small for our purposes and was neglected. Putting $\mathbf{z}=\mathbf{x}+\frac{1}{2}\mathbf{r}$, we obtained

$$I_{pp}=(2\pi)^{\frac{3}{2}}\phi_d(\tfrac{1}{2}\mathbf{K}_0+\mathbf{K}')\int\phi_{K''}(z)\times V(z)e^{i(\mathbf{K}_0+\frac{1}{2}\mathbf{K}')\cdot\mathbf{z}}d\mathbf{z}. \quad (11)$$

Upon evaluation, the contribution to M from I_{pp} turned out to be about 15% of the contribution from I_{np} at 9-Mev bombarding energy. This result is then in agreement with the notion that the main contribution to the high-energy neutron scattering results from the n-p interaction.

Figure 3 gives a comparison of the calculations of this section with experiment at 8.9 Mev. The calculated values are too high, although the discrepancy is not as large as with the F-G model. Figures 4 and 5 illustrate the progressively poorer fits to the magnitude of the peak as the bombarding energy is decreased.

As with the previous calculation, the shape and position of the high-energy peak is given quite accurately. One can conclude from this that the shape and position of this part of the neutron energy spectrum is primarily dependent on the final-state interaction of the two protons—particularly the Coulomb interaction. If there were no Coulomb interaction, as would be the case for the (n,d) reaction, then this peak would occur practically at the maximum-energy position and have a very narrow width. It is the Coulomb interaction which causes the shift in position to roughly 0.5 Mev below the maximum-energy position and increases the width.

It is apparent that as the incident energy is decreased, it will be a less and less good approximation to consider separate energy regions in which either the n-p or p-p final-state interaction is dominant. Rather, both will be effective in distorting the final-state wave function, which, of course, becomes difficult to treat. This is a consequence of the fact that when the velocity of the incoming nucleon becomes equal to, and less (at about 7 Mev) than, the internal velocities of the deuteron, there is a polarization of the deuteron, which must be taken into account in order to obtain any quantitative agreement with experiment. In the method of F-G, these effects seem to be partially taken into account by their use of the impulse approximation. This accounts for the fair agreement of Frank and Gammel's calculation with a substantial part of the experimental results.

ACKNOWLEDGMENTS

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Analysis of $C^{12}+d$ Reactions*

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The $C^{12}(d,d)C^{12}$ scattering is analyzed at two levels in N^{14}, one at 12.42-Mev excitation and the other at 12.60 Mev. The lower level is assigned as 4^- and the upper as 3^+. A nuclear radius is assumed and reduced widths are obtained for the two levels. With the aid of the reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev level), partial level widths for the three modes of decay of N^{14*} are obtained. Both (d,p) reactions are shown to be consistent with the presence of stripping amplitudes and resonant compound nuclear amplitudes and also consistent with the $B^{10}+\alpha$ data of Shire *et al.* for the same excitation region in N^{14}. The (d,p) angular distributions are analyzed to obtain the reduced neutron widths of the ground and first excited states of C^{13}.

An appendix gives an explicit partial-wave expansion of the differential cross section for reactions involving incident spins of 0 and 1 and outgoing spins of $\frac{1}{2}$ and $\frac{1}{2}$.

INTRODUCTION

THE differential cross sections for the $C^{12}(d,d)C^{12}$, $C^{12}(d,p)C^{13}$ (ground state), and $C^{12}(d,p)C^{13*}$ (first excited state) reactions have been measured and published.[1] The present paper gives a partial analysis of those measurements.

Recent attempts to obtain a more complete treatment of the (d,p) and (d,n) reactions than has been provided by the Butler approximation have introduced the Coulomb interaction, the nuclear interaction of incident deuteron with the target and the interaction of liberated particles with the residual nucleus (references contained in I). The formulations of Tobocman,[2] Grant,[3] and also that of Thomas[4] include all of these modifications. The

* Work supported by the U. S. Atomic Energy Commission, and by the Graduate School from funds supplied by the Wisconsin Alumni Research Foundation.
† Now at University of Kentucky, Lexington, Kentucky.
[1] McEllistrem, Jones, Chiba, Douglas, Herring, and Silverstein, Phys. Rev. **104**, 1008 (1956), henceforth referred to as I.
[2] W. Tobocman, Phys. Rev. **94**, 1655 (1954); W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).
[3] I. P. Grant, Proc. Phys. Soc. (London) **A67**, 981 (1954).
[4] R. G. Thomas, Phys. Rev. **100**, 25 (1955).

parameters introduced into the (d,p) and (d,n) amplitudes by the deuteron-target and liberated particle-residual nucleus interactions may be determined from the elastic scattering of deuterons by the target and the elastic scattering of the liberated particles by the residual nucleus, respectively. Therefore, if one should include the elastic scattering of protons by C^{13} with the cross sections of I, one would have enough information to attempt a rather complete analysis of the $C^{12}(d,p)$ reaction,[5] at least away from resonances. Such analysis should yield a reliable value for the neutron reduced width of C^{13}.

Of the formulations mentioned above, Tobocman's[2] neglects the compound-nucleus amplitudes in the (d,p) reaction and treats "resonances" as the result of the scattering of (stripped) protons by the residual nucleus. On the other hand, Thomas's[4] formulation is specifically concerned with both compound-nucleus and stripping amplitudes in (d,p) and (d,n) reactions. Our analysis shows that the cross sections of I are consistent with the point of view adopted by Thomas in the vicinity of resonances. Although the $C^{13}+p$ scattering cross sections are not available in the desired energy range ($\sim$5-Mev protons), we are able to obtain many of the proton parameters from a comparison of the cross sections of I and the $B^{10}(\alpha,p)C^{13}$ cross sections of Shire *et al.*[6] The parameters of the $C^{12}+d$ interaction are obtained at the two most prominent resonances from an analysis of $C^{12}(d,d)C^{12}$. This analysis yields the spins, parities, and deuteron widths of the two corresponding N^{14} levels.

A detailed quantitative analysis of the $C^{12}(d,p)C^{13}$ cross sections has not been attempted. Instead, the neutron reduced widths of the C^{13} ground and first excited states are determined by an approximate analysis of the (d,p) angular distributions. For this analysis, the method of Bowcock[7] is employed. This method is expected to yield widths in which one might place greater confidence than one usually attributes to values obtained from a direct comparison of Butler's formalism and the experimental cross section.

$C^{12}(d,d)C^{12}$ ANALYSIS

The deuteron scattering data present two resonances which are quite strong, and which one might hope to analyze as "isolated" resonances. These occur at $E_d=2.502$ Mev and $E_d=2.735$ Mev. The differential cross section for the elastic scattering of spin 1 particles by spin 0 nuclei has been published.[8] The formulas and notation of reference 8 are used throughout this analysis. In this scattering, levels of a given total angular momentum (J) and parity may be excited by two values of the orbital angular momentum (l) of the deuterons. However, we begin the analysis by assuming that only one l value enters a given resonance, or that orbital angular momentum is conserved as well as total angular momentum. We shall see that this assumption is rather reasonable for these particular resonances.

The apparent complexity of the analysis is greatly mitigated by several features of the scattering formula of reference 8. As mentioned there, for example, if the resonant cross section at 90° should dip below the cross-section level near the resonance, then the compound nuclear state must have even parity. Another restriction is placed on the resonance by the Wigner limit, which specifies that $\gamma_{l,s}^2\leqslant\frac{3}{2}(\hbar^2/\mu a)$, where s denotes a particular mode of formation, μ denotes the reduced mass of the deuteron and a denotes the nuclear radius. Since the penetrability $(1/A_l^2)$ decreases rapidly with increasing l, an upper limit on the possible l value of the resonance is obtained from the experimental width $(\frac{1}{2}\Gamma=k\gamma_l^2/A_l^2)$. Data taken at angles near 180° simplify the analysis because at 180° the spin-orbit coupling terms in the scattering formula, which contain $\sin\theta$ as a factor, vanish. Also the resonant amplitude has its maximum value at 180°. Finally, in this energy range the Coulomb phase shift α_l varies rapidly with l. Consequently the interference of the resonant amplitude with the non-resonant ones changes rapidly with l. By studying the scattering at forward angles, where the interference of the resonant amplitude is quite apparent, one may place quite restrictive limits on the l value of the resonance. In fact, since both the Rutherford amplitude and the Coulomb phase shifts are well determined, the forward-angle data may uniquely fix the l value.

