Neutrons from the $p+d$ Breakup Reaction*

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The high-energy end of the energy spectrum of neutrons from the $p+d$ reaction at incident energies of about 10 Mev is examined. It is found that the structure in the spectrum measured by Nakada and co-workers can be attributed to the final-state interaction of the two protons.

INTRODUCTION

THE disintegration of deuterons by medium-energy ($\sim$10 Mev) neutrons or protons has been examined theoretically by several authors. Bransden and Burhop[1] made an extensive calculation of the process. They quote results only for the total cross sections, and their values are substantially larger than the experimentally measured values. Frank and Gammel[2] have approached the problem from a rather elementary point of view, using zero-range potentials and the impulse approximation. Their calculations of the total cross section and angular distributions of the products agree rather well with the experimental data available to them, although it is not entirely clear why their approach agrees with experiment as well as it does.

Recently Nakada and co-workers[3] have made further experimental studies of the $p+d$ reaction. Using time-of-flight techniques, they have measured the energy distribution of the neutrons coming off at various angles to the incident proton beam. A comparison of their results in the forward direction with some of the numerical results given by Frank and Gammel[2] (F-G) shows qualitative agreement with a substantial part of the neutron energy spectra. However, the F-G theory, as it stands, does not yield the peak that is experimentally observed[3] at the upper end of the neutron energy spectrum (see Fig. 1). This peak can be readily attributed to the final-state interaction of the two protons, which was neglected in the calculations of Frank and Gammel. In the present paper, the effect of this final-state interaction is evaluated, using the F-G theory. As will be seen, this particular aspect of the disintegration process is not properly accounted for in the F-G approximation.

Accordingly, the high-energy peak is examined somewhat further by means of another simple approximation. One can conclude that the position and shape of this peak depend predominantly on the final-state interaction of the two protons, but that an evaluation of the magnitude of the peak requires a detailed analysis of the scattering process.

FRANK AND GAMMEL APPROXIMATION

The calculation of Frank and Gammel proceeds by setting up the problem in Born approximation, making the assumption of zero-range potentials, and replacing a certain integral by an experimentally measured quantity. The F-G formulation takes into account the final-state interaction of the neutron with a proton. For the emission of high-energy neutrons, we assume instead that we can neglect the effect of the final-state n-p interaction and consider only the final-state p-p interaction. Letting the incident proton be represented as particle No. 1, and the deuteron's proton and neutron as particles No. 2 and No. 3, respectively, we can write the matrix element for the process as

$$M=(\chi_f(3)\phi_f(1,2),\ [V_{np}(1,3)+V_{pp}(1,2)]\chi_i(1)\phi_i(2,3)), \quad (1)$$

where $\phi_i(2,3)$ specifies the initial deuteron, $\phi_f(1,2)$ specifies the final state of the two protons, and χ specifies a plane wave. Because of the low relative energy of the two protons, we need to consider their s-state motion only. It will be important, though, to include the effect of the Coulomb potential on this state. The spin sums implicit in the above expression can be readily evaluated, and one obtains for $|M|^2$ the

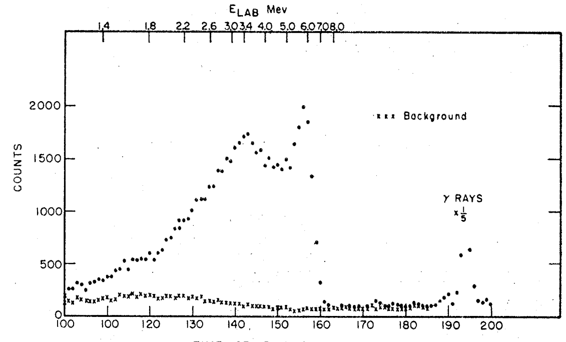

FIG. 1. Experimental 0° time-of-flight spectrum of neutrons from bombardment of deuterium with 8.9-Mev protons.

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[3] Anderson, Gardner, McClure, Nakada, and Wong, University of California Radiation Laboratory Report UCRL–5075, December, 1957; M. P. Nakada *et al.*, Phys. Rev. **110**, 594 (1958).

