Correlations and the Nuclear Magnetic Moment*

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(Received February 18, 1958)

Correlation structure can be introduced into the shell model by means of a product of pair correlation functions, each of which vanishes within the repulsive core radius and then approaches unity with a short range dependent only on the Fermi momentum and the relative energy of the pair. Correlations of this type are independent of the state of a particule within a given shell-model orbit. The additional effect of such scalar, state-independent correlations on the magnetic moment of a "closed shell plus one" nucleus is shown to vanish, leaving only the shell-model value. The proof can be extended to correlations having a scalar spin dependence and to certain more complicated symmetries. The effect of residual state-dependent correlations due to terms in the relative momentum of the correlated pair and to the space-exchange part of the attraction as it appears in the correlations is estimated for O^{17} as a correction to the magnetic moment of 0.002 magneton. There is no modification to the magnetic moment operator due to velocity dependence of the correlation functions, and the expectation value of the ordinary space exchange operator coming from the exchange part of the Hamiltonian is shown to be the same as given by the shell model.

I. INTRODUCTION

THE basis of the independent-particle model, or shell model,[1] of the nucleus has been much solidified by recent advances in the theory of many-particle systems.[2] These studies have improved our understanding of nuclear matter in bulk, particularly in regard to its saturation energy and to its correlation structure. There are good reasons to believe that this correlation structure can be carried over to finite nuclei, in which case it becomes possible to examine the effect of the correlations in finite nuclei on the expectation value of certain dynamical operators.

The shell model is particularly suited to accounting for nuclear properties associated with the symmetries of the wave functions, and has thus been specially successful near closed shells where states of different symmetry are most easily classified and separated. In the independent-particle model in its widest sense, the separation is often brought about through the effects of a two-body interaction between degenerate or nearly degenerate states. This mixing and separating of close configurations of particles outside closed shells will introduce long-range, state-dependent correlations among these particles, and it is well known that correlations of this type are extremely important in determining nuclear properties. Breuckner, Eden, and Francis[3] have studied the introduction of these correlations into the independent-particle system from the point of view of the actual nuclear Hamiltonian. However, there are other correlations in the nucleus which arise when the highly singular two-body interactions are "turned on." These correlations affect strongly all the particles in the nucleus and are much more important in determining the total energy of the system than are the relatively weak correlations among particles outside closed shells. The success of the shell model leads one to expect that there are large classes of predictions unaffected by these correlations, but until now there has only been this pragmatic argument. We shall discuss the effect of the correlations on certain dynamical operators, in particular the magnetic moment, in an attempt to give some theoretical justification to the success.

It is convenient to separate the effect of the short-range correlations affecting all the particles and the long-range, state-dependent correlations introduced by configuration mixing among particles outside closed shells. Since we are interested only in the former, we shall consider the case of a closed shell plus one particle and examine the effect of the correlations of the odd particle with the particles in the core on the magnetic moment. Since the magnetic moment of nuclei with only one particle outside an *L-S* closed shell is well given by the shell model (e.g., O^{17}),[4] and since for more than one particle outside an *L-S* closed shell the deviations from the Schmidt lines can be accounted for in terms of configuration mixing among the particles outside closed shells,[5] we expect to find the effect of the correlations to be small, and this indeed is the case.

The nuclear two-body interaction is extremely singular, consisting of an infinite repulsive core and a short-range, very strong attraction.[6] In order that the total energy of the nucleus be finite with this type of interaction, the total wave function must vanish whenever any internucleon coordinate is within a core radius. Thus the first type of correlation that must be introduced into the shell model consists in the making of

* Supported in part by The National Science Foundation.

[1] A full account of the shell model and a complete list of references are given by J. P. Elliott and A. M. Lane, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1957), Vol. 39.

[2] A comprehensive list of references is given by K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).

[3] Brueckner, Eden, and Francis, Phys. Rev. **99**, 76 (1955).

[4] R. J. Blin-Stoyle, Revs. Modern Phys. **28**, 75 (1956).

[5] R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954); A. Arima and H. Horie, Progr. Theoret. Phys. Japan **11**, 509 (1954).

[6] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957); J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 1337 (1957).

"holes" in the independent-particle wave functions any time two nucleons get closer together than a certain distance. In an infinite nucleus the exclusion principle prevents a nucleon from being excited into a nearby state, except at the top of the Fermi sea. Thus the correlation of two distant particles, which correlation would correspond to very small energy excitation, is forbidden, and the wave function quickly approaches its free-particle uncorrelated value as the two particles separate. The presence of the attractive part of the two-body potential has very little further effect on the correlations, although of course it is very important for determining the binding energy. The range and structure of the correlation is then almost entirely determined by the core radius, the Fermi momentum, and the relative energy of the two particles, and this range turns out to be very small.[7]

It is expected that the structure of the correlations in finite nuclei will be essentially the same as in the infinite case. Firstly the energy gap for excitations due to the Pauli principle is still present in finite nuclei and is in fact accentuated by the separation of the levels. The gap is not present for the degenerate states of particles in unfilled shells, but we are not concerned with that case here. Thus the correlations will still have a very short range. As is pointed out by Brueckner, Gammel, and Weitzner,[8] if the correlation distance is small, then from the point of view of the independent-particle model, the states being admixed to produce this correlation must be highly excited states; in fact excitation energies of order 150 to 250 Mev are typical. The error introduced by using plane-wave states for the particle functions at such high excitations is certainly small, and thus the admixtures introduced in the finite nucleus by these correlations are well approximated by taking the correlation structure of the infinite case. Alternately one can say that as long as the density fluctuations over the correlation distance are negligible, then the correlation structure of the infinite system can be taken over to the finite nucleus.

The correlations to be introduced into the independent-particle system, then, depend largely on the core radius, the Fermi momentum, and the relative energy of the particles being correlated. The first two of these are essentially state-independent factors, that is, the same for all particles in all states. The relative energy of the particles has a weak effect on the range of the correlation. Neglecting spin-orbit splittings in the core, the energy difference between particles depends on the principal quantum number and on the total orbital angular momentum of the particle orbits and is independent of the z component of angular momentum. Thus the bulk of correlations among particles is independent of state within each orbit, as we shall see in a detailed discussion of the correlation function in Sec. II. There are additional correlations arising from momentum dependences and from the attractive part of the potential that are state-dependent, but these are much smaller than the state-independent correlations.

In Sec. III we shall investigate the effect of state-independent correlations on the magnetic moment of a "closed shell plus one" nucleus. There we shall show that so long as the correlation function is a scalar in ordinary space, and state-independent, then there is no additional contribution to the magnetic moment due to correlations. The proof can be extended, under more restrictive conditions, to correlation functions that have scalar as well as second rank tensors for their orbital parts, but of course are still scalars in total space. The proofs do not depend on the correlations being short-range, or being only two-particle correlations. The result may seem surprising since it is certain that the correlations introduce many different excited states into the system, but when taking an expectation value the additional contribution from these excitations cancels leaving only the shell-model value. It might be emphasized here that this does not mean that the effect of the correlations is small, or that the overlap of the shell-model wave function with the actual correlated wave function is large, but rather that the shell-model wave function has certain symmetry properties that are preserved in the presence of state-independent correlations so that the expectation value of the magnetic moment operator is unaffected by the correlations. Since the proof does not depend on the nature of the correlations except for their state independence, there are an infinite number of nuclear wave functions differing from each other only by state-independent correlations all of which have the same magnetic moment. It is clear that we could adjust the overlap of any of these with the true wave function to zero and we would still have the same magnetic moment. This serves to emphasize the *model* nature of the shell-model state, as being a state which preserves certain general symmetries but need not otherwise resemble the actual nuclear wave function.