2.502-MEV RESONANCE

The first problem is the determination of the orbital angular momentum of this resonance. To put an upper limit on the l value, we use the Wigner limit. From a comparison of the (d,d) data and Shire's $B^{10}+\alpha$ data, we see that the most important mode of N^{14} formation at this resonance is deuteron formation. Consequently, we assume that $\Gamma_d/\Gamma\geqslant0.5$. This assumption together with the Wigner limit tells us that for $l=4$, $\Gamma\leqslant12$ kev and for $l=3$, $\Gamma\leqslant175$ kev. The experimental value is $\Gamma_{exp}=47$ kev (lab.) $=40.3$ kev (c.m.); consequently we can certainly say that $l\leqslant4$. With this restriction, the pronounced dip at 59° (c.m.) plus the following arguments permit only $l=3$. First, $l=0$ would not enable us to reproduce the large back-angle (169.3°) cross section, even if $\Gamma_d/\Gamma=1$. For $l=1$, we get both constructive and destructive interference at 59° (c.m.), and the destructive interference is much too weak to fit the data. For $l=2$, we would have very little resonant effect and very little interference at 59°, because $P_2(\theta)=0$ at 55°, and is still very small at 59°. For $l=4$, we would have too large a resonant effect at 90° unless $\Gamma_d/\Gamma\lesssim0.2$; for such a small Γ_d/Γ, the back-angle cross section would be too

[5] This statement requires the additional assumption that all competing reactions other than the $C^{12}(d,p)$ reaction are small compared to the two mentioned elastic-scattering cross sections.
[6] Shire, Wormald, Jones, Lunden, and Stanley, Phil. Mag. 44, 1197 (1953).
[7] J. E. Bowcock, Proc. Phys. Soc. (London) 68, 512 (1955).
[8] A. I. Galonsky and M. T. McEllistrem, Phys. Rev. 98, 590 (1955).

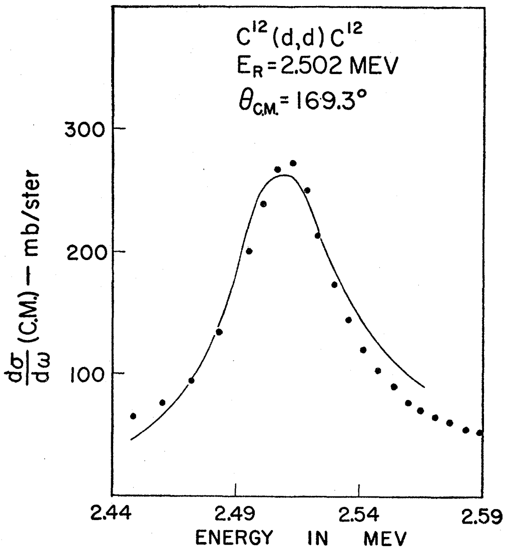

FIG. 1. Deuteron elastic scattering resonance at $\theta=169.3°$ c.m. The solid points represent the data, and the curve is the theoretical fit for $J=4^-$ and pure F-wave excitation.

small by about an order of magnitude. Consequently $l=3$.

The allowed J-values (for an $l=3$ resonance) are 2^-, 3^-, and 4^-. For 2^-, the large back-angle cross section observed cannot be reproduced, even if we assumed $\Gamma_d/\Gamma=1$. As can be seen from the scattering formula of reference 8, there is a large resonant, spin-orbit interaction amplitude for $J=l$ (term labeled E).[8] Such an amplitude cannot interfere with the Coulomb amplitude. At 59° (c.m.), therefore, we find it impossible to obtain the large dip observed if $J=l$. The only remaining single-level possibility for this resonance is $J=4^-$.

The calculated fit to the cross sections for $J=4^-$ is shown for the back angle and for 59° (c.m.) in Figs. 1 and 2. The parameters which have been deduced from the fit to the cross sections are listed in Table I.

2.735-MEV RESONANCE

The Wigner limit in this case limits the l values such that once again $l\leqslant 4$. Here also we obtain qualitative help from the data near 90° (c.m.) (Fig. 9 of I). Since the cross sections at the beginning of the resonance dip well below the value expected from Coulomb+potential scattering, the l value entering this resonance must be even. For $l=0$, if $\Gamma_d/\Gamma=1$, the cross section can just equal the measurements at the back angle. Such a large Γ_d/Γ here is impossible, in view of the fact that the resonant (d,p) cross section to the first excited state of C^{13} is as large as the (d,d) cross section. (See Fig. 10 of I.) In addition, this assumption ($l=0$, $\Gamma_d/\Gamma=1$) would produce a pronounced dip followed by a slight "hump" at 59° (c.m.), neither of which is observed. For $l=4$, the cross section at 96° (c.m.) would begin with a rise or hump followed by a dip, just the opposite of what is observed. The only remaining even l value is 2. This value is further supported by the lack of appearance of the resonance at 59° (c.m.), because we have already noted that the angle factor P_2 is very small at this angle. With $l=2$, there are three possible J-values, 1^+, 2^+, and 3^+. With $J=1^+$, in order to fit the back-angle data we must have $\Gamma_d/\Gamma>0.9$, which (we have already seen) is not possible. For $J=2^+$, we have (at 96°) the difficulty

TABLE I. Level parameters from $C^{12}(d,d)C^{12}$ analysis. Reduced widths are calculated on the assumption that $r_0=4.25$, $\times10^{-13}$ cm.

E_d (Mev)	Γ (c.m.)	$J\pi$	Γ_d/Γ	$\gamma_l^2 \Big/ \left(\frac{3}{2}\frac{\hbar^2}{\mu a}\right)$
2.502	40	4^-	0.55	0.40
2.735	47	3^+	0.63	0.03

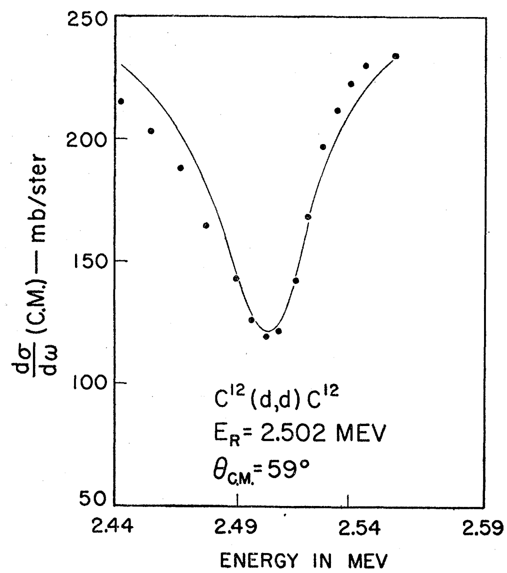

FIG. 2. Deuteron elastic scattering resonance at $\theta=59°$ c.m.. The solid points represent the data, and the curve is the theoretical fit for $J=4^-$ and pure F-wave excitation.

we mentioned for the $J=l$ possibility at the 2.502-Mev resonance, namely, that the large spin-orbit term for this case reduces interference effects at the resonance. In fact, we would need $\Gamma_d/\Gamma=1$ at 96° (c.m.) in order to reproduce the interference observed there, and this ratio is not possible. The only remaining possibility is $J=3^+$. This possibility has been checked at 59° (c.m.), 96° (c.m.), and the back angle 169.3° (c.m.). It is found that it does reproduce the shapes and cross-section variations at these angles. A detailed fit to the cross sections over the region of the resonance has not been calculated; however the parameters of the level (listed in Table I) have been extracted from the magnitude of the cross-section variations at the angles mentioned.

At the beginning of the analysis, we assumed that orbital angular momentum was conserved, and then fixed the two resonances analyzed as $l=3$ and $l=2$ at 2.502 Mev and 2.735 Mev, respectively. If we do not assume conservation of l, then we could have additional contributions from $l=5$ and $l=4$ at 2.502 Mev and 2.735 Mev, respectively. At the lower energy resonance, however, the Wigner limit on the width restricts the $l=5$ contribution to less than 1% of the total, so that l conservation seems a very good assumption here. At the higher energy resonance this restriction would allow the $l=4$ contribution to be $\lesssim 25\%$. For this case, however, we have already noted that an $l=4$ amplitude would be shifted in phase from the $l=2$ amplitude by an angle near 180°. An $l=4$ contribution would then tend to cancel the $l=2$ interference observed at 96°. In order to fit the magnitude of this interference, it was necessary to make Γ_d/Γ quite large. Adding an appreciable $l=4$ term would force us to take Γ_d/Γ appreciably larger to fit 96°, and this does not seem very reasonable.