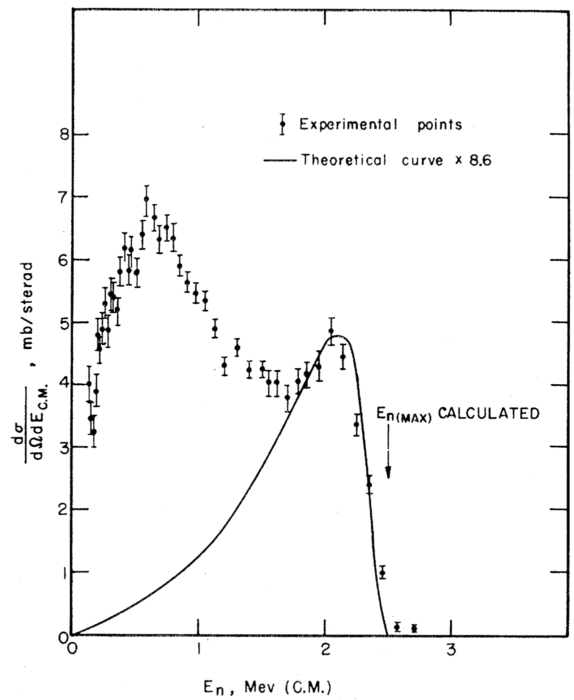

FIG. 2. 0° neutron breakup spectrum plotted on a c.m. energy scale and compared with a peak (smooth curve) calculated from the Frank and Gammel theory. The experimental points have not been corrected for time resolution. The theoretical curve has been multiplied by a normalizing factor of 8.6 to show that the shape is given correctly even though the magnitude is not.

expression

$$|M|^2=\tfrac{1}{16}|({}^1V_{np}{}^+ + {}^3V_{np}{}^+)I_{np}+2\,{}^1V_{pp}{}^+ I_{pp}|^2, \tag{2}$$

where

$$I_{np}=(\chi_f(3)\phi_f(1,2)|U(1,3)|\chi_i(1)\phi_i(2,3)), \tag{3}$$

$$I_{pp}=(\chi_f(3)\phi_f(1,2)|U(1,2)|\chi_i(1)\phi_i(2,3)). \tag{4}$$

We have taken $V(r)=VU(r)$, and have neglected the interactions in the odd-parity states, since with the assumption of very short-range potentials, they will not contribute. The zero-range potential used by F-G can be represented by $U(\mathbf{r})=\delta(|r|)/|r|$. With this potential, the form of the final-state wave function of the two protons is needed only at the origin. For the p-p system the wave function for small r is[4]

$$\phi_K(r)=\frac{e^{-i\delta_K}}{Kr}\left[CKr\cos\delta_K+\frac{1}{C}\sin\delta_K\right], \tag{5}$$

where C is the Coulomb penetration factor and δ_K is the s-wave p-p scattering phase shift. In the Born approximation for the scattering of two nucleons, the zero-range potential yields a value of zero (although there does exist a solution of the Schrödinger equation for this potential). For this reason I_{np} [Eq. (3)] is zero. However, I_{pp} [Eq. (4)] is finite because of the singularity in $\phi_K(r)$. To calculate I_{pp}, we take the coordinate system

$$\mathbf{r}=\mathbf{r}_3-\mathbf{r}_2,\quad \mathbf{x}=\mathbf{r}_1-\mathbf{r}_2.$$

Then

$$I_{pp}=\int e^{-i\mathbf{K}'\cdot(\mathbf{r}-\frac{1}{2}\mathbf{x})}\phi_{K''}{}^*(\mathbf{x})U(\mathbf{x})\phi_d(\mathbf{r})e^{i\mathbf{K}_0\cdot(\mathbf{x}-\frac{1}{2}\mathbf{r})}d\mathbf{r}d\mathbf{x}$$

$$=\int e^{i(\frac{1}{2}\mathbf{K}'+\mathbf{K}_0)\cdot\mathbf{x}}\phi_{K''}{}^*(\mathbf{x})U(\mathbf{x})d\mathbf{x}\int e^{-i(\mathbf{K}'+\frac{1}{2}\mathbf{K}_0)\cdot\mathbf{r}}\phi_d(\mathbf{r})d\mathbf{r}$$

$$=\frac{4\pi\sin\delta_{K''}e^{i\delta_{K''}}}{K''C}\int e^{-i(\mathbf{K}'+\frac{1}{2}\mathbf{K}_0)\cdot\mathbf{r}}\phi_d(\mathbf{r})d\mathbf{r}, \tag{6}$$