In Sec. IV the state-dependent correlations coming from the small momentum dependence of the correlation function and from the exchange parts of the attractive potential as it appears in the correlations are considered. The effect of these is estimated for O^{17}. We find that the additional contribution to the usual moment operator is of the order of $+0.002$ nuclear magneton. This extremely small contribution comes about essentially because of the short range of the correlations. In addition there appears the well-known space-exchange moment operator that comes from the space-exchange potential in the Hamiltonian. This has essentially the same expectation value as it would have in the shell model since the major contribution to the matrix element comes from the long-range, uncorrelated

[7] A detailed discussion of the correlation structure is given by Brueckner and Gammel, reference 2. See particularly their Fig. 5 and Appendix B.

[8] Brueckner, Gammel, and Weitzner, Phys. Rev. **110**, 431 (1958).

part of the wave function. Thus the correlations introduced when the two-body interactions are "turned on" in an independent-particle system have essentially no new effect on the magnetic moment of a "closed shell plus one" nucleus.

II. STRUCTURE OF THE CORRELATION FUNCTION

Brueckner and co-workers[9] have shown that corresponding to the actual nuclear Hamiltonian

$$H=\sum_i T_i+\sum_{i<j} v_{ij}, \tag{1}$$

where T_i is the kinetic energy operator and v_{ij} is the two-body interaction including the repulsive cores, one can write a model Hamiltonian

$$H_M=\sum_i T_i+\sum_i V_i, \tag{2}$$

where V_i is a one-body potential obtained from a self-consistent average of the reaction matrix corresponding to v_{ij}. If the eigenfunctions, Ψ, of H are related to those of H_M, Φ, by the operator F,

$$\Psi=F\Phi, \tag{3}$$

where F is defined through a complicated chain of equations involving the off-diagonal parts of the reaction matrix, then the energies of the model system, calculated with appropriate factors of $\frac{1}{2}$, will correspond almost exactly to the eigenvalues of H. Since H_M is a single-particle Hamiltonian, Φ will be an independent-particle wave function, which can be constructed from determinental wave functions of single-particle states. Ψ, on the other hand, is the fully correlated, actual nuclear wave function. The correlations are introduced through F. For the purposes of calculating binding energies and the like, it is essential that F be fully known since the matrix elements of the highly singular H are very sensitive to small variations in the wave function.[2] On the other hand, an operator like the magnetic moment is not so sensitive to these variations. We make use of this to simplify greatly the form of F.

Firstly we use the fact that the most important part of the correlations occurs over a very short range and therefore, as was discussed above and is treated in much more detailed by Brueckner, Gammel, and Weitzner,[8] we can use the correlation structure of the infinite nucleus in the finite case. The correlation structure of nuclear matter has been discussed by a number of authors.[2,10] In particular Brueckner and Gammel[2] have calculated the correlation structure of nuclear matter at saturation density. They show that the shape of the wave function for the relative motion of two particles in an S state is almost entirely determined by the core, the Fermi momentum, and the relative energy and momentum of the particles. One can thus in first approximation neglect the effect of the attraction on the wave function and write the wave function for the relative motion of two particles in an S state as[11]

$$U(r)=\frac{\sin kr}{kr}-\frac{\sin kr_c\, G(r,r_c)}{kr_c\, G(r_c,r_c)},\quad r>r_c$$
$$U(r)=0,\qquad r<r_c$$

where r_c is the core radius, k the magnitude of the relative momentum wave number for the particles, and r is the relative coordinate. $G(r,r')$ is the Green's function for the propagation of particles in the self-consistent average potential with the effect of the exclusion principle in forbidding excitations to filled states included. $U(r)$ can be written

$$U(r)=\left[1-\frac{r\sin kr_c\, G(r,r_c)}{r_c\sin kr\, G(r_c,r_c)}\right]\frac{\sin kr}{kr},\quad r>r_c$$
$$U(r)=0,\qquad r<r_c.$$

Since $\sin kr/kr$ is the free particle wave function for an S state, this defines a correlation function, B, for the two particles:

$$B(r)=1-\frac{r\sin kr_c\, G(r,r_c)}{r_c\sin kr\, G(r_c,r_c)},\quad r>r_c$$
$$B(r)=0,\qquad r<r_c. \tag{4}$$

This correlation function has the expected features. It vanishes if any two particles get within a core radius of each other, and since $G(r,r_c)$ is a short-range function that goes to zero for $r\gg r_c$, the particles become uncorrelated when they are far apart. Since such a form for the correlations is to be assumed for all pairs of particles, we obtain the total correlation function by approximating to the operator F with the product over all pairs of nucleons of B. This is an excellent approximation if we restrict our attention to nonsingular operators.

In this approximation of no attraction, the Green's function depends on the state of the two particles only through their relative energy, which dependence determines the range of the Green's function. But for the correlations between particles in given orbits in a finite nucleus this energy difference is a constant and thus so is the Green's function. We can take this dependence into account then by changing the range of the Green's function for each orbit but keeping it constant within a given orbit.

The more explicit dependence on the relative momentum, and therefore the state of the particles, occurring in the $\sin kr_c/\sin kr$ term is also easily dealt with. At nuclear saturation densities, $k_F r_c=0.7$, where

[9] See K. A. Brueckner and C. A. Levinson, Phys. Rev. **97**, 1344 (1955).

[10] H. A. Bethe and J. Goldstone, Proc. Roy. Soc. (London) **A238**, 551 (1956); Comes, Walecka, and Weisskopf, Ann. Phys. (to be published).

[11] Correlations will be most important in S states since in these states the nucleons get closest. By extending the S-state correlations to all particles, therefore, we overestimate the effect of the correlations.

k_F is the Fermi momentum. This means that even for the maximum possible relative momentum, $\sin kr_c$ is well given by kr_c. Since $G(r,r_c)$ is a short-range function of r, the expansion of $\sin kr$ is also justified and, if one keeps only the first term, B becomes

$$B=1-\frac{G(r,r_c)}{G(r_c,r_c)}, \quad r>r_c$$
$$B=0, \quad r<r_c. \tag{5}$$

This form of the correlation function has the great advantage of being state-independent, and it is only this state independence and not any detailed knowledge of the correlation structure that is needed in order to prove that the correlations have no effect on the magnetic moment. The ordinary part of the attractive potential will also produce state-independent correlations and hence give nothing new. We shall see in Sec. III that this is true as well of scalar, spin-dependent correlations. The corrections both from higher terms in k and from the space-exchange part of the attraction in the correlation structure are discussed in Sec. IV, where we see that they are extremely small in the case of the magnetic moment. We consider, then, correlations making the total wave function vanish if any pair is inside a core radius and going to one, as pairs separate, in a way dependent only on the relative energy of the correlated pair. For the proof of Sec. III any correlation function having this type of state independence would do, but of course this particular choice has strong physical justification.