We note also that the "potential" phases which we found necessary to fit the data depart radically from the "hard sphere" phases usually employed in this type of analysis. We assume that this is the effect of several nearby resonances which are not specifically treated in the analysis. Since we do not deduce our "potential" phases from hard-sphere scattering or any other potential, we are not able to fix a nuclear interaction distance from the data. For this parameter, we have arbitrarily assumed that $r_0=r_0'A^{\frac{1}{3}}$, where $r_0'=1.22 \times 10^{-13}$ cm. The reduced widths listed in Table I depend on this choice of r_0.

Shire *et al.*[6] and Shire and Edge[9] have observed all of the resonances of I in their study of the $B^{10}+\alpha$ reactions, including the two analyzed above. They were able to assign a spin of 4^- to the lower energy resonance, and our assignment is in agreement with theirs.

(d,p) RESONANCES

We have noted, in I, the close correspondence between the anomalies in the elastic scattering data and in the $C^{12}(d,p)$ reactions (see Fig. 10 of I). In order to test whether these (d,p) "resonances" could be interpreted as the result of single levels in the compound nucleus (N^{14}), the $C^{12}(d,p)C^{13}$ (ground state) data at the 2.735-Mev resonance was separated into a resonant and nonresonant amplitude via the following formula:

$$\frac{d\sigma}{d\omega} \sim \frac{1}{k_d^2}|\mathbf{A}_t+\mathbf{A}_R|^2,$$

where

$$A_R=C(|A_{s,s'}|,s,s')\frac{(\Gamma_p\Gamma_d)^{\frac{1}{2}}}{\Gamma}\sin\beta e^{i\beta}.$$

$\mathbf{A}_t$ is the nonresonant amplitude, defined in terms of the $\mathbf{A}_{s,s'}$ ($\mathbf{A}_{s,s'}$ is the nonresonant amplitude for a given state of incident and outgoing channel spins and polarizations; s, s' are indices specifying the channel spins and polarizations), and $[(\Gamma_p\Gamma_d)^{\frac{1}{2}}/\Gamma]\sin\beta e^{i\beta}$ is the familiar Breit-Wigner amplitude with $\tan\beta=\frac{1}{2}\Gamma/(E_R-E)$. This formula has been shown to be a valid expression for the cross section in the vicinity of a single resonance.[10]

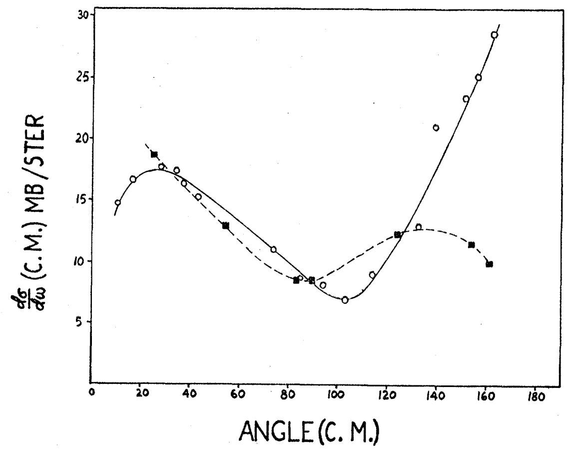

FIG. 3. $C^{12}(d,p)C^{13}$ (ground state) data at $E_d=2.735$ Mev. The "remainder" cross sections $(1/k^2|A_t|^2)$ after the resonant amplitude has been subtracted from the data are shown as box points (dashed curve). The circle points (solid curve) represent the measured cross sections at the 2.735-Mev resonance.

The separation was undertaken with the parameters listed in Table I for this resonance: [$\Gamma=47$ kev (c.m. system), $E_R=2.735$ Mev (lab. system)]. It was found possible to fit the data at all angles to well within the experimental error. Figure 3 shows the $C^{12}(d,p)C^{13}$ angular distribution on the resonance, and also shows a "remainder" distribution, after the resonant amplitude (A_R) has been extracted from the data. The latter distribution is similar to one obtained away from resonances (see Fig. 5 of I).

The angular dependences of the resonances in the $C^{12}(d,p)C^{13*}$ reaction are consistent with the assumption that they are the result of compound-nucleus amplitudes. To discuss this, we must know the orbital angular momentum (l') of the outgoing protons as well as that of the incoming deuterons (l). The intrinsic parity of $C^{13*}+p$ is the same as that of $C^{12}+d$, and there are only two possible outgoing channel spins, $j'=0$ and 1 (for notation and formulas see Appendix). From the two analyzed scattering resonances we found $J=l+1$, and hence in this (d,p) reaction only $j'=1$ can contribute to the resonances. The $j'=1$ part of the cross-section formula has a spin and orbital angular momentum dependence identical to that of the $C^{12}(d,d)C^{12}$ formula.[8] Also the outgoing proton energy is approximately equal to the incoming deuteron energy, and thus the arguments which suggested orbital angular momentum conservation in the resonant deuteron scattering would apply also to this (d,p) reaction.

At the 2.502-Mev resonance we found the assignment

[9] E. S. Shire and R. D. Edge, Phil. Mag. **46**, 640 (1955).

[10] W. Haeberli, Phys. Rev. **99**, 640(A) (1955).

TABLE II. Ratios of partial widths to total widths, compound-nucleus intensity factors, and $(\Gamma_p/\Gamma)^2$.[a]

Reaction	E_{inc} (lab) (Mev)	E_{ex} in N^{14} (Mev)	Γ_d/Γ	Γ_{p1}/Γ	Γ_{p0}/Γ	$(\Gamma_d\Gamma_{p1}/\Gamma^2)$ $\times 10^2$	$(\Gamma_d\Gamma_{p0}/\Gamma^2)$ $\times 10^2$	$(\Gamma_{p1}/\Gamma)^2$ $\times 10^2$	$(\Gamma_{p0}/\Gamma)^2$ $\times 10^2$
$C^{12}+d$	2.502	12.41	0.55	0.04	0.01 (0.007)[b]	2.25	0.55	0.16	0.01
	2.735	12.60	0.63	0.4	0.06	25	3.8	16	0.36
$B^{10}+\alpha$	1.51	12.69	0.065	0.012	0.045	0.078	0.29	0.01	0.20
	1.64	12.78	0.14	0.006	0.013	0.09	0.18	0.004	0.01

[a] E_{inc} (incident energy) $=E_\alpha$ for $B^{10}+\alpha$; $=E_d$ for $C^{12}+d$. p_0—proton group to ground state of C^{13}. p_1—proton group to first excited state of C^{13}.
[b] Obtained from $B^{10}+\alpha$ data and data on elastic scattering of deuterons from C^{12}.

$J=4^-$, $l=3$ from an analysis of the deuteron scattering. For $l'=3$ protons we expect the resonant effects near 140° (c.m.) and also near 90° (c.m.) to be small, because the amplitude of this orbital angular momentum component is zero at these angles. We see that the data (Fig. 8 of I) have this behavior. At the 2.735-Mev resonance we found the assignment $J=3^+$ and $l=2$. For $l'=2$ protons we expect reasonably large resonant amplitudes near 90° (c.m.), larger effects there than at any angles except those well beyond 125° (c.m.). This is observed in the data. That is, the 90° (c.m.) (d,p) data show the resonance more strongly than any other angle except the two largest angles, 141° (c.m.) and 169° (c.m.).

We would like now to discuss the resonant magnitudes in more detail. To do this, we shall need the formulas of the appendix and a notation for partial widths:

Γ_d=partial width for formation by $C^{12}+d$,
Γ_{p0}=partial width for formation by C^{13} (ground state) $+p$,
Γ_{p1}=partial width for formation by C^{13*} (first excited state)$+p$,
Γ_α=partial width for formation by B^{10} (ground state) $+\alpha$,
σ_{p0}=differential cross section for $C^{12}(d,p)C^{13}$ (ground state),
σ_{p1}=differential cross section for $C^{12}(d,p)C^{13*}$ (3.09-Mev state).

The stripping amplitudes indicated in the appendix are not explicitly written down here, but may be obtained from the paper of Thomas[4] or that of Tobocman and Kalos.[2] With the aid of the formulas and the resonant parameters determined in the elastic scattering analysis, we are able to fix the proton partial level widths for the two prominent resonances of I.

