Polarization Effects in Integrated Bremsstrahlung Cross Section

HARIDAS BANERJEE
Indian Institute of Technology, Kharagpur, India
(Received March 12, 1958)

The Born approximation bremsstrahlung cross section, summed over the outgoing electron's spin states and integrated over its momentum directions, is obtained for arbitrary polarizations of the initial electron and the outgoing photon. A brief discussion is given of the way in which the photon polarization depends on that of the incident electron.

CONTRIBUTIONS to the differential bremsstrahlung cross section for forward scattering from longitudinal polarization of the primary electron and circular polarization of the emitted radiation have been obtained by McVoy.[1] In the following the dependence of the integrated bremsstrahlung cross section on the polarization of the incident electron and the emitted photon has been given for the most general case. The recoil energy of the nucleus and the effect of screening of the nuclear Coulomb field, which will be considered in a forthcoming paper, has been neglected throughout. The nuclear Coulomb field has been considered only in the first Born approximation.

The matrix element M for this process can be written in Jauch's notation[2] in the form

$$M=\frac{1}{q^2}\bar{u}(p)Qu(p_0), \tag{1}$$

where

$$Q=e_\mu\gamma^\mu\frac{i(p+k)_\nu\gamma^\nu-m}{2(E-p\cos\theta)}\gamma^0-\gamma^0\frac{i(p_0-k)_\nu\gamma^\nu-m}{2(E_0-p_0\cos\theta_0)}e_\mu\gamma^\mu,$$

$$\mathbf{q}=\mathbf{p}_0-\mathbf{p}-\mathbf{k}.$$

In the above equations θ, θ_0 are the angles between $\mathbf{k}$ and $\mathbf{p}$, $\mathbf{p}_0$, respectively, and $\mathbf{e}$ is the polarization vector of the photon. The transition probability is then proportional to

$$|M|^2=\frac{1}{q^4}\operatorname{Tr}[u(p)\bar{u}(p)Qu(p_0)\bar{u}(p_0)\bar{Q}]. \tag{2}$$

In order to describe the polarization phenomena, it is convenient to introduce the two basic polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ and the direction $\boldsymbol{\zeta}_0$ of the spin angular momentum of the primary electron in the rest system. We take $\mathbf{e}_1$ in the $\mathbf{k}-\mathbf{p}_0$ plane and write $\mathbf{e}=a_1\mathbf{e}_1+a_2\mathbf{e}_2$, with $|a_1|^2+|a_2|^2=1$. Following Tolhoek,[3] the polarization density matrix ρ of the emitted photon can now be written as

$$\rho=\tfrac{1}{2}(1+\boldsymbol{\xi}\cdot\boldsymbol{\omega}), \tag{3}$$

where

$$\xi_1=|a_1|^2-|a_2|^2,\quad \xi_2=a_1a_2^*+a_2a_1^*,$$

and

$$\xi_3=i(a_1a_2^*-a_2a_1^*).$$

Here ξ_1 represents linear polarization with $\mathbf{k}-\mathbf{e}_1$, $\mathbf{k}-\mathbf{e}_2$ as the reference planes of polarization; ξ_2, linear polarization with reference planes of polarization making angles $\frac{1}{4}\pi$ with the $\mathbf{k}-\mathbf{e}_1$ plane; and ξ_3, circular polarization. The 2×2 ω matrices are $\omega_1=\sigma_3$, $\omega_2=\sigma_1$, $\omega_3=\sigma_2$, where the σ's are the Pauli matrices. The projection operator $u(p_0)\bar{u}(p_0)$ as given by Tolhoek[3] in terms of the polarization vector $\boldsymbol{\zeta}_0$ of the electron is

$$u(p_0)\bar{u}(p_0)=\tfrac{1}{4}\left(1-\frac{ip_0^\mu\gamma_\mu}{m}\right)(1-s_0^\nu\gamma_\nu\gamma_5), \tag{4}$$

where

$$s_0^\mu=\left(\boldsymbol{\zeta}_0+\frac{(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0)\mathbf{p}_0}{m(E_0+m)},\frac{(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0)}{m}\right).$$

