downwards from the Σ threshold. For channel angular momentum equal to zero, $k_\Sigma Q_\Sigma$ is equal to $k_\Sigma=\hbar^{-1}[2M(E-E_t)]^{\frac{1}{2}}$, where M is the reduced mass of the Σ, and E and E_t are the energy of the system and the threshold energy, respectively. The Λ^0 production cross section can then

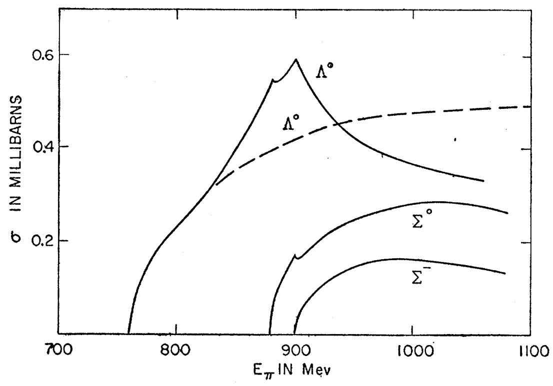

FIG. 1. S-wave production cross sections as a function of π^--meson bombarding energy for $\pi^-+p\rightarrow\Lambda^0+K^0$, labeled Λ^0; for $\pi^-+p\rightarrow\Sigma^0+K^0$, labeled Σ^0; and for $\pi^-+p\rightarrow\Sigma^-+K^+$ labeled Σ^-. The solid curve labeled Λ^0 represents cross section behavior of the type expected if the Σ and Λ^0 have the same parity, while the dashed curve shows a typical Λ^0 cross section under the condition that the Λ^0 and Σ have different parities.

be represented near the Σ threshold energy E_t, by

$$\begin{aligned}\sigma_\Lambda&=A+B(E_t-E)^{\frac{1}{2}}\text{ below threshold,}\\ \sigma_\Lambda&=A+C(E-E_t)^{\frac{1}{2}}\text{ above threshold,}\end{aligned}\qquad(6)$$

where A, B, and C are independent of energy. The coefficients B and C are negative in the approximation of Eq. (2), but are not otherwise closely related. Because of this cusp effect, the normal dependence of the Λ^0 S-wave cross section on the first power of the momentum of the exit channel will hold only very near threshold. Since k_Σ is real above threshold and imaginary below, the Λ^0 reaction amplitude varies sharply in phase as well as magnitude at the Σ thresholds. This can be particularly important when effects in differential cross section or polarization are considered, as the interference of the partial wave in which the cusp occurs, with waves of other angular momentum or parity, must depend upon their relative phases. Examples of the appearance of cusps in reactions at the threshold of a new channel have been observed in the scattering of protons by tritium at the T(p,n) threshold,[10] and in the elastic and inelastic scattering of protons by lithium at the Li(p,n) threshold.[11,12]

A second significant feature is the rapid decrease of this S-wave cross section with increasing energy. In this calculation this results from the rapid increase with energy of the magnitude of the terms $k_\Sigma Q_\Sigma R_{\Sigma\Sigma}$ and $k_\Lambda Q_\Lambda R_{\Lambda\Lambda}$ in the denominator and can be considered as a radiative damping. Serber[13] has pointed out that the increase to be expected in the multiple π production

[9] Brown, Glaser, Meyer, Perl, Vander Velde, and Cronin, Phys. Rev. **107**, 906 (1957).

[10] M. E. Ennis and A. Hemmendinger, Phys. Rev. **95**, 772 (1954).

[11] P. R. Malmberg, Phys. Rev. **101**, 114 (1956).

[12] Newson, Williamson, Jones, Gibbons, and Marshak, Phys. Rev. **108**, 1294 (1957).

[13] R. Serber, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957).

with energy, resulting from the rapid increase in three-body phase space, will also serve to damp the strange-particle production. Our treatment of this effect, expressed in the energy dependence of Q_n, is adequate for a small energy range, but probably underestimates the damping at higher energies. The Λ^0 production cross section at 1100 Mev is only about 0.25 mb,[14] supporting the view that some sort of damping may be important.