where the kinematical equations and definitions of the wave vectors are given in reference 2. The integral which appears here can, of course, be readily evaluated. However, instead of a direct evaluation of this integral, Frank and Gammel[2] write down the expressions for p-d elastic scattering which contains a similar integral, and then evaluate the integral in terms of the experimentally measured elastic scattering cross section. When this procedure is extended to the calculation of I_{pp}, we obtain finally

$$\frac{d\sigma_{\mathrm{in}}}{d\Omega' dE'}=\frac{1}{3}\frac{d\sigma_{\mathrm{el}}}{d\Omega'}\left(\frac{{}^1V_{pp}{}^+}{{}^3V_{np}{}^+}\right)^2\frac{2\pi}{\alpha}\frac{\sin^2\delta_{K''}}{C^2}\frac{M}{(2\pi\hbar)^2}\frac{K'}{K_0K''}, \tag{7}$$

where the elastic scattering cross section is to be evaluated at the angle corresponding to the same momentum transfer as for the inelastic scattering problem. When evaluated in this manner, the F-G theory gives a peak in the neutron energy spectrum of the right shape and position, as shown in Fig. 2. However, the magnitude of the peak is low by an order of magnitude. (Note that the theoretical curve in Fig. 2 has been normalized.) This is not surprising, since physically one would expect the high-energy neutrons to come from the incident proton striking the neutron and knocking it in the forward direction, which is the I_{np} contribution to Eq. (2). However, this particular contribution vanishes in the F-G theory, because of their treatment of the zero-range potential. Accordingly, in the next section, we estimate the contribution of I_{np} in a rather simple manner. This term does then make a larger contribution to the production of high-energy neutrons than the I_{pp} term.

FURTHER APPROXIMATION

In order to obtain a nonvanishing contribution from I_{np} in Eq. (2), we adopted a potential of the form $U(\mathbf{r})=\delta(\mathbf{r})$. With the coordinate system

$$\mathbf{r}=\mathbf{r}_2-\mathbf{r}_3,\quad \mathbf{x}=\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3),$$

[4] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. 22, 77 (1950).

the integral I_{np} becomes

$$I_{np}=\int e^{-i\mathbf{K}'\cdot(\frac{3}{4}\mathbf{r}-\frac{1}{2}\mathbf{x})}\phi_{K''}{}^*(\mathbf{x}+\tfrac{1}{2}\mathbf{r})\times U(\mathbf{x}-\tfrac{1}{2}\mathbf{r})\phi_d(\mathbf{r})e^{i\mathbf{K}_0\cdot\mathbf{x}}d\mathbf{r}d\mathbf{x} \quad (8)$$
$$=\int e^{-\frac{1}{2}i(\mathbf{K}'-\mathbf{K}_0)\cdot\mathbf{r}}\phi_{K''}{}^*(\mathbf{r})\phi_d(\mathbf{r})d\mathbf{r}.$$

We now need to know $\phi_{K''}(r)$ not just at the origin, but in a region $0\leq r\leq$ radius of the deuteron. Outside of the region of nuclear forces, ϕ_K has the form[4]

$$\phi_K(r)=F_K(r)\cos\delta+G_K(r)\sin\delta.$$

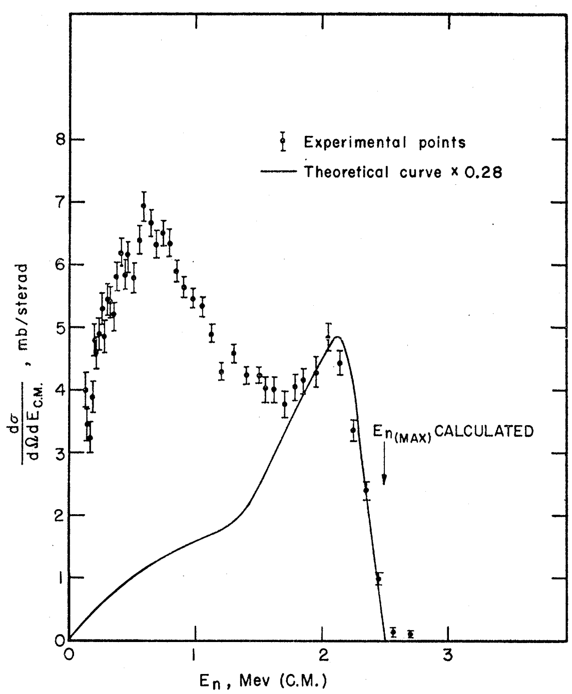

FIG. 3. 0° neutron breakup spectrum plotted on a c.m. energy scale and compared with a peak calculated from the potential $V(\mathbf{r})=V\delta(\mathbf{r})$. The theoretical peak has been multiplied by a normalizing factor of 0.28.