From (2) and (3) we now get

$$|M|^2=\tfrac{1}{2}[(\Sigma_{11}+\Sigma_{22})+\xi_1(\Sigma_{11}-\Sigma_{22})+\xi_2(\Sigma_{12}+\Sigma_{21})-i\xi_3(\Sigma_{12}-\Sigma_{21})], \tag{5}$$

where $\Sigma_{rs}=M_rM_s^*$, M_r being obtained from M by replacing $\mathbf{e}$ by $\mathbf{e}_r$. Summing (5) over the spin states of the final electron and a subsequent integration over its directions of momentum give

$$\int\sum|M|^2d\Omega=\Phi(0)+\Phi(\boldsymbol{\xi})+\Phi(\boldsymbol{\zeta}_0)+\Phi(\boldsymbol{\xi},\boldsymbol{\zeta}_0). \tag{6}$$

For the explicit expression of $\Phi(0)$ and $\Phi(\boldsymbol{\xi})$, one may refer to the paper of Gluckstern and Hull.[4] If we write

$$\eta^{-1}=2(Ze^3m/\pi)^2(p/p_0)kdkd\Omega_0,$$

then

$$\Phi(0)=\eta d\sigma_{\text{I}},\quad \Phi(\boldsymbol{\xi})=\xi_1\eta(d\sigma_{\text{II}}-d\sigma_{\text{III}}),\quad \Phi(\boldsymbol{\zeta}_0)=0, \tag{7}$$

where $d\sigma_{\text{I}}$, $d\sigma_{\text{II}}$, and $d\sigma_{\text{III}}$ are given by Eqs. (4.1), (4.2), and (4.3) of Gluckstern and Hull's paper.

We can write $\Phi(\boldsymbol{\xi},\boldsymbol{\zeta}_0)$ in the form

$$\Phi(\boldsymbol{\xi},\boldsymbol{\zeta}_0)=-(\pi\xi_3/8m\omega)\{(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0/p_0)[\Phi_L+\sin^2\theta_0\Phi_L']+(\boldsymbol{\zeta}_0\cdot\mathbf{n}_0)\Phi_T\}, \tag{8}$$

[1] K. W. McVoy, Phys. Rev. **106**, 828 (1957).

[2] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Cambridge, 1955), p. 364.

[3] H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).

[4] R. L. Gluckstern and M. H. Hull, Phys. Rev. **90**, 1030 (1953).

where $\mathbf{n}_0$ is the unit vector perpendicular to $\mathbf{p}_0$ in the $\mathbf{k}-\mathbf{p}_0$ plane. The quantities Φ_L and Φ_L', contributions from longitudinal polarization of the incident electron, and Φ_T, that from transverse polarization, can now be written as

$$\Phi_L=\frac{1}{mp_0}\left\{\frac{2(2E_0^2\omega-Ep_0^2)}{\omega p_0^2\Delta_0^2}-\frac{4E_0}{p_0^2\Delta_0}+\frac{2E_0p_0(p_0-\omega\cos\theta_0)}{\omega T^2\Delta_0^2}+\frac{m^2L}{p_0p}\times\left[\frac{p_0^2-2E_0\omega}{p_0^2\Delta_0^2}+\frac{2m^2\omega-p_0^2E_0-p_0^2E}{p_0^2\Delta_0^3}\right]+\frac{2p_0\epsilon^T}{pT}\left[\frac{p_0-E_0\cos\theta_0}{\Delta_0^2}+\frac{E_0(p_0-\omega\cos\theta_0)}{T^2\Delta_0}\right]\right\}, \quad (9)$$

$$\Phi_L'=\frac{1}{mp_0}\left\{-\frac{4m^2(2E_0^2+m^2)}{p_0^2\Delta_0^4}+\frac{12m^2E_0}{p_0^2\Delta_0^3}+\frac{L}{p_0p}\left[-\frac{2m^2E_0(3m^2\omega-p_0^2E)}{p_0^2\Delta_0^4}+\frac{6E_0^2m^2\omega+p_0^4\omega-2p_0^2E_0^3}{p_0^2\Delta_0^3}\right]\right\}, \quad (10)$$

$$\Phi_T=\sin\theta_0\left\{\frac{4(3E_0^2\sin^2\theta_0+m^2)}{p_0^2\Delta_0^4}-\frac{16E_0}{p_0^2\Delta_0^3}+\frac{2}{p_0^2\Delta_0^2}+\frac{2}{T^2\Delta_0^2}+\frac{L}{p_0p}\left[\frac{2E_0E-m^2}{p_0^2\Delta_0^2}-\frac{2m^2(E_0+\omega)}{p_0^2\Delta_0^3}+\frac{E_0(m^2E+\sin^2\theta_0[6m^2\omega-p_0^2E])}{p_0^2\Delta_0^4}\right]+\frac{2\epsilon^T}{pT}\left[\frac{1}{\Delta_0^2}+\frac{\omega}{T^2\Delta_0}\right]\right\}. \quad (11)$$