Shire *et al.*[6] have covered the same excitation region of N^{14} in the $B^{10}+\alpha$ reactions as we have in the $C^{12}+d$. They have analyzed their data to obtain N^{14} level parameters for levels near the two we analyzed. The partial widths for four N^{14} levels, the two we analyzed and two analyzed by Shire *et al.*, are tabulated in Table II. We have also tabulated the relevant factor $\Gamma_d\Gamma_p/\Gamma^2$ for compound nucleus formation in the (d,p) reaction.

In the analysis of the elastic scattering, no effort was made to obtain a unique set of potential phases which would insure a fit to the cross sections at all angles, and it is not clear that there is a unique set. Consequently all of the partial level widths in Table II are uncertain by about 10 to 15% of the quoted values. In addition, Γ_{p1}/Γ at the 2.502-Mev resonance was obtained from data at angles such that the "off-resonant" cross section was comparable to the cross section on resonance. These angles were 56°58′ and 122°28′ (see Fig. 8 of I). Since the nature of the interference between the "off resonant" amplitude and the resonant amplitude is not known, the value quoted in Table II (0.04) could be too large by as much as a factor of 2. This value was extracted assuming that the interference mentioned was negligible.

The relative sizes of the Γ_d/Γ from $B^{10}+\alpha$ analysis and those from $C^{12}+d$ analysis explains why the former two resonances appear so weakly in the $C^{12}(d,d)C^{12}$ reaction (at $E_d=2.954$ and 2.986 Mev, Fig. 10 of I). The compound-nucleus factors for the (d,p) reactions also explain the fact that the two higher-energy resonances are not even identified in the (d,p) reactions.

A quantitative comparison can be made between the partial widths obtained for the two levels analyzed by us from the $C^{12}+d$ data and the data of Shire *et al.* for the same two levels, although the $B^{10}+\alpha$ data have not been analyzed at those levels. We have noted earlier that at these two resonances the spins and parities are such that the spin and orbital angular momentum dependences of the $C^{12}(d,d)C^{12}$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev level) are the same. For the same reasons the spin and orbital angular momentum dependences of the $B^{10}(\alpha,d)C^{12}$ and $B^{10}(\alpha,p)C^{13*}$ must also be the same. Consequently the ratio of these two $B^{10}+\alpha$ cross sections at 90° (c.m.) gives us directly the ratio of their "compound-nucleus intensity factors." For $B^{10}(\alpha,d)C^{12}$, the factor is $\Gamma_\alpha\Gamma_d/\Gamma^2$, and for $B^{10}(\alpha,p)C^{13*}$ the factor is $\Gamma_\alpha\Gamma_{p1}/\Gamma^2$. These ratios are $\Gamma_{p1}/\Gamma_d=0.082$ at the 2.502-Mev resonance and $\Gamma_{p1}/\Gamma_d=0.92$ at the 2.735-Mev resonance. From the $C^{12}+d$ partial widths in Table II, extracted assuming compound nucleus formation, we find $\Gamma_{p1}/\Gamma_d=0.073$ at 2.502 Mev and $\Gamma_{p1}/\Gamma_d=0.64$ at 2.735 Mev. Since our ratios are uncertain by as much as 20 to 30%, and since there is some uncertainty in the $B^{10}+\alpha$ data,[6] these two sets of ratios are in good agreement. We may make one additional test of our compound-nucleus assumption for the (d,p) resonances. We may obtain a value for Γ_{p0}/Γ at the 2.502-Mev resonance from the data of Shire *et al.* for $B^{10}(\alpha,p)C^{13}$ (yield at 90° and angular distribution) and $B^{10}(\alpha,d)C^{12}$ [yield at 90° and angular distribution if we normalize his $B^{10}(\alpha,d)C^{12}$ yield to our Γ_d/Γ (Table II)]. In this way we obtain $\Gamma_{p0}/\Gamma=0.007$ without referring to the $C^{12}(d,p)C^{13}$ data. This value agrees well with our value of $\Gamma_{p0}/\Gamma=0.01$ obtained from the $C^{12}(d,p)C^{13}$ data.

The agreement between our resonant parameters for $C^{12}+d$ and the data of Shire for the $B^{10}+\alpha$ reactions suggests that the same mechanism (compound nucleus formation) is the dominant one for both sets of resonances. We should like to turn now to the (d,p) cross sections "away" from resonances. The experimental angular distributions show significant deviations from the Butler stripping calculations at large angles. (See Figs. 4 and 5.) In fact, the $C^{12}(d,p)C^{13}$ (ground state data is about 30 times the Butler cross section at large angles. If these large-angle deviations from stripping are caused by compound nucleus formation, we might be able to relate the cross sections to the $B^{10}+\alpha$ reaction cross sections. To do this, we must select the $C^{12}(d,p)C^{13}$ (ground state) distribution and the $C^{13}(d,p)C^{13*}$ (3.09-Mev level) distribution at or quite near the same energy. Such distributions near $E_d=2.88$ Mev are found in Figs. 5 and 11 of I. The ratio of the large-angle cross sections of these distributions $(\sigma_{p1}/\sigma_{p0})$ has a minimum of $\sim$1.9 at 135° and a maximum of $\sim$3.2 at 100°. We can also obtain this ratio $(\sigma_{p1}/\sigma_{p0})$ from the $B^{10}+\alpha$ data, for E_α equal to such an energy that the compound-nucleus (N^{14}) excitation corresponds to $E_d=2.88$ Mev.

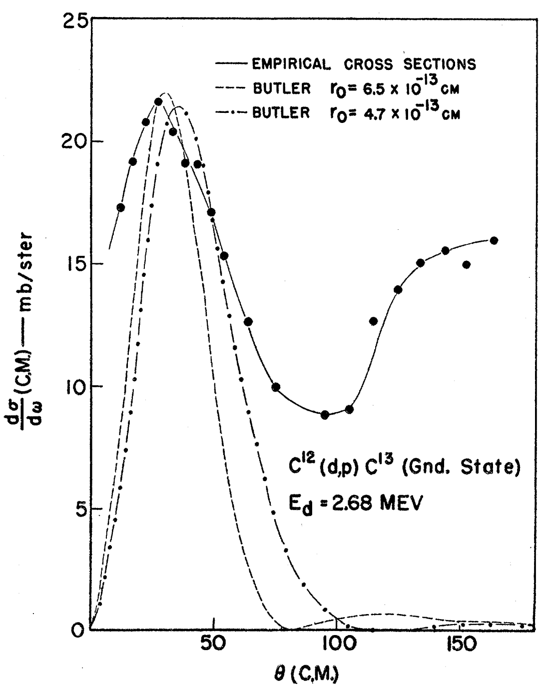

FIG. 4. $C^{12}(d,p)C^{13}$ (ground state) data at $E_d=2.68$ Mev. The solid points (solid curve) represent the data. r_0=nuclear radius. The measurements were actually made at $E_d=2.656$ Mev, but the cross sections are flat enough so that this is also a representation of the distribution at $E_d=2.68$ Mev.

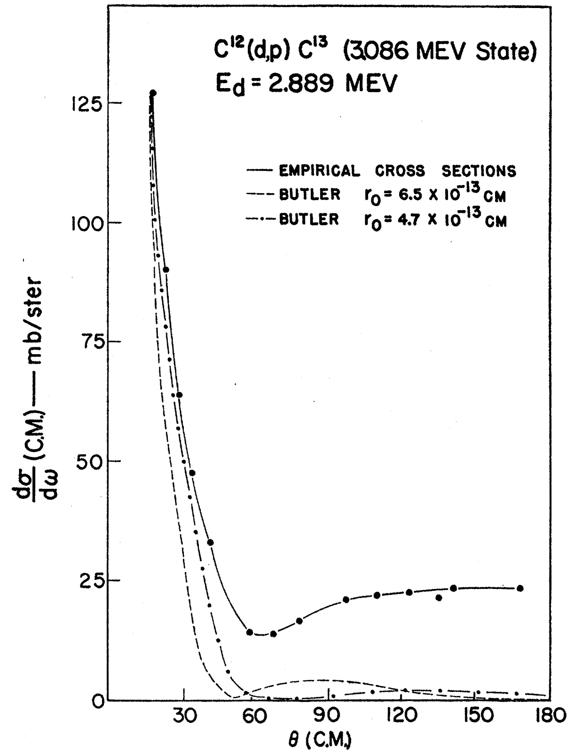

FIG. 5. $C^{12}(d,p)C^{13*}$ (3.086-Mev level) data at $E_d=2.889$ Mev. The solid points (solid curve) represent the data. r_0=nuclear radius.