III. STATE-INDEPENDENT CORRELATIONS AND THE MAGNETIC MOMENT

If the total Hamiltonian for a nucleus is given by (1), and if the two-body interactions contain no space-exchange part or other velocity dependence, then it is well known that the operator for the magnetic dipole moment can be written[4]

$$\boldsymbol{\mu}=\frac{eh}{4\pi Mc}\left(\sum_{k=1}^{A} g_l^{(k)}\mathbf{l}^{(k)}+\sum_{k=1}^{A} g_\sigma^{(k)}\boldsymbol{\sigma}^{(k)}\right), \tag{6}$$

where $\mathbf{l}^{(k)}$ and $\boldsymbol{\sigma}^{(k)}$ are the orbital angular momentum and spin operators for the kth nucleon and $g_l^{(k)}$ and $g_\sigma^{(k)}$ are the orbital and spin gyromagnetic ratios. The factor $eh/4\pi Mc$ is the nuclear magneton. It should be noted that it is the magnetic moment of the actual nucleus that concerns us. We introduce the electromagnetic field in the usual way into the actual Hamiltonian, and therefore it is the real nucleon mass that appears in the magneton and not some effective mass.[12] Upon using the transformation (3), the magnetic dipole moment of a nucleus in some state becomes

$$\langle\boldsymbol{\mu}\rangle=\frac{\langle\Psi|\boldsymbol{\mu}|\Psi\rangle}{\langle\Psi|\Psi\rangle}=\frac{\langle\Phi|F^\dagger\boldsymbol{\mu}F|\Phi\rangle}{\langle\Phi|F^\dagger F|\Phi\rangle}.$$

It might be thought that if F is velocity-dependent, there will be additional contributions to the magnetic moment operator with respect to the model state coming from the currents generated by this velocity dependence. Thus we might say that Eq. (3) is only the transformation for zero electromagnetic field, so that in the presence of a field the energy of the system, E, is not given by

$$E=\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}=\frac{\langle\Phi|F^\dagger HF|\Phi\rangle}{\langle\Phi|F^\dagger F|\Phi\rangle} \tag{7}$$

unless the field is included in F as well as in H. This is of course correct but no new parts of $\boldsymbol{\mu}$ will arise from this. The terms contributing to $\boldsymbol{\mu}$ are identified as those in the expansion of the energy which are linear in the electromagnetic field. Since $F|\Phi\rangle$ is an eigenfunction of H, the energy is stationary with respect to changes in the eigenfunction, and thus there are no terms coming from F in the expansion of the energy that are linear in the electromagnetic field. Therefore, although state-dependent correlations may change the expectation value of $\boldsymbol{\mu}$ as given in Eq. (6), (see Sec. IV), there will be no contribution to the magnetic moment operator arising from velocity-dependent correlations. This can easily be verified directly in Eq. (7). If one puts the electromagnetic field into F and then makes a small field expansion, the linear terms from the numerator will just be cancelled by those from the normalization denominator.

Upon using the well-known properties of the gyromagnetic ratios and measuring the magnetic moment in units of the nuclear magneton, $\boldsymbol{\mu}$ for diagonal expectation values can be written

$$\mu_z=\sum_{\substack{k=1\\ \text{protons}}}^{Z} l_z^{(k)}+g_p\sum_{\substack{k=1\\ \text{protons}}}^{Z}\sigma_z^{(k)}+g_n\sum_{\substack{k=1\\ \text{neutrons}}}^{N}\sigma_z^{(k)},$$

where n and p refer to neutrons and protons. Writing $\boldsymbol{\mu}$ in this form we see that our problem reduces to showing that the correlations do not change the expectation value of the z component of the proton orbital angular momentum, or the value of the z component of the proton or neutron spin angular momentum separately. Since the state for a doubly closed L-S shell plus one is an eigenstate of each of these three operators,[13] the additional contribution to the magnetic moment from the correlations will be given by the commutator of

[12] Bell, Eden, and Skyrme, Nuclear Phys. 2, 586 (1956/57); J. S. Bell, Nuclear Phys. 4, 295 (1957).

[13] We consider the case in which the odd-particle total angular momentum $j=l+\frac{1}{2}$, and as usual for the magnetic moment fix our attention on the azimuthal state for which $m_j=j$. For this case the appropriate Clebsch-Gordan coefficient is unity and the state is an eigenstate of the operators. The extension of our discussions to other couplings for the odd particle is straightforward.

the operator μ_z with F. Thus, for example, the expectation value of the z component of the proton orbital angular momentum for the case in which the model state is a closed L-S shell plus one particle is

$$\langle l_z^{(P)}\rangle=\frac{\langle\Phi|F^\dagger l_z^{(P)}F|\Phi\rangle}{\langle\Phi|F^\dagger F|\Phi\rangle}=\frac{\langle\Phi|F^\dagger F l_z^{(P)}|\Phi\rangle}{\langle\Phi|F^\dagger F|\Phi\rangle}+\frac{\langle\Phi|F^\dagger[l_z^{(P)},F]|\Phi\rangle}{\langle\Phi|F^\dagger F|\Phi\rangle}, \quad (8)$$

where we have set $l_z^{(P)}=\sum_{k,\text{ protons}} l_z^{(k)}$. The first term in Eq. (8) is the shell model value and the second is the additional contribution from the correlations. We shall call this Δ_l. There are clearly similar expressions for $\Delta_{\sigma P}$ and $\Delta_{\sigma N}$.

We consider the case in which F is approximated by a correlation function of the general type discussed in Sec. II, which function is the product of two-body, state-independent correlation functions containing no exchange. That is F is considered to be some sort of product of simple scalar "hole makers" for the nucleus. For a nucleus with a double closed shell plus one nucleon, the wave function can be split into a spin part and an orbital part, and the orbital part can be further factored into a neutron function, Ω_N, and a proton function, Ω_P. Since we are assuming that the correlations are spin-independent, the spin part of Φ in the evaluation of Δ_l will just be cancelled by the normalization and we need only consider the orbital part. Putting the normalization denominator equal to one, Δ_l can be written

$$\Delta_l=\langle\Omega_N\Omega_P|F^\dagger[l_z^{(P)},F]|\Omega_N\Omega_P\rangle. \quad (9)$$

In order to evaluate this, we expand the scalar F into a product of tensor operators for the protons and neutrons.[14] This can be one so long as the neutron and proton parts commute, and for our form of the correlations they do. Thus we write

$$F=\sum_{\lambda,\mu}(-1)^\mu P_{\lambda,\mu}N_{\lambda,-\mu},$$

where $P_{\lambda,\mu}$ is a tensor operator of rank λ with z component μ for the protons and $N_{\lambda,-\mu}$ is the similar operator for the neutrons. As was shown by Racah,[14] the tensor operators can be defined in terms of their commutation relations with the angular momentum operators; thus

$$[l_z^{(P)},F]=\sum_{\lambda,\mu}[l_z^{(P)},P_{\lambda,\mu}](-1)^\mu N_{\lambda,-\mu}=\sum_{\lambda,\mu}(-1)^\mu P_{\lambda,\mu}N_{\lambda,-\mu}\mu.$$

Substituting into Eq. (9), we get

$$\Delta_l=\sum_{\lambda,\mu,\lambda',\mu'}\langle\Omega_N|N_{\lambda,-\mu}{}^\dagger N_{\lambda',-\mu'}(-1)^{\mu+\mu'}|\Omega_N\rangle\times\langle\Omega_P|P_{\lambda,\mu}{}^\dagger P_{\lambda',\mu'}|\Omega_P\rangle\mu'. \quad (10)$$

[14] G. Racah, Phys. Rev. **62**, 438 (1942).