Here

$$L=\ln\left[\frac{EE_0-m^2+pp_0}{EE_0-m^2-pp_0}\right],$$

$$T=|\mathbf{p}_0-\mathbf{k}|,\quad \Delta_0=E_0-p_0\cos\theta_0,\quad \omega=|\mathbf{k}|,$$

$$\epsilon^T=\ln\left[\frac{T+p}{T-p}\right].$$

Thus, for bremsstrahlung in the forward direction,

$$\Phi(\boldsymbol{\xi},\boldsymbol{\zeta}_0)=-(\pi\xi_3/8m\omega)(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0/p_0)\Phi_L.$$

From the above, the following conclusions about polarization phenomena in bremsstrahlung can immediately be made:

(i) If the primary electron beam be unpolarized the emitted radiation will be linearly polarized, since ξ_3, which represents circular polarization, is absent in the expression for $\Phi(\boldsymbol{\xi})$. Further, from the dependence of $\Phi(\boldsymbol{\xi})$ on ξ_1 it follows that the emitted radiation will be polarized in the $\mathbf{k}-\mathbf{e}_1$ and $\mathbf{k}-\mathbf{e}_2$ planes and there will be no polarization in planes making angles $\frac{1}{4}\pi$ with these planes.

(ii) There is no correlation between the polarization of the primary electron and linear polarization of the emitted radiation, $\Phi(\boldsymbol{\xi},\boldsymbol{\zeta}_0)$ being dependent on ξ_3 only. Thus with a polarized incident electron beam the bremsstrahlung will, in general, be elliptically polarized.

(iii) The contributions from transverse polarization of the primary electron vanishes for $\theta_0=0$, and hence it cannot be detected by analyzing the circular polarization of the forward bremsstrahlung.

The results for the phenomenon of pair production can be obtained directly from the above by replacing p and E by $-p_+$ and $-E_+$ respectively, where $(\mathbf{p}_+,E_+)$ are the momentum and energy of the positron.

Note added in proof:—The parameter $\boldsymbol{\xi}$ in terms of which the transition probability w [which is proportional to the right-hand side of Eq. (6)] has been expressed is, in fact, the Stokes parameter for the polarization detector of the bremsstrahlung. Following Tolhoek[3] and using the density matrix formalism one can show that if the transition probability w be expressed in the form

$$w\sim\Phi(0)(1+\boldsymbol{\xi}\cdot\boldsymbol{\xi}'), \quad (12)$$

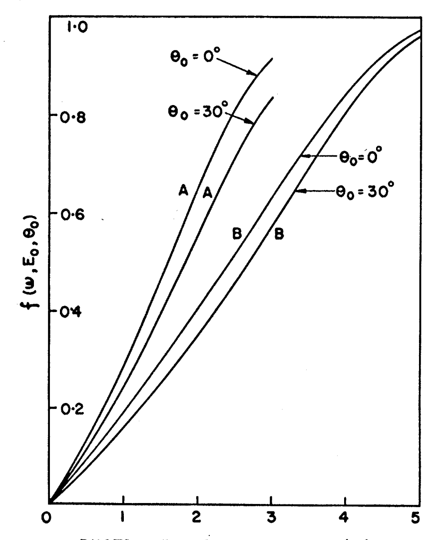

FIG. 1. $f_L(\omega,E_0,\theta_0)$ defined by Eq. (13) as a function of the photon energy for two values of the scattering angle $\theta_0=0$, 30°. The graphs A and B correspond to primary energy $E_0=4m$ and $6m$, respectively.