If we interpolate between (α,p) angular distribution data of Shire *et al.*[6] at $E_d=1.51$ Mev and 1.64 Mev and make use of their 90° excitation curves, we can estimate σ_{p1}/σ_{p0}. From the (α,p) data for the two reactions $B^{10}(\alpha,p)C^{13}$ (ground state) and $B^{10}(\alpha,p)C^{13*}$ (3.09-Mev level), we find $\sigma_{p1}/\sigma_{p0}\cong 2.6\pm 0.5$, and we note that the ratio is quite insensitive to angle. Thus, between resonances, the (d,p) data at large angles behave in a manner consistent with the presence of compound-nucleus amplitudes.

Tobocman and Kalos[2] have suggested that certain (d,p) resonances may be interpreted as distortions of the stripping amplitude, and that the resonance occurs in the scattering of the stripped protons by the residual nucleus. Their calculations show that the (d,p) angular distributions are most seriously distorted by the resonance near the angles which correspond to the stripping peaks of the undistorted stripping theory. We see that the (d,p) data of I do not behave in this manner near the two prominent resonances. At the 2.735-Mev resonance, for example, the largest effect is at very large angles, where the stripping amplitude would be expected to be small. The resonant distortions expected at large angles would be proportional to $(\Gamma_p/\Gamma)^2$ if they were caused by the scattering of the stripped protons. These factors are tabulated in Table II. On this basis, we would not have seen the $E_d=2.502$ Mev resonance in the ground-state

(d,p) reaction, and the resonance expected at $E_d=2.954$ Mev should have been as prominent as the 2.735-Mev resonance. In $C^{12}(d,p)C^{13*}$ (3.09 Mev), the 2.735-Mev resonance should have been 100 times as important as the 2.502-Mev resonance, which would mean that we should not have seen the lower energy resonance in this reaction either. Since these expectations are all inconsistent with the data of I, we do not think that our resonances could be explained in this manner. To explain the relative magnitudes of the resonant effect in the (d,p) data of I, it seems necessary that they be proportional to $\Gamma_d\Gamma_p$.

ANGULAR DISTRIBUTION ANALYSIS AND C^{13} REDUCED WIDTHS

The stripping cross section has been shown to be proportional to the reduced nucleon width of the residual nucleus.[11] The calculations of Tobocman and Kalos[2] have shown that the extraction of these widths from the measured angular distributions can result in large errors if the relatively simple formalism of Butler is the basis for the extraction. Recently Bowcock[7] has introduced a method for extracting the widths which would tend to remove some of the uncertainty noted by Tobocman and Kalos. Bowcock's method is an approximation which is based on the assumption that only low-angular-momentum components of the reaction amplitude are appreciably distorted, either by nuclear interaction between the outgoing proton and the residual nucleus or by compound nucleus formation. In order to make use of this assumption, it is necessary to obtain the reaction amplitude from the experimental cross sections. This is not simple, since in general the amplitude will be complex, and the cross section will be the square of its magnitude. Bowcock has developed an approximate method for extracting the real part of the reaction amplitude from the data. He then expands this amplitude and the Butler amplitude (which is real) in terms of the angular momenta of the outgoing protons. Instead of comparing the total extracted amplitude with the total Butler amplitude, only the higher angular momentum components are compared with each other.

For deuteron energies not well above the Coulomb barrier, the Coulomb distortion of both deuteron and proton wave functions will also be important. The most useful calculations of this effect for light nuclei are the approximate calculations of Yoccoz.[12] He finds that the principal part of the effect is a reduction in magnitude of cross sections at all angles, with only a slight distortion of the angular distribution. Yoccoz writes an expression for the reduction factor f: $f_d=\rho(\eta_d,\theta)[\eta_d e^{-\pi\eta_d}/\sinh(\pi\eta_d)]$, where $\eta=Zze^2/\hbar v$ and the dependence on θ represents the distortion of the distribution. Neglecting the θ dependence, ρ may be expanded as a power series in η^2. We find empirically that if we set $\rho=\eta^2$, the factor f correctly extrapolates from one of Yoccoz' numerical calculations ($\eta\approx0.7$) to another ($\eta\approx1$). Since our η's were within the range defined by the η's of Yoccoz calculations, we have interpolated between his calculated reduction factors for $Be^9(d,p)$ and $Be^9(d,n)$. We used f with $\rho=\eta^2$ for the interpolation. In addition there is the assumption that, aside from the difference in η, the Coulomb effects for the $Be^9(d,p)$ reaction will be the same as those for $C^{12}(d,p)$.

We have used Bowcock's methods in analyzing the $C^{12}(d,p)C^{13}$ (ground state), $C^{12}(d,p)C^{13*}$ (3.09-Mev level) and $C^{12}(d,n)N^{13}$ (ground state) angular distributions. Figure 5 shows the data and two Butler distributions for the $C^{12}(d,p)C^{13*}$ reaction. It is clear that the distribution for the smaller radius provides the better fit. The (approximate) real part of the reaction amplitude $e_R(\theta)$ and the Butler amplitude $b(\theta)$ normalized to the data at forward angles are shown in Fig. 6. The expansion of $e_R(\theta)$ in terms of the outgoing proton angular momenta (l') is

$$e_R=-0.17P_0+7.67P_1+3.25P_2+0.35P_3+0.95P_4+0.39P_5+0.26P_6+0.30P_7,$$

and the expansion for $b(\theta)$ is

$$b=0.80P_0+4.16P_1+3.88P_2+2.06P_3+1.08P_4+0.55P_5+0.39P_6+0.45P_7,$$

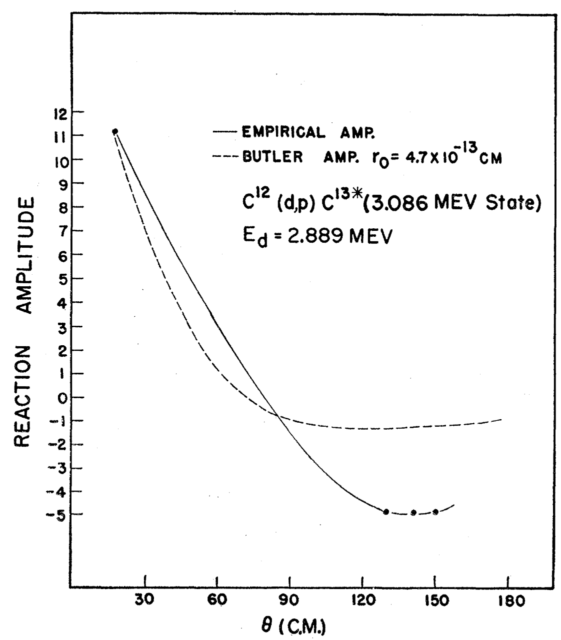

FIG. 6. Approximate real part of the reaction amplitude [in unit of $(\text{mb/sterad})^{\frac{1}{2}}$] and Butler amplitude of Fig. 5, normalized to the data at the smallest angle. The solid points represent the data extractions and the solid curve has been drawn to join three points smoothly.

[11] R. Huby, *Progress in Nuclear Physics* (Butterworths-Springer, London, 1953), Vol. 3, p. 177; F. L. Friedman and W. Tobocman, see reference 2; S. Yoshida, Progr. Theoret. Phys. (Japan) **10**, 1, 370 (1953).

[12] J. Yoccoz, Proc. Phys. Soc. (London) **A67**, 813 (1954).

where the $P_i \equiv P_i(\cos\theta)$ are Legendre polynomials. Bowcock suggests that a criterion for the validity of his assumptions and approximations is the constancy of the ratio of coefficients of corresponding terms in the two expansions beyond some minimum value of l'. We see that for $l' \leqslant 3$, the coefficients are certainly not in constant ratio, but for $l' \geqslant 4$, the ratio is reasonably constant. The average of the ratios for $l' \geqslant 4$ is $\bar{b}/\bar{e}_R = 1.38$. Thus, if we extract the reduced width of the C^{13} ground state from the total Butler angular distribution normalized to the total experimental cross section at the Butler peak, then this width must be divided by $(1.38)^2 = 1.9$ to allow for the high angular-momentum comparison.

