Test of the Nature of the Vector Interaction in β Decay*

MURRAY GELL-MANN

California Institute of Technology, Pasadena, California

(Received March 3, 1958)

An experiment is proposed to test the theory recently advanced by Feynman and Gell-Mann on the vector interaction in β decay. According to the theory, the Fermi coupling constant is not appreciably altered by renormalization, in agreement with experiment. A further property of the theory is exploited here, that analogous γ and β transitions in light nuclei have proportional matrix elements, as far as the V interaction is concerned. In particular, the V interaction gives rise to "weak magnetism" analogous to the magnetic effects that induce the emission of M1 photons. This "weak magnetism" obeys Gamow-Teller selection rules and interferes with the A coupling, distorting the spectra of high-energy β transitions with $\Delta J = 1$ (no).

It is suggested that the β spectra of B^{12} and N^{12} be measured accurately and compared. Departure from linearity of the Kurie plot should be noticed in each case; the ratio of the spectra can be calculated with confidence on the basis of the theory, which predicts a 20% effect. The measured γ-ray width of the 15.11-Mev state in C^{12} is used in the calculation.

I. THE PROBLEM OF THE VECTOR INTERACTION

IT has been proposed[1,2] on theoretical grounds that the weak interactions possess a universal vector-axial vector form as well as a universal strength.[3] Recent experimental evidence[4-6] has tended to confirm this assertion. In particular, it appears that the interaction in nuclear β decay is of the type $V-A$, with a left-handed longitudinal neutrino. The effective coupling may be written, in an obvious notation, as follows:

$$G[\bar{p}\gamma_\mu(1+\lambda\gamma_5)n]\left[\bar{e}\gamma_\mu\frac{1+\gamma_5}{\sqrt{2}}\nu\right]+\text{Herm. conj.} \quad (1)$$

The positive sign of λ (corresponding to the negative sign in $V-A$) is determined by the results of reference 5. The magnitude of λ is the ratio of Gamow-Teller and Fermi coupling strengths and is found by studying the ft values of β transitions in light nuclei where matrix elements are known. The value $|\lambda|=1.14$ is quoted by Winther and Kofoed-Hansen.[7]

The Fermi constant G may be compared with the corresponding constant G_μ in the muon-decay coupling:

$$G_\mu[\bar{\nu}\gamma_\mu(1+\gamma_5)\mu]\left[\bar{e}\gamma_\mu\frac{1+\gamma_5}{\sqrt{2}}\nu\right]+\text{Herm. conj.} \quad (2)$$

It was pointed out in reference 1 that $G=G_\mu$ to within

the one- or two-percent accuracy of present experiments. This is of course in excellent agreement with the notion of universality, except possibly for one important point. The muon seems to possess no strong couplings, while the nucleon is strongly coupled to pions and to strange particles. If the universality applies, as we are accustomed to thinking it does, to the bare particles, then we would expect a considerable renormalization of G in the case of the dressed nucleon.

A possible explanation of the equality $G=G_\mu$ was given in reference 1. (See also earlier work by Gershtein and Zeldovich.[8]) The starting point is the realization that in electrodynamics the universality of coupling strength is not affected by renormalization, since the current density j_μ obeys the conservation law $\partial j_\mu/\partial x_\mu=0$. It is then suggested that for the vector interaction in β decay a similar principle holds: the vector quantity that is coupled is not merely $\bar{p}\gamma_\mu n$, but a conserved quantity of which this is one term. We make use of the charge independence of the strong interactions and consider the total isotopic-spin current

$$\mathfrak{J}_\mu=\tfrac{1}{2}i\bar{N}\tau\gamma_\mu N+\pi\times(\partial\pi/\partial x_\mu)+\cdots. \quad (3)$$

Apart from small electromagnetic corrections, we have the conservation law

$$\partial\mathfrak{J}_\mu/\partial x_\mu=0. \quad (4)$$

The following replacement is then made for the vector part of the weak interaction:

$$i\bar{p}\gamma_\mu n \rightarrow \mathfrak{J}_{\mu+}\equiv\mathfrak{J}_{\mu x}+i\mathfrak{J}_{\mu y}. \quad (5)$$

With the new coupling, the equality $G=G_\mu$ is unaffected by renormalization, except for electromagnetic corrections.