The dashed line on Fig. 1 illustrates the Λ^0 cross section computed in a similar manner under the assumption that the cross section is primarily due to S-wave emission and that the Σ parity is odd. The Σ channel, with the same angular momentum and parity as the S-wave Λ^0 channel, will now have one unit of orbital angular momentum. It is then evident from Table I that the function $k_\Sigma Q_\Sigma R_{\Sigma\Sigma}$ will not vary sharply near the Σ threshold, and no cusp in the S-wave Λ^0 cross section will be evident. A cusp will then occur in the $P_{\frac{1}{2}}$ partial cross section but it should be rather small.

Anomalous behavior can also be expected in elastic $\pi^- - p$ scattering at both the Λ^0 and Σ thresholds. Even if the assumption of Eq. (4) is valid, the anomalies will not necessarily take the form of cusps. Since the amplitude varies in phase as well as magnitude, the partial wave which is affected by the opening of the S-wave channel may interfere constructively with the incoming wave, and with scattered waves of different angular momentum on one side of the threshold, and destructively on the other side. The differential cross section as a function of energy will then again have the form of Eq. (6) but B and C may have different signs. This is a particular case of an effect pointed out and treated more generally, by Breit.[2] If the hyperons and K mesons have the same parity, the $\pi - p$ $P_{\frac{1}{2}}$ wave will be absorbed to produce an S-wave $Y-K$ system and the $P_{\frac{1}{2}}$ scattered wave will be affected at threshold. However, if the hyperon and K meson have the opposite parity, the S-wave $\pi - p$ channel will feed the S-wave $Y-K$ channel and the S-wave scattering will exhibit an anomalous behavior at the hyperon threshold. It is not possible to make a reliable calculation of the character of the scattering anomaly, but a rough estimate leads to deviations in kA, where k is the wave number and A the $\pi^- - p$ scattering amplitude, of about 0.05. This should lead to effects of the order of 10% in the $\pi - p$ differential scattering cross section at most angles, and from an investigation as a function of angle it might be possible to determine whether the S or $P_{\frac{1}{2}}$ wave was affected and measure the relative $Y-K$ parity.

It is clear from inspection of Eq. (2) that reliable quantitative estimates of the cross-section behavior cannot be made if the compound assumption of Eq. (4) breaks down seriously. Indeed this approximation was made primarily because it leads to definite results. It is unlikely, however, that the coefficient of $|E_t - E|^{\frac{1}{2}}$ be small on both sides of the threshold, and the qualitative aspects of these conclusions should be valid. Some insight into the behavior of the R-matrix elements and the adequacy of the compound assumption can be gained by considering the work of Wigner and Eisenbud.[5] Since these authors were interested in low-energy processes, their formal analysis is nonrelativistic and may not be completely relevant to this problem as the $\pi - p$ channel velocities are large. They find that the R-matrix element R_{ij} can be written as $R_{ij} = \sum_\lambda \gamma_{i\lambda}\gamma_{j\lambda}/(E_\lambda - E)$. When one term or one intermediate state, λ', dominates, the familiar Breit-Wigner single level formula results and the compound condition is fulfilled.

This may be particularly pertinent as the $T=\frac{1}{2}$ $\pi - p$ cross section exhibits a resonance-like bump near 900-Mev π^- energy.[15] Although this bump is too large to be ascribed to a $j=\frac{1}{2}$ state, it could conceivably be related to some kind of collective phenomena which would enhance several partial waves.

The relation of Eq. (4) results also from the less stringent condition that the strange-particle interaction alone be dominated by a single intermediate state. Matthews and Salam[16] and Landovitz and Leitner[17] have suggested that the production of particles with total angular momentum $\frac{1}{2}$ takes place primarily through the nucleon as an intermediate state. Then $R_{p\Sigma} = \gamma_p\gamma_\Sigma/(E_\lambda - E)$, $R_{\Sigma\Sigma} = \gamma_\Sigma\gamma_\Sigma/(E_\lambda - E)$, and $R_{\Lambda\Lambda} = \gamma_\Lambda\gamma_\Lambda/(E_\lambda - E)$, while $R_{pp} = \gamma_p\gamma_p/(E-E) + R_{pp}'$, where E_λ will be an energy near the nucleon mass, and R_{pp}' represents effects from all other interaction modes. It can be seen that, although the $\pi - p$ elastic scattering is not dominated by the intermediate state, Eq. (2) still reduces to Eq. (4).