We take

$$F=C\left[\sin Kr+\frac{Kr^2}{2R}\cos\left(\frac{\sqrt{2}}{3}Kr\right)\right],$$
$$G=\frac{1}{C}\left[\cos Kr+\frac{\sin Kr}{Kr}+\frac{r}{R}\left(\ln\frac{r}{R}+2\gamma+h(\eta)-1\right)\right], \quad (9)$$

where $R=e^2/mc^2$, $\eta=e^2/\hbar v$, and $h(\eta)$ is defined in reference 4. This representation is reasonably good over the required range of integration. We have used this form down to $r=0$, although it should be modified inside the range of nuclear forces. Calculations showed that the modification is not important for these considerations. It turns out that the first terms in the F-G ex-

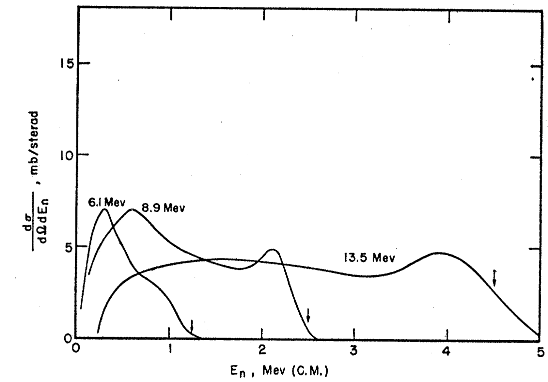

FIG. 4. 0° experimental neutron spectra at several bombarding energies, plotted in c.m. and uncorrected for time resolution. The arrows indicate calculated maximum energies for the spectra. The data are those of Nakada and co-workers.[3]

pansions are dominant, so that the approximations made in the second terms are sufficiently accurate for our purposes. The integrals that one obtains from Eqs. (8) and (9) can be solved analytically, but the results are lengthy and it seems hardly worthwhile to quote them. The potentials ${}^1V_{np}{}^+$ and ${}^3V_{np}{}^+$ were chosen by calculating the n-p scattering cross section, using $U(r)=\delta(\mathbf{r})$, in the Born approximation, at the same incident energy, and choosing the potentials to give the experimental n-p cross section.

The integral I_{pp} in Eq. (2) was also evaluated:

$$I_{pp}=\int e^{-i\mathbf{K}'\cdot(\frac{3}{4}\mathbf{r}-\frac{1}{2}\mathbf{x})}\phi_{K''}(\mathbf{x}+\tfrac{1}{2}\mathbf{r})\times U(\mathbf{x}+\tfrac{1}{2}\mathbf{r})\phi_d(\mathbf{r})e^{i\mathbf{K}_0\cdot\mathbf{x}}d\mathbf{r}d\mathbf{x}. \quad (10)$$

Since the potential $U(\mathbf{r})=\delta(\mathbf{r})$ would cause a divergence in I_{pp}, we used a square-well potential[4]:

$$V(r)=-13.2 \text{ Mev},\quad r<2.63\times10^{-13} \text{ cm}$$
$$V(r)=0,\quad r>2.63\times10^{-13} \text{ cm}.$$

The Coulomb contribution to $V(r)$ was calculated to be

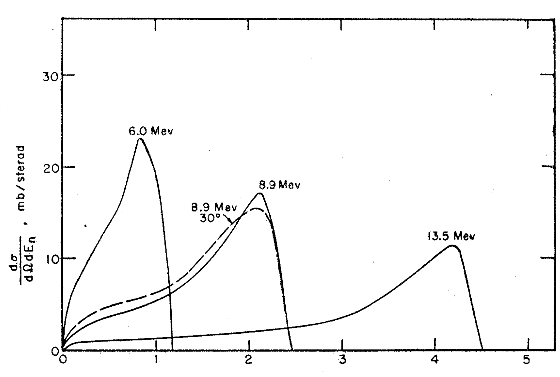

FIG. 5. 0° neutron peaks calculated in Born approximation, which are to be compared with the experimental results shown in Fig. 4. A 30° curve (c.m.) at 8.9 Mev is also shown.