Since we are considering the case of a doubly closed shell plus one particle, either the neutron or proton wave functions must be spherically symmetric, that is an S state. The matrix element between S states of a tensor operator vanishes unless that operator is a scalar. A scalar can be constructed only from two tensor operators of the same rank, so the double sum in Eq. (10) reduces to a single sum with $\lambda=\lambda'$ and $\mu=\mu'$. The correlations actually depend on the orbits of the particles through the weak dependence of the Green's function on relative energy. But the condition for the combination of tensor operators between the closed shell states being a scalar is that the combination be constant as we sum over each of the $2l+1$ azimuthal quantum numbers of each filled orbit l in the core, and this is in fact satisfied. In other words, we can label the correlation function by the orbital angular momentum of the orbits, but they will be constant as we sum over the azimuthal quantum numbers in each orbit, and it is the fact that all the $2l+1$ azimuthal states for each orbit are full that makes the closed shell an S state.

The Hermitian conjugate of a tensor operator is defined by[15]

$$T_{\lambda,\mu}{}^\dagger=(-1)^{\lambda+\mu}T_{\lambda,-\mu},$$

so that Eq. (10) becomes

$$\Delta_l=\sum_{\lambda,\mu}\mu\langle\Omega_N|N_{\lambda,\mu}N_{\lambda,-\mu}|\Omega_N\rangle\langle\Omega_P|P_{\lambda,-\mu}P_{\lambda,\mu}|\Omega_P\rangle.$$

Each of the operator combinations in the matrix elements is symmetric under change of μ to $-\mu$, since the operators commute. Thus the sum is odd in μ, and since it runs from $\mu=-\lambda$ to $\mu=+\lambda$, it must vanish. Thus for this case, $\Delta_l=0$.†

If the correlations do not depend on spin at all, then clearly there can be no $\Delta_{\sigma P}$ or $\Delta_{\sigma N}$. If the correlation function is a product of a scalar in spin space and a scalar in orbit space, we can factor the wave function of a closed shell plus one into a spin part and an orbital part and the matrix element of the scalar spin correlation functions between the factorized spin wave functions will just be cancelled by the normalization and therefore the proof that $\Delta_l=0$ goes as before. The proof that for a closed shell plus one particle $\Delta_{\sigma P}$ and $\Delta_{\sigma N}$ must vanish for scalar, commuting, spin correlations is completely analogous to the one for $\Delta_l=0$. Thus if the correlations contain a $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ term making them different in singlet and triplet states, this will not affect the magnetic moment since the spin operators for protons and neutrons commute and we can carry out the tensor operator proof.

[15] A. R. Edmonds, Cern Report 55–26, 1955 (unpublished).

† For this case J. Bell, T. H. R. Skyrme, and E. J. Squires have suggested an alternative, simple proof. The choice of Hermitian conjugation properties for the tensor operators means that F is Hermitian. Thus we have

$$\Delta_l=\langle F^+[l_z^{(P)},F]\rangle=\langle F[l_z^{(P)},F]\rangle=\langle[F,l_z^{(P)}]F\rangle=\tfrac{1}{2}\langle[F,[l_z^{(P)},F]]\rangle=0.$$

For F a function of coordinates only the double commutator is clearly zero.

This concludes the demonstration that there are no additional contributions to the magnetic moment of a "closed shell plus one" nucleus from correlations so long as these are state-independent correlations, scalar in spin and orbital parts separately. It is interesting to note that since the proton (and clearly neutron) spin and orbital angular momenta are separately unaffected by the correlations, so the total spin and total orbital angular momentum must be unaffected. This is a result of interest in deuteron stripping where the orbital angular momentum of a state is "measured." Similarly, inasmuch as the nucleon operators in β decay are the same as those in the magnetic moments, the above proof applies to β-decay calculations.

The techniques used with correlations that are separately scalar in spin and in orbital parts can be extended to investigations in which this is no longer the case. We consider then an arbitrary scalar correlation function, F, which we expand in tensor operators $B_{\lambda,\mu}$ for the orbital part and $S_{\lambda,-\mu}$ for the spin part. That is we consider F to have the form

$$F=\sum_{\lambda,\mu}(-1)^{\mu}B_{\lambda,\mu}S_{\lambda,-\mu}. \tag{11}$$

We shall assume here and in all further expansions that all the parts commute. If we wish to consider Δ_l, then we need further to expand $B_{\lambda,\mu}$ into a proton part, p, and a neutron part, n. This can be done by using a Clebsch-Gordan coefficient[16]:

$$B_{\lambda,\mu}=\sum_{k,k',m}p_{k,m}n_{k',\mu-m}\langle kk'm\mu-m\,|\,kk'\lambda\mu\rangle.$$

Then, if χ is the total spin wave function of the system, we can write

$$\langle\Phi|F^\dagger[l_z{}^{(P)},F]|\Phi\rangle=\sum_{\lambda\lambda'\mu m}\sum_{kk'k''k'''}\langle\Omega_N|n_{k''m-\mu}n_{k'''\mu-m}|\Omega_N\rangle$$
$$\times\langle\chi|S_{\lambda\mu}S_{\lambda'-\mu}|\chi\rangle\langle\Omega_P|p_{k-m}p_{k'm}|\Omega_P\rangle m(-1)^{k''+k+\lambda}$$
$$\times\langle kk''\lambda\mu\,|\,kk''m\mu-m\rangle\langle k'k'''m\mu-m\,|\,k'k'''\lambda_1\mu\rangle, \tag{12}$$

where we have made use of the Hermitian conjugation properties of the tensor operators and where we have used the fact that since we are taking diagonal matrix elements, the total z component of the operators must be zero. If we assume the odd particle is a proton, then the neutron wave function is an S state and so we must put $k'''=k''$ and form a scalar of the n operators. Let us call this scalar $N_0(k'')$. The matrix element of $N_0(k'')$ is independent of the z components so it can be taken out of the z-component sum. The product of spin operator tensors can be coupled to form a sum over single tensor operators using a Clebsch-Gordan coefficient:

$$S_{\lambda\mu}S_{\lambda'-\mu}=\sum_L\langle\lambda\lambda'\mu-\mu\,|\,\lambda\lambda L0\rangle\mathcal{S}_L(\lambda\lambda'),$$

with the restriction that L must be even if $\lambda=\lambda'$. This restriction arises because if $\lambda=\lambda'$ we can interchange the operators on the left without changing their matrix elements, whereas on the right this interchange multiplies the expression by $(-1)^L$ by virtue of the conjugation properties of the Clebsch-Gordan coefficients. Similarly, we can couple the product of the proton operators:

$$p_{k-m}p_{k'm}=\sum_\kappa\langle kk'-mm\,|\,kk'\kappa0\rangle\mathcal{P}_\kappa(kk'),$$

with again the restriction that if $k=k'$, then κ must be even. Substituting these back into (12) we can do the sum over μ immediately. The sum over m can also be done if we notice that

$$m=[x(x+1)]^{\frac{1}{2}}\langle x1m0\,|\,x1xm\rangle$$

for nonzero x. One finds finally for Eq. (12)

$$\sum_{\kappa\lambda\lambda'}\sum_{kk'k''L}\langle\Omega_N|N_0(k'')|\Omega_N\rangle$$
$$\times\langle\chi|\mathcal{S}_L(\lambda\lambda')|\chi\rangle\langle\Omega_P|\mathcal{P}_\kappa(kk')|\Omega_P\rangle$$
$$\times W(Lk\kappa k;k'1)W(k\lambda k'\lambda';k''L)\langle L100\,|\,L1\kappa0\rangle, \tag{13}$$

where factors of $2\lambda+1$, phase factors, etc., have been absorbed into the definition of the matrix elements and where $W(abcd;ef)$ is the usual Racah coefficient.[14]