HIGH ANGULAR MOMENTA AND POLARIZATION

The utility of discussing reactions by the use of the R-matrix formalism rests largely in the clarification, which results, of the form of variation of reaction amplitudes with energy. While this leads naturally to an understanding of the S-wave contribution to the strange-particle production cross section as a function of energy, since a reasonable estimate can be made of S-wave cross sections at one energy, the lack of a reliable partial-wave analysis of the strange-particle production cross section precludes this type of extrapolation for higher angular momenta. Furthermore, the magnitude of the scattering matrix element depends explicitly upon the value of a, the channel radius, for channels in which the orbital angular momentum is greater than zero, and the variation of $|U_{ab}|$ with energy depends particularly upon the values chosen for the radii of channels a and b.

It has been suggested[3] that the very large S-wave cross section for the production of strange particles is an indication that the interaction strength leading to the production of strange particles by $\pi - p$ collisions is very large, and that the small cross section at higher energies

[14] Brown, Glaser, and Perl, Phys. Rev. **108**, 1036 (1957).

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is indicative of a singular interaction rather than a weak interaction. The small cross section would then be attributed to the suppression of strange-particle emission in states of higher angular momentum by the centrifugal barrier. There are plausible reasons for expecting that the production of strange particles by π-nucleon collisions takes place only at small distances. For example, one possible basic production process would be described by the transition of the nucleon to a state consisting of a virtual hyperon and K meson, one of the strange particles then absorbing the incoming π^-. Such a process would take place at a range less than $\hbar/M_K c$. Though a complete description of the process would probably require a more intricate description of the interaction, the range would still be small. Strong attractive long-range initial- and final-state interactions could, however, refract the incoming and outgoing beams so as to magnify this region.

Since the complexity of the angular distribution of reaction products is limited by the values of angular momenta contributing to the reaction, and the maximum values of angular momenta which can be important are in turn related to the size of the interaction radius, it should be possible to make an estimate of this radius by an inspection of the angular distribution. If we should assume that the partial cross section for the production of a final state with total angular momentum, j, is determined predominantly by the overlap of initial- and final-state wave functions in an interaction volume, we would find that the partial cross section would be proportional to $P_i(a_i)P_f(a_f)$ where the P are penetration factors for initial- and final-state channels, and are closely approximated by the expressions of Table I. Since the penetration factors are functions of a, the channel radius, and the orbital angular momentum of the channel, ratios of partial cross sections for different angular momenta could serve to provide an estimate of the values of a. An analysis in this spirit, but somewhat better suited for strong interactions where damping is important, is provided by the use of Eq. (4). The angular distribution will take the form

$$d\sigma/d\Omega=\pi k_p^{-2}(|A|^2+|B|^2),$$

where

$$\begin{aligned} A=&U_{\frac{1}{2}}^{+}+(2U_{\frac{3}{2}}^{-}+U_{\frac{1}{2}}^{-})\cos\theta \\ &+(3U_{\frac{5}{2}}^{+}+2U_{\frac{3}{2}}^{+})(\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2})+\cdots, \quad (7) \\ B=&(U_{\frac{3}{2}}^{-}-U_{\frac{1}{2}}^{-})\sin\theta+(U_{\frac{5}{2}}^{+}-U_{\frac{3}{2}}^{+})3\sin\theta\cos\theta+\cdots, \end{aligned}$$

where θ is the angle of production in the center-of-mass system of the K meson, and the symbols U represent the scattering matrix elements according to Eq. (4); the subscripts represent the total angular momentum, the superscripts the parity. The K-meson parity is taken to be even, following other considerations.[18] In general the

[18] R. K. Adair (to be published).