small for our purposes and was neglected. Putting $\mathbf{z}=\mathbf{x}+\frac{1}{2}\mathbf{r}$, we obtained

$$I_{pp}=(2\pi)^{\frac{3}{2}}\phi_d(\tfrac{1}{2}\mathbf{K}_0+\mathbf{K}')\int\phi_{K''}(z)\times V(z)e^{i(\mathbf{K}_0+\frac{1}{2}\mathbf{K}')\cdot\mathbf{z}}d\mathbf{z}. \quad (11)$$

Upon evaluation, the contribution to M from I_{pp} turned out to be about 15% of the contribution from I_{np} at 9-Mev bombarding energy. This result is then in agreement with the notion that the main contribution to the high-energy neutron scattering results from the n-p interaction.

Figure 3 gives a comparison of the calculations of this section with experiment at 8.9 Mev. The calculated values are too high, although the discrepancy is not as large as with the F-G model. Figures 4 and 5 illustrate the progressively poorer fits to the magnitude of the peak as the bombarding energy is decreased.

As with the previous calculation, the shape and position of the high-energy peak is given quite accurately. One can conclude from this that the shape and position of this part of the neutron energy spectrum is primarily dependent on the final-state interaction of the two protons—particularly the Coulomb interaction. If there were no Coulomb interaction, as would be the case for the (n,d) reaction, then this peak would occur practically at the maximum-energy position and have a very narrow width. It is the Coulomb interaction which causes the shift in position to roughly 0.5 Mev below the maximum-energy position and increases the width.

It is apparent that as the incident energy is decreased, it will be a less and less good approximation to consider separate energy regions in which either the n-p or p-p final-state interaction is dominant. Rather, both will be effective in distorting the final-state wave function, which, of course, becomes difficult to treat. This is a consequence of the fact that when the velocity of the incoming nucleon becomes equal to, and less (at about 7 Mev) than, the internal velocities of the deuteron, there is a polarization of the deuteron, which must be taken into account in order to obtain any quantitative agreement with experiment. In the method of F-G, these effects seem to be partially taken into account by their use of the impulse approximation. This accounts for the fair agreement of Frank and Gammel's calculation with a substantial part of the experimental results.

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Analysis of $C^{12}+d$ Reactions*

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The $C^{12}(d,d)C^{12}$ scattering is analyzed at two levels in N^{14}, one at 12.42-Mev excitation and the other at 12.60 Mev. The lower level is assigned as 4^- and the upper as 3^+. A nuclear radius is assumed and reduced widths are obtained for the two levels. With the aid of the reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev level), partial level widths for the three modes of decay of N^{14*} are obtained. Both (d,p) reactions are shown to be consistent with the presence of stripping amplitudes and resonant compound nuclear amplitudes and also consistent with the $B^{10}+\alpha$ data of Shire *et al.* for the same excitation region in N^{14}. The (d,p) angular distributions are analyzed to obtain the reduced neutron widths of the ground and first excited states of C^{13}.

An appendix gives an explicit partial-wave expansion of the differential cross section for reactions involving incident spins of 0 and 1 and outgoing spins of $\frac{1}{2}$ and $\frac{1}{2}$.

INTRODUCTION

THE differential cross sections for the $C^{12}(d,d)C^{12}$, $C^{12}(d,p)C^{13}$ (ground state), and $C^{12}(d,p)C^{13*}$ (first excited state) reactions have been measured and published.[1] The present paper gives a partial analysis of those measurements.

Recent attempts to obtain a more complete treatment of the (d,p) and (d,n) reactions than has been provided by the Butler approximation have introduced the Coulomb interaction, the nuclear interaction of incident deuteron with the target and the interaction of liberated particles with the residual nucleus (references contained in I). The formulations of Tobocman,[2] Grant,[3] and also that of Thomas[4] include all of these modifications. The

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