We wish now to investigate the conditions under which (13) will vanish. Before doing this we note one further restriction that can be imposed on the quantities. For a closed shell plus one, the total spin eigenfunction, χ, will correspond to a state of spin $\frac{1}{2}$. Since the matrix element must be a scalar, the fact that χ corresponds to spin $\frac{1}{2}$ restricts L to have the values one or zero. The Clebsch-Gordan coefficient in (13) requires that if $L=0$, $\kappa=1$. But if $L=0$ the triangle condition imposed by the Racah coefficient on the triad $(kk'L)$ will require that $k=k'$. This means that κ must be even, which conflicts with the condition that $\kappa=1$, so the contribution from $L=0$ must vanish. Of course the proof for scalar spin and scalar orbital correlations is a special case of this. The remaining possibility is $L=1$, with the restriction that $\lambda\neq\lambda'$. The triad $(\lambda\lambda'L)$, which for this case is $(\lambda\lambda'1)$, must satisfy the triangle conditions. If we assume that the nonscalar part of the spin correlations are entirely two-body correlations, then the highest-rank tensor we can form from two spin vectors is 2, and thus $\lambda,\lambda'=0,1,2$. If we exclude $\lambda,\lambda'=1$, then we cannot satisfy the triad $(\lambda\lambda'1)$ and $\lambda\neq\lambda'$ with $\lambda,\lambda'=0,2$, and hence we must get zero for Δ_l. It is clear that the particularization to the case of odd proton can easily be reversed, and Δ_l will still vanish if the odd particle is a neutron. Thus, even if the two-body correlations contain a tensor part of the same type as the ordinary nuclear tensor interaction (that is of rank 2), there will be no additional contribution to the z component of the orbital angular momentum of the protons.

By a similar procedure one can examine the effect of correlations with nonscalar orbital parts on the z

[16] The Clebsch-Gordan, or vector addition, coefficients are taken to agree in phase and normalization with those of E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1935).

component of the total orbital angular momentum of the system. In this case we find that for a closed shell plus one particle, the correlations do not change the shell-model value so long as the correlations are only scalar and tensor of rank two in the orbital part, and there is no need to make the additional stipulation that the total spin of the system is $\frac{1}{2}$.

Analogous procedures can be used to investigate the spin angular momentum. We find that the z component of the total spin angular momentum of the system is unchanged by correlations if the expansion (11) only involves $\lambda=0$, 2. The proton and neutron z components of spin angular momentum are separately unaffected if the condition of $\lambda=0$, 2 is combined with the restriction of total spin of the system of $\frac{1}{2}$.

Thus we see that the magnetic moment of a "closed shell plus one" nucleus is unaffected by correlations so long as the correlations when expanded into orbital and spin tensor operators only contain scalar and tensor of rank two parts. This restriction is not very serious. We have seen in Sec. II that the most important correlations are scalar in the orbital part and spin-independent. The introduction of a scalar spin dependence has no effect on the magnetic moment. The higher order parts if present will come in from the attractive part of the interaction, and hence will be much weaker than the correlations from the repulsive core and the Pauli principle. For these weak correlations, it is certainly a good approximation to consider only the two-body correlations, and for these only $\lambda=0$, 1, and 2 can exist. $\lambda=0$ gives nothing new. $\lambda=2$ comes from the ordinary tensor force. $\lambda=1$ would arise from some vector part of the two-body interaction like the spin-orbit force. Velocity-dependent forces like the spin-orbit force no doubt are present in the two-body interaction, but their effect on the correlations is probably quite small. In addition to making $\Delta_l\neq 0$, the spin-orbit forces would modify the magnetic moment operator, through their velocity dependence. We do not consider any of these effects here.

IV. STATE-DEPENDENT CORRELATIONS

We saw in Sec. II that one can reduce the correlation function to a form dependent only on the relative energy of the correlated pair, which form is a good approximation to the actual correlation function. In Sec. III we saw that such correlations do not affect the expectation value of the magnetic moment. We now consider corrections to this due to the state-dependent correlations, but since these are quite small and since the bulk of the correlation structure gives no effect, we can simplify considerably the correlation structure in studying these corrections.

To take into account the state dependence of the correlations introduced through the attraction and through higher order terms in the relative momentum from the term in $\sin kr_c/\sin kr$, one must have an explicit form for the correlation function. This can be obtained by following Brueckner and Gammel.[2] If one approximates the effect of the Pauli principle in allowing only transitions to states above the Fermi surface by adding some mean excitation energy to the energy denominators in the Green's function, then one finds

$$G(r,r')=-\frac{M}{8\pi\lambda rr'}\times[\exp(-\lambda|r-r'|)-\exp(-\lambda|r+r'|)], \quad (14)$$

where M is the nucleon mass, and λ is a parameter depending on the Fermi momentum and on the relative energy of the particles. In the finite nucleus it is λ, the range of the Green's function, that we take to be a constant for a given orbit. Upon using this form for the Green's function, B in Eq. (5) becomes

$$\begin{aligned} B&=1-\frac{r_c}{r}\exp[-\lambda(r-r_c)], \quad r>r_c \\ B&=0, \quad r<r_c. \end{aligned} \quad (15)$$

A plot of this function is shown in Fig. 1, where it is compared with a Gaussian correlation function of the form $[1-\exp(-\gamma r^2)]$ for all r. The parameter γ is a function of λ and r_c and is chosen to give the best correspondence to the form of Eq. (15). This Gaussian function retains the major features of Eq. (15) and is in an easily managed form.

We first consider the effect on the correlation function of the higher order terms coming from the expansion of the sine term in Eq. (4). We need only go to the next higher term in kr since $\sin kr_c$ is always well approximated by kr_c, and the short range of the Green's function suppresses the higher order contributions to $\sin kr$. Furthermore, as we shall see, all the state-dependent corrections make very small contributions to the magnetic moment and thus we need not estimate

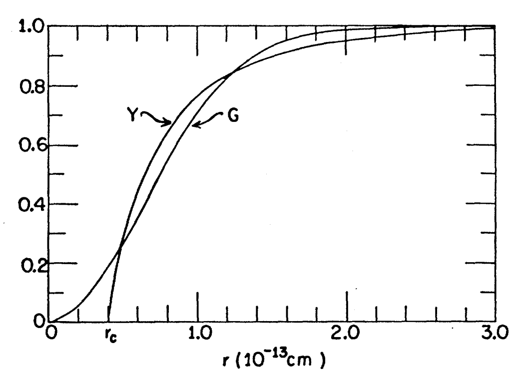

FIG. 1. The Yukawa form for the correlation function, $Y=1-(r/r_c)\exp[-\lambda(r-r_c)]$ for $r>r_c$ and $Y=0$ for $r<r_c$, compared with the Gaussian form, $G=1-\exp(-\gamma r^2)$ for all r, with the parameters $\lambda=0.87\times10^{13}$ cm^{-1}, $\gamma=1.25\times10^{26}$ cm^{-2}, and $r_c=0.4\times10^{-13}$ cm.