For the axial vector interaction, no scheme has been invented that prevents the coupling strength from being altered by renormalization. Presumably, then, the

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] R. E. Marshak and E. C. G. Sudarshan, Nuovo cimento (to be published).

[3] For the original hypothesis of universality, see G. Puppi, Nuovo cimento **5**, 505 (1948); O. Klein, Nature **161**, 897 (1948); Lee, Rosenbluth, and Yang, Phys. Rev. **75**, 905 (1949); J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 144 (1949).

[4] F. Boehm and A. H. Wapstra, Phys. Rev. **109**, 456 (1958).

[5] Burgy, Epstein, Krohn, Novey, Raboy, Ringo, and Telegdi, Phys. Rev. (to be published).

[6] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958).

[7] A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. (to be published).

[8] S. S. Gershtein and J. B. Zeldovich, Zhur. Eksptl. i Teort. Fiz. U.S.S.R. **29**, 698 (1955) [translation: Soviet Phys. JETP **2**, 576 (1957)].

parameter λ, estimated experimentally as 1.14, is the renormalization constant. It is not understood why the constant should be so close to unity, nor why it should exceed unity. (In the static form of meson theory, for example, it is always <1.) The somewhat mysterious nature of this situation for the axial vector interaction makes it especially desirable to check on whether the vector interaction is correctly described by the theory referred to above. In what follows, we shall describe a possible experiment to test the theory.

II. ANALOGY WITH ELECTROMAGNETISM

We begin by remarking that the electromagnetic interaction Hamiltonian density may be written as the sum of an isotopic scalar part $\mathcal{K}_{el}{}^{S}$ and an isotopic vector part $\mathcal{K}_{el}{}^{V}$, where

$$\mathcal{K}_{el}{}^{V} = -e\mathfrak{J}_{\mu z} A_{\mu}. \tag{6}$$

But according to our theory the vector part of the β-decay interaction is given by the coupling Hamiltonian density

$$-G\mathfrak{J}_{\mu+}i\left(\bar{e}\gamma_{\mu}\frac{1+\gamma_{5}}{\sqrt{2}}\nu\right) + \text{Herm. conj.} \tag{7}$$

From (6) and (7) we see immediately that any electromagnetic interaction of a nuclear system has its analog in a weak vector interaction with leptons, and that the matrix elements for the two are strictly proportional insofar as charge independence holds. We must make the replacements

$$\mathfrak{J}_{\mu z} \to \mathfrak{J}_{\mu+}, \quad A_{\mu} \to \frac{G}{e}i\left(\bar{e}\gamma_{\mu}\frac{1+\gamma_{5}}{\sqrt{2}}\nu\right) \tag{8}$$

for β^{-} decay. For β^{+} decay we have to take the Hermitian conjugate expressions.

Let us expand the electromagnetic interaction of a nuclear system in powers of the momentum or energy transferred to the field. We shall ignore the recoil of the nucleus as a whole. In zeroth order, we have just the static interaction of the nucleus with a constant static potential. The isotopic vector part of the coupling Hamiltonian is just

$$eI_{z}A_{0}, \tag{9}$$

where $\mathbf{I}$ is the total isotopic spin vector and A_{0} is the scalar potential of the electromagnetic field evaluated at the nucleus. Let us now make use of the instructions given in Eq. (8) in order to find the effective Hamiltonian in vector β^{-} decay to zeroth order in the momentum transferred to the leptons. We obtain

$$GI_{+}\left(e^{\dagger}\frac{1+\gamma_{5}}{\sqrt{2}}\nu\right), \tag{10}$$

where $I_{+}=I_{x}+iI_{y}$ and $e^{\dagger}$ is the Hermitian conjugate of e as distinct from the Dirac adjoint $\bar{e}$. Again the field operators are evaluated at the nucleus. For β^{+}

decay we obtain, of course,

$$GI_{-}\left(\nu^{\dagger}\frac{1+\gamma_{5}}{\sqrt{2}}e\right). \tag{11}$$

The results (10), (11) are rigorous apart from corrections to charge independence and they have already been given in reference 1. It was, in fact, suggested that (11) be tested for the decay $\pi^{+} \to \pi^{0}+e^{+}+\nu$, the matrix element of I_{-} being $\sqrt{2}$ for this case. Such an experiment[9] is unfortunately very difficult, because of the slowness of this decay compared to $\pi^{+} \to \mu^{+}+\nu$.