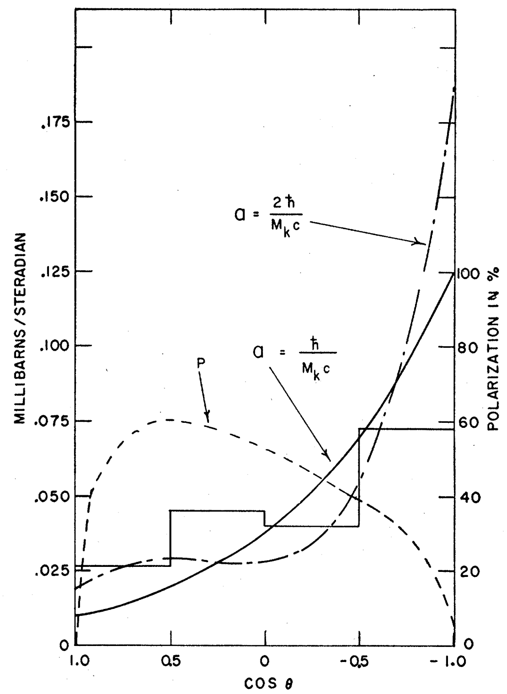

FIG. 2. Differential cross section and polarizations expected from the reaction $\pi^-+p\rightarrow\Lambda^0+K^0$. The solid and dotted curves represent angular distributions calculated using radii of interaction, a, equal to $\hbar/M_K c$ and $2\hbar/M_K c$, respectively, while the histogram represents the experimental values from reference 3. The dashed curve presents the polarization calculated using the radius $\hbar/M_K c$. The quantity θ represents the angle of production of the Λ^0 in the center-of-mass system.

Λ^0 will be polarized. This polarization will be equal to[19,20] $2\,\mathrm{Im}(A^*B)/(AA^*+BB^*)$.

These equations and Eq. (5) were then used to calculate the angular distribution and polarization, using the same R matrix for each partial wave. As in the previous section, the values for the R-matrix elements were determined by the total cross section at 960 Mev. The solid line of Fig. 2 shows the angular distribution calculated using a channel radius of $\hbar/M_K c$ for all channels, while the dotted line shows the polarization of the Λ^0 as a function of angle. The angular distribution is in good agreement with the experimental results shown as a histogram, while the value of the average polarization $\bar{P}$ of 0.38 is also in good agreement with the values $\bar{P}\geq 0.44 \pm 0.10$ and 0.52 ± 0.11 found[21,22] at nearby energies. The dotted line represents a calculation of the angular distribution using a channel radius of $2\hbar/M_K c$. Clearly, within the limits of the approximations made, the

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[22] Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Phys. Rev. **108**, 1102 (1957).

smaller radius best represents the experimental distribution.

If the K meson is scalar, the $P_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$ production amplitudes might be expected to differ considerably as the $P_{\frac{1}{2}}$ wave is fed by the $S_{\frac{1}{2}}$ $\pi-p$ partial wave while the Λ^0-K $P_{\frac{3}{2}}$ wave is fed from the incident D wave. Since the $D_{\frac{3}{2}}$ $\pi-p$ partial wave will encounter a larger centrifugal barrier than the S wave, the Λ^0-K $P_{\frac{3}{2}}$ amplitude will be smaller than the $P_{\frac{1}{2}}$ amplitude. The phases of the different partial waves will also exhibit different retardations due to the different centrifugal barriers which they meet. It is then the splitting and phase shifts resulting from the centrifugal barriers which leads naturally to the Λ^0 polarization shown in Fig. 2. This polarization is in the direction of $-(k_\pi\times k_K)/|k_\pi\times k_K|$. Measurements of the decay asymmetry of Λ^0 hyperons produced by the π^--p interaction at this energy[3,17,18] show that the protons from the Λ^0 decay tend to go in the opposite direction. This suggests, then, that the protons from decaying Λ^0 hyperons tend to be emitted opposite to the direction of the Λ^0 spin. It must be emphasized that this results only from the effects of the centrifugal barrier. In particular, singularly different values for the R matrix in the different angular momentum states would obviate this conclusion.