them with great accuracy. Expanding then, we find

$$\frac{r\sin kr_c}{r_c\sin kr}\approx\frac{1}{1-\dfrac{(rk)^2}{3!}}\approx 1+\frac{(rk)^2}{3!}=1+\frac{2[(\mathbf{r}\cdot\mathbf{k})^2]_{\text{Av}}}{3!},\quad(16)$$

where the factor of 2 comes from taking the average of the cosine squared that enters when $(rk)^2$ is replaced by $(\mathbf{r}\cdot\mathbf{k})^2=[rk\cos(\mathbf{r},\mathbf{k})]^2$. Equation (16) may be written

$$1-\frac{2}{3}\left[1-\frac{(\mathbf{r}\cdot\mathbf{k})^2}{2!}-i\mathbf{r}\cdot\mathbf{k}+i\frac{(\mathbf{r}\cdot\mathbf{k})^3}{3!}-1\right]_{\text{Av}},\quad(17)$$

as long as we consider terms linear in these corrections, since then the odd terms in $\mathbf{r}\cdot\mathbf{k}$ will be averaged out and the even term will have its cosine appropriately averaged when expectation values are taken. To third order in $\mathbf{r}\cdot\mathbf{k}$, Eq. (17) becomes

$$1+\tfrac{2}{3}-\tfrac{2}{3}\exp(-i\mathbf{r}\cdot\mathbf{k})=(5/3)-\tfrac{2}{3}P_M,$$

where use has been made of the fact that r and k both represent relative coordinates to replace $\exp(-i\mathbf{r}\cdot\mathbf{k})$ by P_M, the Majorana or space-exchange operator.[17,18] For k a c number, as it is above, this replacement only holds for plane wave states, but if $\mathbf{k}$ is an operator the replacement is an operator identity. Since it is in the spirit of our approximation to assume that the form of the correlations for infinite nuclei can be taken over to the finite case, we shall assume that the correlations can be written in the finite nucleus with the space-exchange operator expressing the momentum dependence arising from this term. Since the corrections arising in the magnetic moment from this part of the correlation will turn out to be very small, we believe that no serious error is made by this approximation. Putting the space exchange form for the momentum dependence into the correlation function, and using the Gaussian correlation form for all r discussed above, one gets

$$B=1-(5/3)\exp(-\gamma r^2)+\tfrac{2}{3}P_M\exp(-\gamma r^2).\quad(18)$$

In the limit of very short range for the Gaussian, for which limit P_M approaches one, this goes over into the exchange-independent correlation function we had previously. It should be noted that in using Eq. (18) care must be taken to consider only terms linear in P_M, since those quadratic in the P_M part are excluded by the nature of the approximation and are actually of a very much higher order.

Since the attraction has little effect on the correlation structure, it may be introduced in lowest order as a correlation proportional to the interaction itself, multiplied by the Green's function propagator. The Green's function factor is important in reducing the range of the correlation effect due to the attraction. The nonexchange part of the attraction, if introduced in this way, is state-independent and hence yields no corrections to the magnetic moment. We saw in Sec. III that this is true for the spin exchange as well since it can be written in terms of $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. Thus we need only consider the space-exchange part of the attraction.

If the space-exchange part of the attractive potential has strength U and radial dependence $f(r)P_M$, then the correlation function may be modified to take this into account by multiplying the Green's function term in B by the factor $[a+bf(r)P_M]$, where a and b are constants chosen to give the desired magnitude of correction as well as maintain the vanishing of B at the core. If we make the simplification of taking a Gaussian form for the interaction, $f(r)=\exp(-\eta r^2)$, so that $f(0)=1$, then using Eq. (18) we can write the correlation function as

$$B=1-(1-\beta)[(5/3)\exp(-\gamma r^2)-\tfrac{2}{3}P_M\exp(-\gamma r^2)]\\-\beta f(r)P_M\exp(-\gamma r^2),\quad(19)$$

where we have neglected the relative momentum dependence expressed in Eq. (18) in the term involving the attractive interaction. The effect of the attraction on the correlation is measured by β. We estimate β by putting $P_M=1$, plotting Eq. (19), and requiring that the attraction have the same relative effect on the correlations as is found by Brueckner and Gammel.[2] This gives $\beta=0.6$ with the Gaussian form for $f(r)$. This relatively large value of β may seem to belie the claim that the attraction has a small effect on the correlations, but actually it is β times the variation in $f(r)$ over the range of the Green's function that represents the effect on the correlations, and since the attraction in its Gaussian form is very slowly varying over the short range of the Green's function, this product is in fact small. We can collect Eq. (19) into the form

$$B=C+DP_M,\quad(20)$$

where $C=1-(5/3)(1-\beta)\exp(-\gamma r^2)$ and $D=[\tfrac{2}{3}(1-\beta)-\beta f(r)]\exp(-\gamma r^2)$.

The effect of these various state dependences on the magnetic moment will be entirely on the orbital part, and therefore we need only consider Δ_l. The contributions to Δ_l from such state-dependent correlations must, from symmetry considerations, involve the odd particle. The leading term will involve the correlation of the odd particle with one core particle at a time. The next term will involve three-particle correlations, etc. This is essentially an expansion in powers of the ratio of the correlation length to the interparticle spacing. For nuclear saturation densities this parameter is about $\frac{1}{2}$. Thus we can determine the order of magnitude of the corrections to the magnetic moment if we keep only the two-body correlations. This is adequate since we shall see that the corrections are quite small. In this approximation, then, we replace the product of correlation functions appearing in F by a sum over functions

[17] J. A. Wheeler, Phys. Rev. **50**, 643 (1936).
[18] R. G. Sachs, Phys. Rev. **74**, 433 (1948).

of the form in Eq. (20). The normalization change introduced by such an approximation to F is of the order of the ratio of correlation volume to total volume per particle. This is $(1/2)^3$, and therefore negligible for order-of-magnitude estimates. Thus, in this approximation to F we take $\langle\Phi|F^\dagger F|\Phi\rangle=\langle\Phi|\Phi\rangle=1$.

Since we are considering a two-body interaction with a space-exchange part, there will arise space-exchange currents from the term $Uf(r)P_M$ in the Hamiltonian, and these will modify the magnetic moment operator.[18] Upon taking these into account, the orbital part of the magnetic moment becomes

$$M_N\langle\Phi|F^\dagger l_z^{(P)}F|\Phi\rangle+U\langle\Phi|F^\dagger \boldsymbol{\varrho} F|\Phi\rangle, \qquad (21)$$

where M_N is the nuclear magneton. $\boldsymbol{\varrho}=(i\pi/hc)\sum_{j>k}\times(e_j-e_k)(\mathbf{r}_k\times\mathbf{r}_j)f(r_{jk})P_M(jk)$ is the exchange moment operator arising from the exchange potential in the Hamiltonian. As we discussed at the beginning of Sec. III, there are no exchange operators from the exchange dependence of F.