Equations (10) and (11) have, of course, been tested for nuclei, especially for $0^{+} \to 0^{+}$ transitions in which the axial vector coupling plays no role. The trouble is that for a nucleus the relations (10) and (11) are not an incisive test of the theory. They follow in practically any theory of β decay if the nucleus is regarded as a collection of dressed nucleons, each with its meson cloud undisturbed by its neighbors. The success of (10) and (11) for β decay of nuclei might be regarded, therefore, merely as a test of such a nuclear model.

To find an experiment in nuclear physics that really tests the new theory of the vector interaction, we must look beyond the "allowed" approximation in which the momentum and energy transferred to the leptons are neglected. Let us go on to first order in these quantities. For electromagnetism this corresponds to considering electric or magnetic dipole interactions, depending on whether the nuclear system changes its parity or not. For electric dipole interactions, practically the same objection is encountered as for monopole interactions, namely that the predictions for β decay are ones that would also have followed from a simple theory of the nucleus. We therefore take up the magnetic dipole case.

Consider, for example, a transition from a $J=1^{+}$, $I=1$ level in a self-conjugate nucleus to one with $J=0^{+}$, $I=0$. (Here J is total angular momentum and I is total isotopic spin.) For simplicity let us take the $J_{z}=0$ component of the initial state. Then the effective Hamiltonian for electromagnetic interaction, to first order in the momentum radiated, is

$$\frac{-\mu e}{2M}[\boldsymbol{\nabla}\times\mathbf{A}]_{z}, \tag{12}$$

where μ is the transition magnetic moment in units of the proton Bohr magneton $e/2M$ and where $\boldsymbol{\nabla}\times\mathbf{A}$ is, of course, evaluated at the nucleus. Since I changes by one in the transition, only the isotopic vector part of the current contributes. Now suppose the $J=1^{+}$, $I=1$ level is the ground state of the isobar with $I_{z}=-1$, so that β^{-} decay takes place from this level to the state with $J=0^{+}$, $I=0$. This β^{-} decay obeys Gamow-Teller selection rules and is dominated by the allowed axial vector interaction. There is, however, also a contribu-

[9] E. Feenberg and H. Primakoff (to be published).

tion from the vector interaction, and we may calculate it from (12) by using the instructions given in (8). We note that the matrix element of $\mathfrak{J}_{\mu+}$ from the $I_z=-1$ state is equal to $\sqrt{2}$ times that of $\mathfrak{J}_{\mu z}$ from the $I_z=0$ state. For the effective Hamiltonian in β^- decay, we thus obtain

$$-\frac{\mu}{\sqrt{2}M}G\left[\nabla\times\left(e^\dagger\alpha\frac{1+\gamma_5}{\sqrt{2}}\nu\right)\right]_z, \quad (13)$$

as the contribution from the vector interaction.

The effect described by (13) bears the same relation to the allowed Fermi coupling that magnetism bears to electricity. We might refer to it as "weak magnetism." It is by means of this effect that we propose to test the new theory of the vector interaction. We note that the transition magnetic moment μ contains not only contributions from Dirac and orbital magnetic moments of the nucleons but also larger contributions from their anomalous magnetic moments. All of these are taken over into β decay according to (13). Now in a conventional theory of β decay the meson cloud is not coupled to leptons and the large "anomalous moment" contributions would be lacking in the analog of (13). Thus weak magnetism, if it can be measured experimentally, will serve to distinguish the new theory from the old one.

III. EFFECT OF WEAK MAGNETISM ON β SPECTRA

In a β transition such as we have been discussing, the leading term in the decay amplitude is supplied by the allowed axial vector interaction, which gives an effective Hamiltonian

$$-G\lambda\mathfrak{M}\left[e^\dagger\sigma\frac{1+\gamma_5}{\sqrt{2}}\nu\right]_z, \quad (14)$$

where $\mathfrak{M}$ is the Gamow-Teller matrix element, the z component of the quantity often written as $\int\sigma$. We wish to consider, however, corrections of first order in the gradient of the lepton fields and we must therefore expand the axial vector as well as the vector interaction to this order. The only possible terms of first order are

$$iB\left[\nabla\left(e^\dagger\gamma_5\frac{1+\gamma_5}{\sqrt{2}}\nu\right)\right]_z \quad (15)$$

and

$$iC\left[\frac{\partial}{\partial t}\left(e^\dagger\sigma\frac{1+\gamma_5}{\sqrt{2}}\nu\right)\right]_z. \quad (16)$$

The coefficients B and C are required to be real by CP invariance.