An examination of Eq. (2) discloses that the momentum dependences of the various Λ^0-K partial waves near threshold is determined[1,2] by the behavior of the functions $k_\Lambda^{\frac{1}{2}}$, $P_\Lambda^{\frac{1}{2}}$, and δ_Λ. This leads to the proportionalities,

$$\begin{aligned} U_{p\Lambda}(S_{\frac{1}{2}}) &\approx k_\Lambda^{\frac{1}{2}} \exp[i(\alpha-k_\Lambda a_\Lambda)], \\ U_{p\Lambda}(P_{\frac{1}{2}}) &\approx k_\Lambda^{\frac{3}{2}} \exp(i\beta), \\ U_{p\Lambda}(P_{\frac{3}{2}}) &\approx k_\Lambda^{\frac{3}{2}} \exp(i\beta'), \end{aligned} \tag{8}$$

where α, β, and β' are constants. These proportionalities differ from the dependences discussed by Lee *et al.*[19] in the inclusion of the momentum-dependent term in the phase of the S-wave production amplitude. If the phase angle α is nearly the same as the phase of the term $[U_{p\Lambda}(P_{\frac{3}{2}})-U_{p\Lambda}(P_{\frac{1}{2}})]$, it is clear from inspection of Eqs. (7) and (8) that the polarization of the Λ^0 near threshold will vary as k^2. Otherwise the polarization will be more nearly proportional to k. Similar considerations, of course, hold for $\Sigma-K$ production.

PHOTOPRODUCTION OF STRANGE PARTICLES

In the previous discussion electromagnetic interactions have been neglected. They can be introduced by adding a row $R_{\gamma i}$ and a column $R_{j\gamma}$ to the R matrix, where the subscript γ refers to the channel $n+\gamma$. Since the electromagnetic interaction is weak, the wave is nearly undistorted by the interaction and the channel radius, $a=0$, leads to the simplest and most easily interpreted values of $R_{j\gamma}$ and $R_{\gamma i}$. A choice of the function Q_γ is desirable which reflects the energy dependence of the multipole radiation. It seems plausible that this should be proportional to $\int_0^a j_l^2(k_\gamma r)r^2dr$; here j_l is the spherical Bessel function of order l, where l is the multipole moment of the radiative transition, k is the photon wave number, and a is the radius of interaction which may be about $\hbar/M_Kc$. This expression is closely approximated by $P_l(a)$ of Table I. The cross section for radiative capture of the π^- by the proton in a state of angular momentum j and parity Π then takes the form $\sigma_{p\gamma}=\frac{1}{2}(2j+1)\pi k^{-2}|U_{p\gamma}|^2$, where $U_{p\gamma}(j,\Pi)$ is similar in structure to $U_{p\Lambda}(j,\Pi)$ and $U_{p\Sigma}(j,\Pi)$. In particular, the denominator functions D are identical and the capture cross section will show cusps, and be affected by radiative damping in the same way as the strange-particle cross sections.

Cross sections for the reactions $\Lambda^0+K^0\rightarrow n+\gamma$ and $\Sigma+K\rightarrow n+\gamma$ in the $T=\frac{1}{2}$ state, and the inverse photoproduction processes can also be written in terms of the R matrix and will also have a structure similar to the π^-+p reactions. This statement is essentially equivalent to the observation that since the $\gamma+n$ and π^-+p reactions share the same final-state interaction, one might expect related behaviors if the final-state interactions are strong.

Since the incident photon beam has different angular momentum properties from the π^- beam, the angular distribution and polarization formulas take different forms. The $\gamma+p\rightarrow\Lambda^0+K^+$ production amplitude can be written as[23]

$$\begin{aligned} A=\tfrac{1}{2}k_\gamma^{-1}\sum_{\Pi}\sum_{j}\sum_{m_\gamma}\sum_{m_p}\sum_{m_K} & U(l_\gamma j\Pi)(s_pl_\gamma,jm_j|s_pm_p,lm_\gamma) \\ & \times D_{l_\gamma}^{m_\gamma}(0)(s_\Lambda l_K,jm_j|s_\Lambda m_\Lambda,l_Km_K) \\ & \times Y_{l_K}^{m_K}(\vartheta\varphi)\chi_\Lambda^{m_\Lambda}\epsilon_{m_\gamma}\epsilon_{m_p}', \end{aligned} \tag{9}$$