The shell-model part of the orbital magnetic moment can be removed from (21) by taking the commutator of $l_z^{(P)}$ with F in the first term. The term in U gives corrections only. In considering F to be a sum of terms of the form (20), we go only to first order in D since the matrix elements depend on a high power of the range of the correlations and D^2 has a much shorter range than D. To first order in D, and considering only two-body matrix elements so that all the operators in the matrix element refer to the same pair of nucleons, one finds

$$\begin{aligned}\Delta_l&=2M_N\langle\Phi|DP_M[l_z^{(P)},C]|\Phi\rangle+U\langle\Phi|C\boldsymbol{\varrho} C|\Phi\rangle\\&=\Delta_{ll}+U\Delta_{lU}.\end{aligned} \qquad (22)$$

Use has been made of the results of the previous section to set the expectation value of $C[l_z^{(P)},C]=0$. The terms involving D and $\boldsymbol{\varrho}$ go out since P_M anticommutes with $\mathbf{r}_j\times\mathbf{r}_k$ and $P_M^2=1$.

From symmetry considerations it is clear that one of the pair of nucleons referred to by the operators in (22) must be the odd particle. Since $\boldsymbol{\varrho}$ depends on the difference in charge between the particles, it will vanish if the core particle and the odd particle have the same charge. In other words, there is no net current set up by the exchange of two protons or two neutrons. Similarly, if both particles of the pair have the same charge, the commutator in the first term will vanish since C is a scalar. This also means that no net current will be set up by correlations between particles of the same charge. Thus in the two-body matrix elements occurring in (22) there will be no exchange terms from antisymmetry.

Since the matrix elements are diagonal, only the z component of $\boldsymbol{\varrho}$ will contribute, and one can make use of the relation

$$(\mathbf{r}_1\times\mathbf{r}_2)_z=\tfrac{4}{3}\pi i|r_1|\,|r_2|\times(Y_{1,-1}(1)Y_{1,1}(2)-Y_{1,1}(1)Y_{1,-1}(2)).$$

The phases and normalizations of the spherical harmonics, $Y_{l,m}$, are taken to agree with those of Condon and Shortley.[16] The matrix elements for a "closed shell plus one" state may now be evaluated by converting the sum over coordinates into a sum over particle states in the core, and by using the addition theorem to expand the dependence on r_{jk} in terms of r_j and r_k. One finds

$$\Delta_{lU}=\langle\Phi|C^2\rho_z|\Phi\rangle=\frac{\pm el_0}{2hc(2l_0+1)}\sum_{l,k}F_k(l_0lll_0;C^2f)(2k+1)\times(\langle l_0-1k00|l_0-1kl0\rangle^2-\langle l_0+1k00|l_0+1kl0\rangle^2), \qquad (23)$$

where l_0 is the orbital angular momentum of the odd particle, and the sum over l goes over the filled orbits in the core for protons if the odd particle is a neutron and over neutrons if the odd particle is a proton. The upper sign is for odd neutron and the lower for odd proton. As before we consider the odd-particle state with $j=l_0+\frac{1}{2}$ and $m_j=j$. The radial integral in (23) is defined as

$$F_k(abcd;E)=\int_0^\infty\int_0^\infty R_a(1)R_b(2)R_c(1)R_d(2)E_k(1,2)r_1^3r_2^3dr_1dr_2,$$

where

$$E(r_{12})=\sum_{k,\mu}E_k(r_1,r_2)Y_{k,\mu}(1)Y_{k,\mu}^*(2),$$

and where R_a is the single-particle radial function in the state a.

The commutator of $l_z^{(P)}$ with C is easily evaluated for the Gaussian form of correlations since in general

$$[l_z^{(P)},\exp(-\delta r_{12}^2)]=-\delta\exp(-\delta r_{12}^2)[l_z^{(P)},r_{12}^2];$$

but since

$$r_{12}^2=r_1^2+r_2^2-2r_1r_2\cos\theta_{12}=r_1^2+r_2^2-\frac{8\pi r_1r_2}{3}\sum_m Y_{1,m}(1)Y_{1,m}^*(2),$$

we get

$$[l_z^{(P)},r_{12}^2]=-(8/3)\pi r_1r_2\sum_m mY_{1,m}(1)Y_{1,m}^*(2)$$

if particle one is a proton and two a neutron. Evaluating as before, one finds

$$\Delta_{ll}=\pm\frac{10}{3}(1-\beta)\frac{l_0\gamma}{(2l_0+1)}\times\sum_{l,k}F_k(ll_0l_0l;D\exp(-\gamma r^2))(2k+1)\times(\langle l_0+1k00|l_0+1kl0\rangle^2-\langle l_0-1k00|l_0-1kl0\rangle^2),$$

where the upper sign is for odd neutron and the lower for odd proton.

These corrections are most easily evaluated in light nuclei where the number of orbits is small and where harmonic oscillator radial functions can be used. The best candidate is O^{17}, with one neutron in a $d_{\frac{5}{2}}$ state outside the extremely stable O^{16} doubly closed LS shell, and with a magnetic moment deviating by only 0.0193 nuclear magneton from the single-particle value.[4] Furthermore, if we take $f(r)$ to have a Gaussian form, $f(r)=\exp(-\eta r^2)$, then the interaction functions all become simple sums of Gaussians and these with harmonic oscillator radial functions reduce the radial integrals to known forms.[19,20]

In evaluating the matrix elements for O^{17}, we shall neglect the dependence of the Green's function, and therefore of the correlation length, on the energy difference and use the length for the p shell in the s shell. This is not serious since the population factors greatly favor the p-shell contribution, and further since the corrections increase with increasing correlation length, this approximation only serves to overestimate what seems to be a small effect. Taking harmonic oscillator radial functions with a characteristic length b, one finds for O^{17}

$$\Delta_{ll}=-(40/3)\gamma(1-\beta)\times b^2[\tfrac{2}{3}(1-\beta)A(2\gamma b^2)-\beta A((2\gamma+\eta)b^2)]M_N,$$

$$\Delta_{lU}=\frac{2\pi eb^2}{hc}\left[A(\eta b^2)-\frac{10}{3}(1-\beta)A((\eta+\gamma)b^2)\right],$$

where

$$A(a)=\frac{9+(3/a)+a^{-2}}{a^{5/2}(2+a^{-2})^{9/2}}.$$

We have neglected the term in $\exp(-2\gamma r^2)$ coming from C^2.

The harmonic-oscillator length parameter b can be obtained from a knowledge of the nuclear radius in oxygen, and one finds $b=1.6\times10^{-13}$ cm.[19] The range of the Green's function, λ [see Eq. (14)], for particles at the bottom of the Fermi sea is given by Brueckner and Gammel[2] in the approximation of no attraction as $\lambda=k_f(2/5)^{\frac{1}{2}}$, where k_f is the wave number of the Fermi momentum. The range of the Green's function is not expected to increase very rapidly as one moves up in the Fermi sea until one is very close to the top. Thus for the correlations between the odd particle and the p shell we take $\lambda=0.5k_f$. At saturation density, $k_fr_c=0.7$, and with a core radius $r_c=0.4\times10^{-13}$ cm, this gives $\lambda=0.87\times10^{13}$ cm^{-1}. For this value of λ, the best fit as in Fig. 1 is obtained with a Gaussian correlation function with $\gamma=1.25\times10^{26}$ cm^{-2}.