Figure 4 contains the angular distribution to the ground state of C^{13} as well as two Butler curves for the radii shown. Here, the larger radius seems to fit best. This radius is also the one used by Holmgren *et al.*[13] to fit their data. It would seem surprising that the ground state of C^{13} should require a much larger radius than the first excited state, especially since the relative cross sections to the two states indicate that the neutron width is much larger for the excited state. Benenson *et al.*[14] found that the $C^{12}(d,n)N^{13}$ (ground state) reaction is well fitted with a radius of 4.7×10^{-13} cm. Many stripping reactions in the light nuclei have been fitted with radii of 5×10^{-13} cm or less, including this one [$C^{12}(d,p)C^{13}$ (ground state)] at a bombarding energy $E_d = 8$ Mev.[15] In order to determine what could cause a need for a larger radius at $E_d = 2.5$ Mev, a proton angular momentum expansion was carried out for both Butler curves of Fig. 4. For $r_0 = 6.5\times 10^{-13}$ cm, we obtain

$$b = 0.75P_0 + 2.78P_1 + 1.70P_2 - 0.32P_3 - 0.99P_4 - 0.88P_5 - 0.65P_6 - 0.52P_7.$$

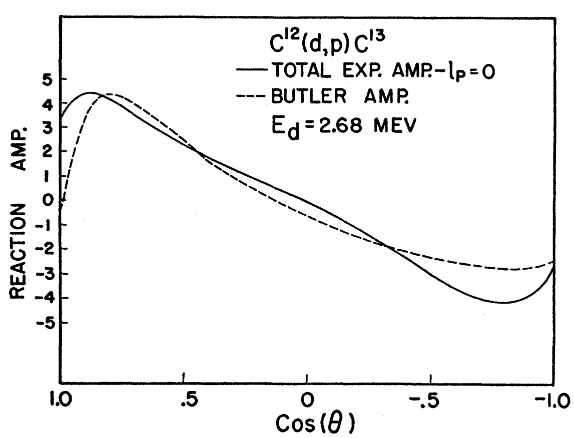

FIG. 7. $C^{12}(d,p)C^{13}$ (ground state) amplitudes. The solid curve is the approximate real part of the reaction amplitude minus the $l'=0$ term. The dashed curve is the Butler amplitude minus the $l'=0$ term.

[13] Holmgren, Blair, Simmons, Stratton, and Stuart, Phys. Rev. **95**, 1544 (1954).
[14] Benenson, Jones, and McEllistrem, Phys. Rev. **101**, 308 (1956).
[15] J. Rotblat, Nature **167**, 1027 (1951).

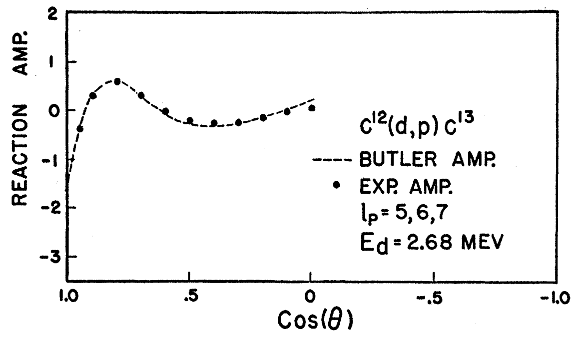

FIG. 8. Amplitudes of Fig. 7 with all $l' \leq 4$ terms subtracted from both, and Butler remainder renormalized to experimental amplitude remainder. The experimental amplitude remainder is plotted as solid points.

For $r_0 = 4.7\times 10^{-13}$ cm, we obtain

$$b = 1.49P_0 + 2.85P_1 - 0.88P_2 - 0.88P_3 - 0.90P_4 - 0.77P_5 - 0.58P_6 - 0.45P_7.$$

The principle difference between the two curves is in the components for $l' \leqslant 3$. These are just the ones expected to be strongly affected by compound-nucleus amplitudes and nuclear distortion of the incoming- and outgoing-particle wave functions. We note also that Butler curves for $r_0 = 4.7\times 10^{-13}$ cm fit both reactions which prefer this radius [$C^{12}(d,n)N^{13}$ and $C^{12}(d,p)C^{13*}$] better than any Butler curve fits $C^{12}(d,p)C^{13}$. For the reasons outlined we have assumed that the correct radius for all three reactions is $r_0 = 4.7\times 10^{-13}$ cm. The proton momentum expansion yields, for the distribution of Fig. 4,

$$e_R = 0.36P_0 + 5.43P_1 + 0.23P_2 - 0.55P_3 + 0.36P_4 - 0.96P_5 - 0.17P_6 - 0.505P_7.$$

Bowcock has mentioned that if the cross section at large angles is large compared to the Butler cross section, as is often the case for $l_n \neq 0$ (l_n = angular momentum of the captured neutrons), the individual expansion terms may not agree well, even for large l'. He then adopts the alternative procedure of subtracting terms for small l' until the remainder curves agree well with one another. This procedure was adopted here. Figure 7 shows the remainder curves (renormalized to one another at the peak) after the $l'=0$ terms have been subtracted from both expansions. Good agreement was not obtained until all of the terms for $l' \leqslant 4$ had been subtracted from both curves. Figure 8 shows this final agreement at forward angles. We require agreement at only forward angles, since we expected that most of the cross section at large angles was not stripping. A similar analysis was carried out for the $C^{12}(d,p)C^{13}$ angular distribution at $E_d = 3.26$ Mev.

The two $C^{12}(d,n)N^{13}$ angular distributions of Benenson *et al.*[15] were measured at $E_d = 2.68$ Mev and $E_d = 3.26$ Mev. Figure 9 shows the (approximate) real part of the reaction amplitude extracted from their data and also the Butler amplitude, normalized to their data at the

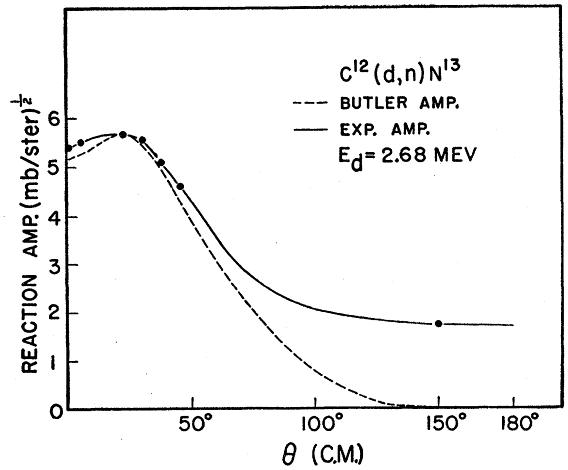

FIG. 9. $C^{12}(d,n)N^{13}$ (ground state) amplitudes. The solid curve is the approximate real part of the reaction amplitude and the dashed curve is the Butler amplitude. The Butler amplitude is normalized to the data at the stripping peak. The "point" at 150° is not a measurement, but an extrapolation from the trend of the data below 90°.

Butler peak. The two curves are brought into good agreement (Fig. 10) by subtracting a fixed amount of the $l'=0$ term from the experimental data and renormalizing the Butler curve to the remainder of the data. The angular distribution at $E_d=3.26$ Mev is treated in the same way, except at this energy it is necessary to adjust both the $l'=0$ and $l'=1$ terms to obtain agreement between the Butler curve and the remainder of the data. Table III contains the results of the angular distribution analysis, the reduced widths in Mev-cm, the fraction they represent of the Wigner limit, the Coulomb factors ($C=1/f$), and the normalization factor (n) resulting from the application of Bowcock's method. The factor n is the ratio of the width which would have resulted from a direct comparison of the data to Butler's theory to the width as modified by Bowcock's analysis. The Coulomb factors (f) and the

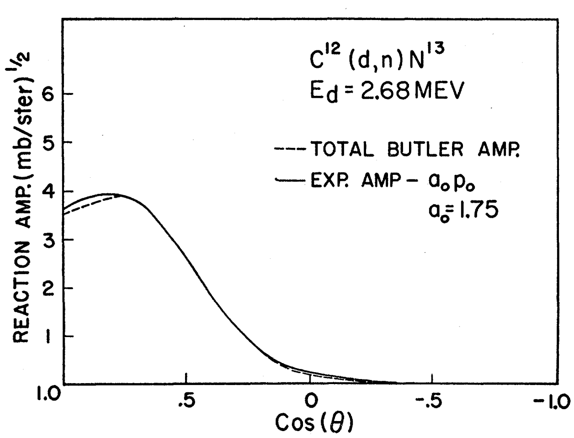

FIG. 10. Experimental amplitude of Fig. 9 with a fixed amount of the $l'=0$ term subtracted out, and the renormalized total Butler amplitude.

angular momentum analysis factors (n) partially cancel each other. A value for both the C^{13} and N^{13} ground state reduced widths of $(0.09\pm0.03)\times\frac{3}{2}(\hbar^2/\mu a)$ would include all of the four ground state measurements. In addition, the average of the two reduced width measurements for each nucleus yields the same value, (0.09) $\times\frac{3}{2}(\hbar^2/\mu a)$. As we can see from the table, the procedures for obtaining the individual width measurements yield results which are uncertain by approximately a factor of two. The measurements of the ground state widths do not indicate a difference between the mirror nuclei C^{13} and N^{13}, and are therefore in agreement with the assumption of charge symmetry. Our measurement of the ground state reduced width is in excellent agreement with the value obtained by Bowcock[7] from the $C^{12}(d,p)C^{13}$ angular distribution of Rotblat *et al.*[15] at $E_d=8.0$ Mev. Bowcock quoted $(0.09)\times\frac{3}{2}(\hbar^2/\mu a)$ for this width.