where D and Y are the vector and ordinary spherical harmonics, l is the multipole moment of the γ ray, l_K the orbital angular momentum of the $\Lambda-K$ channel, j is the total angular momentum of the pertinent state, and Π is its parity. The s_Λ and s_p represent the spin of the Λ^0 and proton, respectively, and the m represent the components of angular momentum in the beam direction. The ϵ_{m_γ} and ϵ_{m_p}' are orthogonal sets of unit vectors such that for unpolarized γ rays, $\epsilon_a\epsilon_b=\delta_{ab}$, and $\epsilon_a'\epsilon_b'=\delta_{ab}$ for unpolarized protons. The $\chi_\Lambda^{m_\Lambda}$ represents the Λ^0 spin function and the U are the elements of the scattering matrix. This equation also is valid for π-meson production, for Σ production, and for photoneutron processes, with obvious appropriate changes.

If we consider only production near threshold, we need only include S and P waves of the Λ^0-K system and the angular distribution takes the form

$$d\sigma/d\Omega=(8k^2)^{-1}(|A|^2+|B|^2+|C|^2), \tag{10}$$

where

$$\begin{aligned} A &= [\tfrac{3}{2}U(E_1,P_{\frac{3}{2}})-\tfrac{1}{2}\sqrt{3}U(M_2,P_{\frac{3}{2}})]\sin\vartheta, \\ B &= [U(E_1,P_{\frac{3}{2}})+(2/\sqrt{3})U(M_2,P_{\frac{3}{2}}) \\ &\qquad -U(E_1,P_{\frac{1}{2}})]\cos\vartheta+U(M_1,S_{\frac{1}{2}}), \\ C &= [\tfrac{1}{2}U(E_1,P_{\frac{3}{2}})+\tfrac{1}{2}\sqrt{3}U(M_2,P_{\frac{3}{2}})+U(E_1,P_{\frac{1}{2}})\sin\vartheta, \end{aligned}$$

[23] D. R. Hamilton, Phys. Rev. 58, 122 (1940).

and the polarization of the Λ^0 will be equal to[24,25]

$$P=-2(\mathrm{Im}B^*C)/(|A|^2+|B|^2+|C|^2) \tag{11}$$

in the direction $(k_\gamma \times k_K)/|k_\gamma \times k_K|$. The arguments of the scattering matrix elements, U, represent the multipole order of the photon interaction and the orbital angular momentum and total angular momentum of the $\Lambda-K$ system. The K meson was again chosen to be scalar. If the K meson is pseudoscalar, the formula (10) remains essentially unchanged but all electric moments are changed to magnetic moments and vice versa.

The matrix elements U are closely related to the elements for π-nucleon production. It is useful to examine a particular partial wave in detail in order to illustrate some general consequences of this. It is particularly convenient to discuss the S-wave Λ^0-K^+ production very near threshold, and assume the K meson is pseudoscalar. Then, neglecting small quantities in comparison with large quantities and for simplicity neglecting multiple meson production, we have from Eq. (2) and Table I,

$$\begin{aligned} U_{\gamma\Lambda}(E_1,S_{\frac{1}{2}}) &= \frac{2ik_\gamma{}^{\frac{1}{2}}P_\gamma{}^{\frac{1}{2}}k_\Lambda{}^{\frac{1}{2}}[R_{\gamma\Lambda}-ik_p(R_{pp}R_{\gamma\Lambda}-R_\gamma R_{p\Lambda})]\exp(-ik_\Lambda a_\Lambda)}{1-ik_pR_{pp}-ik_\Lambda R_{\Lambda\Lambda}-k_pk_\Lambda(R_{pp}R_{\Lambda\Lambda}-R_{p\Lambda}R_{\Lambda p})}, \\ U_{\Lambda\Lambda}(S_{\frac{1}{2}}) &= \left[1+\frac{2ik_\Lambda(R_{\Lambda\Lambda}-ik_p[R_{pp}R_{\Lambda\Lambda}-R_{\Lambda p}R_{p\Lambda}])}{1-ik_pR_{pp}-ik_\Lambda R_{\Lambda\Lambda}-k_pk_\Lambda(R_{pp}R_{\Lambda\Lambda}-R_{p\Lambda}R_{\Lambda p})}\right]\exp(-2ik_\Lambda a_\Lambda). \end{aligned} \tag{12}$$