then $\boldsymbol{\xi}'$ is the Stokes parameter for the emitted photon beam. An alternative procedure for arriving at the same result consists in making w maximum subject to the condition

$$\xi_1^2+\xi_2^2+\xi_3^2=(|a_1|^2+|a_2|^2)^2=1,$$

which the parameter $\boldsymbol{\xi}$ satisfies from definition. The Stokes parameter $\boldsymbol{\xi}'$ of the photon beam is then related to this $\boldsymbol{\xi}$ of the detector by the simple equation $\boldsymbol{\xi}'=\tau\boldsymbol{\xi}$ where τ, the degree of polarization, is given by

$$\tau=(w_{\max}-w_{\min})/(w_{\max}+w_{\min}).$$

Because of its relative importance in the study of the polarization of β rays which are known to be longitudinally polarized, we consider only the contribution ξ_{3L}' to the circular polarization of the bremsstrahlung from longitudinally polarized primary electron beam. We find from Eqs. (6), (8), and (12)

$$\begin{aligned}\xi_{3L}'&=-(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0/p_0)\{\pi(\Phi_L+8m^2\theta_0\Phi_L')/8m\omega\Phi(0)\}\\&=(\boldsymbol{\zeta}_0\cdot\mathbf{p}_0/p_0)f_L(\omega,E_0,\theta_0).\end{aligned}\tag{13}$$

In Fig. 1, f_L has been plotted as a function of the photon energy ω. Since f_L is found to be positive, it is apparent from Eq. (13) that the sense of circular polarization (right or left) of the bremsstrahlung is the same as that of the polarization (parallel or antiparallel to the momentum) of the primary beam. The amount of circular polarization rapidly increases with the photon energy and near the upper end of the bremsstrahlung spectrum it is $\sim$90% for completely polarized primary beam. These results are in perfect agreement with the experimental findings of Goldhaber et al.[5]

ACKNOWLEDGMENT

The author is thankful to Professor S. Gupta for his keen interest in this work and many helpful discussions.

[5] Goldhaber, Grodzines, and Sunyar, Phys. Rev. **106**, 826 (1957).

PHYSICAL REVIEW VOLUME 111, NUMBER 2 JULY 15, 1958

Spins of Some Radioactive Iodine Isotopes*

HUGH L. GARVIN, THOMAS M. GREEN, AND EDGAR LIPWORTH
Radiation Laboratory, University of California, Berkeley, California
(Received March 26, 1958)

The spins of I^{123}, I^{124}, and I^{131} have been measured by the method of atomic beams; the spins are $\frac{5}{2}$, 2, and $\frac{7}{2}$, respectively. The result $I=\frac{7}{2}$ for I^{131} confirms an earlier measurement by a different method.

INTRODUCTION

THIS paper reports the first results of a program to determine the nuclear spins, magnetic moments, and quadrupole moments of some of the available radioactive halogens by the method of atomic beams. These quantities are of interest because an extension of measurements in a region of the periodic table where collective effects are not expected to dominate will serve to broaden the experimental basis of the single-particle shell model and lead to further tests of the model itself. In addition to the results of spin determinations of three iodine nuclei, this paper contains a description of an atomic-beam apparatus that appears particularly suited to the study of radioactive substances.

METHOD

The method used, an atomic-beam "flop-in" type of experiment, was first proposed by Zacharias *et al.*[1,2] In recent years there has been considerable application of this technique to the measurement of the spins and moments of radioactive nuclides,[3,4] and only a brief description of the method is given here.

The ground state of all halogen atoms is $P_{\frac{3}{2}}$. Thus (with normal ordering of the F levels) there are two observable flop-in transitions at low frequency. For $I>0$ these are

$$(F=I+\tfrac{3}{2},\,M_F=-I+\tfrac{1}{2})\rightarrow(F=I+\tfrac{3}{2},\,M_F=-I-\tfrac{1}{2})$$

and

$$(F=I+\tfrac{1}{2},\,M_F=-I+\tfrac{3}{2})\rightarrow(F=I+\tfrac{1}{2},\,M_F=-I+\tfrac{1}{2}),$$

where F is the total angular-momentum quantum number of the atom, I the nuclear-spin quantum number, and M_F the projection of the total angular momentum along the direction of quantization. Figure 1 shows the relevant energy-level diagram for a halogen atom with $I=\frac{5}{2}$. The two transitions are indicated by arrows; they will be referred to as (+) and (−) transitions, respectively. A measurement of the frequencies of either or both of these transitions in the linear

* This work was performed under the auspices of the U. S. Atomic Energy Commission.
[1] J. R. Zacharias, Phys. Rev. **61**, 270 (1942).
[2] Davis, Nagle, and Zacharias, Phys. Rev. **76**, 1068 (1949).
[3] William A. Nierenberg, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 349.
[4] Kenneth F. Smith, Progr. Nuclear Phys. **6**, 52 (1957).