The value obtained here for the C^{13*} (3.09-Mev level) reduced width is $(0.3)\times\frac{3}{2}(\hbar^2/\mu a)$. This may be compared with the value $(0.53)\times\frac{3}{2}(\hbar^2/\mu a)$ obtained by Jackson and Galonsky[16] for the mirror level in N^{13}. They extracted this width from an analysis of the scattering of

TABLE III. Reduced nucleon widths of residual nuclei as inferred from a Bowcock-type analysis of stripping. C = Coulomb factors; n is the ratio of width from simple stripping theory to that from Bowcock's prescription.

Residual nucleus	Deuteron energy (Mev)	C	n	$\gamma^2\times10^{13}$ (Mev-cm)	$\gamma^2\left(\frac{3}{2}\frac{\hbar^2}{\mu a}\right)^{-1}$
C^{13}	2.68	3.58	1.28	1.77	0.12
	3.26	2.98	1.58	0.77	0.05
	8.0			1.56	0.09
N^{13}	2.68	3.58	2.08	0.96	0.07
	3.26	2.98	1.0	1.6	0.11
C^{13*} (3.09 Mev)	2.89	10.4	1.9	3.9	0.3

protons by C^{12}. Within the accuracy of our analysis, these two results are not in disagreement. This is especially so since the Coulomb factor is very large in this case (10.4) and obtained by an approximate method.

NOTES ON FURTHER ANALYSIS

We have analyzed two resonances in the $C^{12}(d,p)$ reaction. For this analysis we have relied upon the magnitude and energy dependence of the cross sections at a few angles. We have also shown that the angular dependence of each resonance is in rough agreement with our analysis. For the $C^{12}(d,p)C^{13*}$ (3.09-Mev level), additional information from the angular dependence of the resonances can be easily obtained. Since this reaction proceeds with $l_n=0$, it can be shown that a stripping amplitude can only enter the terms F and G of the formula in the appendix. Thus the importance of interference between stripping and compound-nucleus (resonant) amplitudes can be estimated. We are especially interested in data at two angles approximately equi-

[16] H. L. Jackson and A. I. Galonsky, Phys. Rev. 89, 370 (1953).

distant from 90°. At such angles, if the interference between the resonant and stripping amplitudes does not change much, the size and shape of the resonance ought to be quite similar. The data of I (Fig. 8 of I) at 56°58′ and at 122°28′ provide a test of this suggestion.

At the 2.502-Mev resonance, $l'=3$, and the resonant amplitudes of F and G contain the factor $P_3(\cos\theta)$. Thus the amplitudes change sign from one angle to the other, but retain the same magnitudes. Figure 6 shows the Butler amplitude for this reaction, and we see that it also changes sign from one angle to the other, but retains approximately the same magnitude. Consequently we might expect that the interference at the two angles would be similar. The data show that the shape and size of the resonance at the two angles is quite the same, although the "off-resonant" cross section changes from one angle to the other by the factor 1.5.

At 2.735 Mev, the resonant $l'=2$ and the amplitude is proportional to $P_2(\cos\theta)$. This amplitude has the same sign at angles equidistant from 90°. Thus the relative phase of the stripping and resonant amplitudes might change by something like 180° from 57° to 122.5°, with quite large changes in their interference. The data at this energy (see Fig. 8 of I) show a resonance size which changes by a factor of three for these symmetric angles. However, $P_2(\cos\theta)=0$ at 125.2° and 54.8°, and is not very large at our angles. Thus little of the resonance can result from the terms F and G, which alone contain stripping amplitudes. We have made detailed calculations to see if the interference can be great enough so that changes in it could be used to interpret the data.

The resonance parameters needed for the calculation have been determined in our analysis of the $C^{12}(d,d)C^{12}$ scattering and the analysis of $C^{12}(d,p)C^{13*}$ at the back angle (168.7°). We have found these parameters to be consistent with the $B^{10}+\alpha$ data of Shire *et al.*[6] We therefore regard the resonant amplitude at both angles as a known quantity. The calculations show that stripping amplitudes of such size are required that the "off-resonance" cross sections would be 4.7 times as large as they are. We conclude that it is not possible to interpret the data entirely in terms of a single compound nuclear resonant amplitude plus a stripping amplitude. A possible explanation for the data would be the presence of compound-nucleus amplitudes other than the single resonant amplitude. They would result from the presence of resonances observed in the data of I "near" this resonance. This suggestion would be consistent with our earlier observation that the ratio of the two (d,p) reaction cross sections "away" from the analyzed resonance was that to be expected from compound-nucleus formation. We had "predicted" the ratio from the data of Shire *et al.*

SUMMARY

The analysis of the data of I has yielded spins, parities, and partial level widths for two levels in N^{14}. With the assumption of a nuclear radius, we have also obtained N^{14} reduced widths. The resonances analyzed have been shown to be consistent with the assumption of compound-nucleus resonant amplitudes in the (d,p) reactions. An approximate analysis of the (d,p) angular distributions has yielded reduced widths for N^{13} and C^{13} ground states and the 3.09-Mev level in C^{13}. Our values are all consistent with the assumption of charge symmetry in nuclear forces.

ACKNOWLEDGMENTS

The author gratefully acknowledges the advice and encouragement of Professor H. T. Richards, who suggested this problem and advised the author throughout the investigation.

APPENDIX

We wish to express the differential reaction cross sections for the reactions $C^{12}(d,p)C^{13}$ (ground state) and $C^{12}(d,p)C^{13*}$ (3.086-Mev level). The differential scattering cross section for $C^{12}(d,d)C^{12}$ has already been expressed in reference 8. Certain quantities contained in both reaction formulas are defined:

$A_{s,s'}{}^{j'}$; $B_{s,s'}{}^{j'}$ are stripping vectors for given outgoing channel spin (j') and states of incoming and outgoing polarizations (s,s'). See Thomas (reference 4).

$U_{l,l'}{}^{J}$ are components of the collision matrix and are obtained from the one-level approximation of the dispersion formalism.

$$\rho_l = \sum_{s=1}^{l} \text{arc}\tan(\eta/s) = \tfrac{1}{2}\alpha_l.$$

α_l is that of reference 8.[17]

$\tan(\varphi^l) = -(F_l/G_l)_a$ for the incident particle. $\varphi^{l'}$ is the corresponding phase for the reaction product.

All other quantities are defined in reference 8.

In general, the reaction cross sections contain contributions from outgoing channel spins of 0 and 1. For the first excited state (d,p) reaction, only resonances whose $J=l$ can contribute to the channel of spin 0. Therefore, since $J=l+1$ for both resonances analyzed in this work, only the channel of spin 1 is important in the (d,p) to the first excited state. This limitation is of importance, because it means that the spin dependencies of the $B^{10}(\alpha,d)C^{12}$ and $B^{10}(\alpha,p)C^{13*}$ (3.086-Mev level) are the same.