The parameters for the exchange part of the attractive potential can be obtained from the potential of Gammel and Thaler.[6] They find a triplet force that is almost pure Serber mixture, that is, zero in odd states. Neglecting the different dependence of the singlet force because of its small weight and because as we saw in Sec. III, terms in the interaction of the form $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ do not affect the moment, we can write the central interaction for all states as

$$-(1+P_M)\frac{438}{2.09r}\exp(-2.09r)\text{ Mev},$$

with r in units of 10^{-13} cm, and with a radius for the repulsive core of 0.4×10^{-13} cm. To cast this in the form of a Gaussian interaction we notice that since a turns out to be 2 or greater, the leading dependence of $A(a)$ on the range of the force comes from the term in $a^{-\frac{5}{2}}$, and this corresponds to a dependence on the fifth power of the range. We thus require that the integral $\int V(r)r^4dr$ be the same for the Gaussian and the Yukawa potentials. Because of the form of the correlations we carry the Gaussian integrals from 0 to ∞, but for the Yukawa potential we take the integrals from r_c to ∞. In actual fact this makes little difference since the r^4 weighting makes the major part of the integral come from the region in which B is essentially one. In order to get U and η separately, we require that the two potentials have the same effect in WKB approximation for zero binding energy. This requires the $\int[V(r)]^{\frac{1}{2}}dr$ be the same for both potentials. These two conditions give a Gaussian with $U=-64$ Mev and $\eta=0.84\times10^{26}$ cm^{-2}. The effect of the attractive interaction on the correlations is represented by taking $\beta=0.6$.

Putting these parameters into the equations we find $\Delta_{ll}=0.002M_N$ and $\Delta_{lU}=0.00285M_N$ Mev^{-1}, and thus $\Delta_l=-0.18M_N$. This relatively large effect is entirely due to Δ_{lU} since Δ_{ll} is negligible compared with Δ_{lU}. Δ_{ll} represents the change in the expectation value of the ordinary magnetic moment operator due to state-dependent correlations. It is extremely small because it contains D and therefore $\exp(-2\gamma r^2)$ in every term. Thus since the effect of the attraction in the correlations appears multiplied by the Green's function, all the contribution to Δ_{ll} comes from within the correlation volume, and this is very small. Δ_{lU} is much larger than this, but by far the largest part of it comes from the effect of the attraction in the Hamiltonian in the region of no correlation. That is, because of the r^4 weighting most of the contribution comes from the region in which B is unity. Thus the result for Δ_{lU} is almost exactly what one would have obtained from an ordinary shell-model calculation for the exchange magnetic moment in an uncorrelated-shell-model state. In view of the small deviation of the O^{17} magnetic moment from the single-particle value, the exchange moment is too large. However, it is well known that there are a number of other exchange effects, etc.,[4] that are expected to give corrections of a similar order of magnitude. The calculations presented here are not an attempt seriously to evaluate any of these corrections but rather to show that the correlations in the wave functions introduce

[19] W. J. Swiatecki, Proc. Roy. Soc. (London) **A205**, 238 (1951).
[20] R. D. Amado, Phys. Rev. **108**, 1462 (1957).

essentially nothing that is not already present in a full shell-model calculation.‡

V. CONCLUSION

The singular nature of the two-nucleon interaction assures us that the actual nuclear state is highly correlated, but the success of the shell model leads us to the conclusion that these correlations are unimportant for finding the expectation value of a large class of operators. The paradigm of this success is the magnetic moment of nuclei with one particle outside an LS doubly closed shell. The correlation structure of nuclei consists in the vanishing of the wave function if two particles come within the radius of the repulsive core followed by a rapid "healing" of the wave function to an uncorrelated state as the pair separates. Since the range of the correlation is short because the exclusion principle forbids transitions to nearby states, the additional effect of the correlations from the relatively long-range, attractive part of the interaction is very small. Thus the bulk of the correlation structure can be introduced into the shell model as a product of pair correlation functions each of which vanishes within the core and then approaches one, with a short range dependent only on the Fermi momentum and on the relative energy of the correlated pair. In a finite nucleus with no spin-orbit splitting, this relative energy depends only on orbital quantum numbers of the correlated pair and not on azimuthal quantum numbers. Therefore correlations of this type are independent of the state of a particle within a given orbit.

To consider the effect of such correlations on the expectation value of the magnetic moment of a "doubly closed shell plus one" nucleus, we notice that the moment operator is a combination of the z component of the proton orbital angular momentum, and the z component of spin of the protons and neutrons separately. A "doubly closed shell plus one" nucleus with $j=l+\frac{1}{2}$ and $m_j=j$, is an eigenstate of each of these operators, and hence the contribution of the correlations to the magnetic moment can be found by taking the commutator of each of these operators with the correlation function. For the scalar, state-independent, correlations discussed above, the expectation value of this commutator is shown to vanish. Since correlations between identical particles will produce no net current, any contribution to the magnetic moment must come from correlations between unlike particles. The correlation function is therefore conveniently expanded into tensor operators for neutrons and protons separately and the fact that the proton or neutron wave function is an S-state requires that the tensor operators in this space combine to form a scalar. This requirement reflects back on the operators in the space of odd particle and makes the commutator vanish. The proof can be extended to correlations having a scalar spin dependence and to certain more complicated symmetries.

The proof does not show that the shell-model wave function is a "good" wave function if we choose to measure this by the overlap of the shell-model state with the actual state, since this overlap is in fact very small. We see rather that the shell model exhibits certain symmetry properties that are unaffected by correlations so long as these are state independent, and this is so independent of what particular physical model we choose for the correlations. It is well known that the shell model is particularly suited to the prediction of properties associated with wave function symmetries, and it is suggested that it is the state independence of the correlations that accounts for this success.

There is a residual state-dependent part of the correlations due to higher order terms in the relative momentum of the correlated pair and due to the space-exchange part of the attractive potential as this appears in the correlations. In estimating the effect of these, we make a number of approximations since we know that these are small refinements to a state-independent correlation function the effect of which we know to be zero. These additional corrections are computed for O^{17} and are found to give a correction to the magnetic moment of order $+0.002$ nuclear magneton. This very small result arises because of the very short correlation length.

The space part of the attraction in the Hamiltonian will also modify the magnetic moment operator giving the well known space exchange operator. The expectation value of this turns out to be essentially the same as it would be in a calculation using uncorrelated shell-model wave functions since the major part of the expectation value comes from the long-range part of the attraction where the correlation function has gone to one. The numerical result is too large to account for the small deviation of the O^{17} moment from the Schmidt line, but this is well known from shell-model calculations as well. No similar space-exchange operator is found from the exchange dependence of the correlation function since the correlated wave function is an eigenfunction of the nuclear Hamiltonian and thus there are no terms in the energy of the system coming from first order changes in the eigenfunction. It is therefore correct to take the magnetic moment operator from the actual nuclear Hamiltonian, with the real nucleon mass.

‡ The correction to the magnetic moment of O^{17} from the Gammel-Thaler spin-orbit force has been calculated. The spin-orbit part of the magnetic moment operator gives $+0.15$ nuclear magneton, thus canceling most of the contribution from the space exchange moment operator. This cancellation appears fortuitous and would probably not occur in other nuclei of very different structure. The presence of spin-orbit correlations gives an entirely negligible contribution.

VI. ACKNOWLEDGMENTS

The author wishes to thank Professor K. A. Brueckner for many valuable discussions on all phases of this work, and in particular on the structure of the correlation function.