When only the Λ^0 channel is open, we can set $R_{jp}=R_{pi}=0$ for all j, i. Then $U_{\Lambda\Lambda}=\exp[2i(\tan^{-1}k_\Lambda R_{\Lambda\Lambda}-k_\Lambda a_\Lambda)]$ and the S-wave Λ^0-K scattering amplitude will equal $k^{-1}\exp[i(\alpha+\beta)]\sin(\alpha+\beta)$, where $\alpha=\tan^{-1}(k_\Lambda R_{\Lambda\Lambda})$ and $\beta=-k_\Lambda a_\Lambda$. The $\gamma+p\rightarrow\Lambda^0+K$ S-wave photoproduction amplitude will be equal to

$$(1/\sqrt{2})k_\gamma{}^{\frac{1}{2}}P_\gamma{}^{\frac{1}{2}}(R_{\gamma\Lambda}/R_{\Lambda\Lambda})\sin\alpha\exp[i(\alpha+\beta)].$$

These relationships differ from the conclusions of Watson[26,27] and Brueckner in the proportionality of the production amplitude to $\sin\alpha$ instead of $\sin(\alpha+\beta)$. However, if other channels are open, e.g., the $\pi-p$ channel, Watson's theorem does not hold. In particular, inspection of Eq. (9) shows that the phase of the amplitude will not necessarily be purely imaginary[28] even very near threshold. Since all of these conclusions hold equally for other angular momenta, the phases of the various partial waves can be expected to vary widely even at threshold. Since the magnitudes of the various partial-wave amplitudes can also be expected to be strongly affected by the final-state interactions, it seems unlikely that the detailed predictions of perturbation theory calculations[29–32] concerning angular distributions and coupling constants can be reliable.

The qualitative momentum dependences of the various strange-particle partial waves will be determined by the final-state wave function in the same way as for the production by $\pi-p$ interactions, and will be identical in form to that presented in Eqs. (10). The polarization will be proportional to $k_\Lambda{}^2$ if α is very nearly the same as the phase of term C in Eq. (7), a possibility discussed by Sakurai[25]; otherwise the polarization very near threshold will vary as k_Λ. A comparison of Eqs. (7) and (8) with Eqs. (5) and (6) show that the angular distributions and polarizations of the strange particles produced by π-nucleon and photonucleon reactions have no very close connection.[21] In particular, the potential barrier arguments which suggested definite angular distributions and polarizations for the Λ^0 lead to no such striking effects for $\gamma-p$ productions.

Since the $\gamma+p\rightarrow\Lambda^0+K^+$ and the $\pi^-+p\rightarrow\Lambda^0+K^0$ reactions both take place through the $T=\frac{1}{2}$ state, the R-matrix elements are the same for the two reactions. The anomalous behavior in the $\pi^--p\rightarrow\Lambda^0+K^0$ cross section near the $\Sigma-K$ threshold should also occur in the $\gamma+p\rightarrow\Lambda^0+K^+$ reaction near the $\gamma+p\rightarrow\Sigma+K$ thresholds, and should also help to determine the parity of the Σ relative to the Λ. The $\gamma+p\rightarrow\pi^++n$ and $\gamma+p\rightarrow\Lambda^0+p$ should also exhibit cusps at both the Λ^0 and Σ^0 threshold. Again the cusp will affect the $S_{\frac{1}{2}}$ partial wave if the K meson is pseudoscalar and the $P_{\frac{1}{2}}$ wave if the K is scalar.

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Several years ago Dr. Gyo Takeda informed me of calculations which he made concerning the effect of hyperon production on $\pi+p$ cross sections near the hyperon thresholds. The calculations in this paper are made in a similar spirit and some of the conclusions may duplicate the results of unpublished work by Dr. Takeda.

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