For the (d,p) cross section to the 3.086-Mev level, we require the formula for incident spins of 0^+, 1^+, and outgoing spins of $\frac{1}{2}^+$, $\frac{1}{2}^+$. For the outgoing channel spin

[17] The α_l are erroneously defined in reference 8, but the numerical values of them used in the analysis of the He^4+d scattering were obtained from the correct expression given above.

of 1 (i.e., $1/2^{+}+1/2^{+}=1^{+}$) the reaction cross section in terms of the collision matrix is formally identical to the scattering formula[8] with the Rutherford amplitude (R) excluded. In addition there is the outgoing channel spin 0 contribution. The entire cross-section formula is written below.

$$3k^2(d\sigma/d\omega)=\tfrac{1}{8}|F|^2+\tfrac{1}{4}|G|^2+\tfrac{1}{4}\sin^2(\theta)\{|H|^2+|I|^2\}+\tfrac{1}{8}\sin^4(\theta)|J|^2+\sin^2(\theta)|K|^2.$$

K is the contribution of outgoing channel spin $=0$.

$$F=\sum_l e^{i\rho_l}\{e^{i\rho_l}P_l[(l+2)U_{l,l}{}^{l+1}+(2l+1)U_{l,l}{}^{l}+(l-1)U_{l,l}{}^{l-1}]-e^{i\rho_{l+2}}P_{l+2}[(l+1)(l+2)]^{\frac{1}{2}}U_{l,l+2}{}^{l+1}-e^{i\rho_{l-2}}P_{l-2}[l(l-1)]^{\frac{1}{2}}U_{l,l-2}{}^{l-1}\}+B_{1,1}{}^{1},$$

$$G=\sum_l e^{i\rho_l}\{e^{i\rho_l}P_l[(l+1)U_{l,l}{}^{l+1}+lU_{l,l}{}^{l-1}]+e^{i\rho_{l+2}}P_{l+2}[(l+1)(l+2)]^{\frac{1}{2}}\times U_{l,l+2}{}^{l+1}+e^{i\rho_{l-2}}P_{l-2}[l(l-1)]^{\frac{1}{2}}U_{l,l-2}{}^{l-1}\}+B_{0,0}{}^{1},$$

$$H=\sum_l e^{i\rho_l}\left\{\frac{e^{i\rho_l}P_l'}{l(l+1)}[l(l+2)U_{l,l}{}^{l+1}-(2l+1)U_{l,l}{}^{l}-(l^2-1)U_{l,l}{}^{l-1}]+e^{i\rho_{l+2}}P_{l+2}'\left[\frac{(l+1)}{(l+2)}\right]^{\frac{1}{2}}U_{l,l+2}{}^{l+1}-e^{i\rho_{l-2}}P_{l-2}'\left[\frac{l}{(l-1)}\right]^{\frac{1}{2}}U_{l,l-2}{}^{l-1}\right\},$$

$$I=\sum_l e^{i\rho_l}\left\{e^{i\rho_l}P_l'[U_{l,l}{}^{l+1}-U_{l,l}{}^{l-1}]-e^{i\rho_{l+2}}P_{l+2}'\left[\frac{(l+1)}{(l+2)}\right]^{\frac{1}{2}}U_{l,l+2}{}^{l+1}+e^{i\rho_{l-2}}P_{l-2}'\left[\frac{l}{(l-1)}\right]^{\frac{1}{2}}U_{l,l-2}{}^{l-1}\right\},$$

$$J=\sum_l e^{i\rho_l}\left\{\frac{e^{i\rho_l}P_l''}{l(l+1)}[lU_{l,l}{}^{l+1}-(2l+1)U_{ll}{}^{l}+(l+1)U_{l,l}{}^{l-1}]-\frac{e^{i\rho_{l+2}}P_{l+2}''}{[(l+1)(l+2)]^{\frac{1}{2}}}U_{l,l+2}{}^{l+1}-\frac{e^{i\rho_{l-2}}P_{l-2}''}{[l(l-1)]^{\frac{1}{2}}}U_{l,l-2}{}^{l-1}\right\},$$

$$K=\sum_l e^{2i\rho_l}\frac{(2l+1)}{[l(l+1)]^{\frac{1}{2}}}U_{l,l}{}^{l}P_l',$$

where

$$U_{l,l'}{}^{J}=\frac{\pm[\Gamma_d{}^{l}\Gamma_{p1}{}^{l'}]^{\frac{1}{2}}}{\Gamma}2i\sin\beta^J\exp(i\beta^J)\exp[i(\varphi_d{}^{l}+\varphi_{p1}{}^{l'})].$$

For the (d,p) cross section to the ground state, we need the formula for incident spins of 0^+ and 1^+, and outgoing spins of $\frac{1}{2}^+$ and $\frac{1}{2}^-$:

$$3k^2(d\sigma/d\omega)=\tfrac{1}{8}|L|^2+\sin^2(\theta)\{\tfrac{1}{2}|M|^2+\tfrac{1}{4}|N|^2\}+\tfrac{1}{8}\sin^4(\theta)|O|^2+\tfrac{1}{2}|P|^2+\tfrac{1}{2}|Q|^2\sin^2(\theta).$$

P and Q are contributions of outgoing channel spin $=0$.

$$L=\sum_l e^{i\rho_l}\{e^{i\rho_{l+1}}[[l(2l+1)]^{\frac{1}{2}}P_{l+1}U_{l,l+1}{}^{l}+[(l+2)(2l+3)]^{\frac{1}{2}}P_{l+1}U_{l,l+1}{}^{l+1}]-e^{i\rho_{l-1}}[[(l+1)(2l+1)]^{\frac{1}{2}}P_{l-1}U_{l,l-1}{}^{l}+[(l-1)(2l+1)]^{\frac{1}{2}}P_{l-1}U_{l,l-1}{}^{l-1}]\}+A_{1,1}{}^{1},$$

$$M=\sum_l e^{i\rho_l}\left\{e^{i\rho_{l+1}}\left[\frac{[l(2l+1)]^{\frac{1}{2}}}{l+1}P_{l+1}'U_{l,l+1}{}^{l}+\frac{[2l+3]^{\frac{1}{2}}}{(l+1)[l+2]^{\frac{1}{2}}}P_{l+1}'U_{l,l+1}{}^{l+1}\right]+\left[\frac{[(l+1)(2l+1)]^{\frac{1}{2}}}{l}P_{l-1}'U_{l,l-1}{}^{l}-\left[\frac{(2l-1)}{(l-1)}\right]^{\frac{1}{2}}\frac{P_{l-1}'}{l}U_{l,l-1}{}^{l-1}\right]\right\}+A_{1,0}{}^{1},$$

$$N=\sum_l e^{i\rho_l}\left\{e^{i\rho_{l+1}}\left[\frac{(2l+3)}{(l+2)}\right]^{\frac{1}{2}}P_{l+1}'U_{l,l+1}{}^{l+1}+\left[\frac{(2l-1)}{(l-1)}\right]^{\frac{1}{2}}P_{l-1}'U_{l,l-1}{}^{l-1}e^{i\rho_{l-1}}\right\}+A_{0,1}{}^{1},$$

$$O=\sum_l e^{i\rho_l}\left\{e^{i\rho_{l+1}}\left[\left[\frac{(2l+1)}{l}\right]^{\frac{1}{2}}\frac{P_{l+1}''}{l+1}U_{l,l+1}{}^{l}-\left[\frac{(2l+3)}{l+2}\right]^{\frac{1}{2}}\frac{P_{l+1}''}{l+1}U_{l,l+1}{}^{l+1}\right]-e^{i\rho_{l-1}}\left[\left[\frac{(2l+1)}{l+1}\right]^{\frac{1}{2}}\frac{P_{l-1}''}{l}U_{l,l-1}{}^{l}-\left[\frac{(2l-1)}{(l-1)}\right]^{\frac{1}{2}}\frac{P_{l-1}''}{l}U_{l,l-1}{}^{l-1}\right]\right\},$$

$$P=\sum_l e^{i\rho_l}\{e^{i\rho_{l+1}}[(2l+3)(l+1)]^{\frac{1}{2}}P_{l+1}U_{l,\,l+1}{}^{l+1}+e^{i\rho_{l-1}}[(2l-1)l]^{\frac{1}{2}}P_{l-1}U_{l,\,l-1}{}^{l-1}\},$$

$$Q=\sum_l e^{i\rho_l}\left\{e^{i\rho_{l+1}}\left[\frac{(2l+3)}{(l+1)}\right]^{\frac{1}{2}}P_{l+1}'U_{l,\,l+1}{}^{l+1}-e^{i\rho_{l-1}}\left[\frac{(2l-1)}{l}\right]^{\frac{1}{2}}P_{l-1}'U_{l,\,l-1}{}^{l-1}\right\}+A_{1,\,0}{}^{0},$$

where

$$U_{l,\,l'}{}^{J}=\frac{\pm[\Gamma_d{}^{l}\Gamma_{p0}{}^{l'}]^{\frac{1}{2}}}{\Gamma}2i\sin\beta^J\exp(i\beta^J)\exp[i(\varphi_d{}^{l}+\varphi_{p0}{}^{l'})].$$