By contrast with the vector interaction, there has been no suggestion that the axial vector coupling involves the meson cloud. The mechanism that makes the term (13) large is thus absent for (15) and (16). Nevertheless, we shall take these terms into account in our calculations. They are, of course, not predictable in magnitude since there is no analogy with electromagnetism in the case of the axial vector coupling.

The term (16) may be regarded merely as providing a numerical correction to the allowed Gamow-Teller amplitude given in (14), since $\partial/\partial t$ merely brings down a factor ik_0, where k_0 is the total energy transferred to the leptons and equals the energy difference between initial and final nuclear states. Since the matrix element $\mathfrak{M}$ cannot in any case be predicted with high accuracy, we may ignore the term (16) from now on.

The total effective Hamiltonian out to first order in gradients is thus given by the sum of (13), (14), and (15). The gradient may be replaced by $-i\mathbf{k}$, where $\mathbf{k}$ is the momentum given to the leptons. Let us set

$$a\equiv\frac{\mu}{\sqrt{2}M}\frac{1}{\lambda\mathfrak{M}} \quad (17)$$

and

$$b\equiv\frac{B}{G\lambda\mathfrak{M}}. \quad (18)$$

Then we have

$$H_{\text{eff}}=-G\lambda\mathfrak{M}\left[e^\dagger\{\sigma-ia\mathbf{k}\times\alpha-b\mathbf{k}\gamma_5\}_z\frac{1+\gamma_5}{\sqrt{2}}\nu\right]. \quad (19)$$

We may now use (19) to calculate first-order corrections to the spectrum and β-ν angular correlation of the β^- transition. The angular correlation comes out

$$1-\frac{1}{3}\frac{p}{E}\cos\theta\{1+8a(E-\tfrac{1}{2}k_0)-2bk_0\}, \quad (20)$$

where p and E are the momentum and energy of the electron, respectively. The maximum value of E, is, of course, k_0.

The spectrum is far more suitable for a crucial experiment since it is much easier to measure. The transition probability is proportional to

$$p^2(k_0-E)^2dp$$
$$\times\left\{1+\frac{8}{3}a\left(E-\tfrac{1}{2}k_0-\frac{1}{2}\frac{m^2}{E}\right)+\frac{2}{3}b\left(k_0-\frac{m^2}{E}\right)\right\}. \quad (21)$$

The terms $\frac{2}{3}bk_0$ and $-\frac{4}{3}ak_0$ are constant and give no first-order change in the spectrum. The terms in m^2/E become very small for a high-energy β transition, for which the correction factor in the spectrum reduces to

$$1+(8/3)aE, \quad (22)$$

not involving b.

So far we have considered the β^- transition only. Let us now turn to the analogous β^+ decay. Everything must of course be the same except possibly for the sign of the effect. We are dealing with an interference effect between the V and A couplings and we know that the interaction $V-A$ looks like $V+A$ if it is written in terms of positrons and antineutrinos instead of electrons and neutrinos. It is clear, therefore, that the correction term in (22) due to weak magnetism changes sign as we go from β^- to β^+ decay. This is of the highest

importance for a possible experiment, since many systematic errors (and corrections such as are discussed below) can be eliminated by comparing β^- and β^+ spectra for analogous transitions. In the ratio, moreover, the effect is doubled.

Let us now discuss in detail the calculation of a from the rate of the analogous γ transition. Using the coupling (13), we obtain for the rate

$$\Gamma_\gamma = \frac{\mu^2}{3(137)} \frac{\omega^3}{M^2},\qquad(23)$$

where ω is the energy of the γ ray (roughly equal to k_0, of course). The rate of the β^- decay is controlled (to zeroth order) by the quantity $\lambda\mathfrak{M}$ and can be used to determine it. The simplest formula makes use of a comparison with O^{14}, which is a pure Fermi transition and has a known matrix element of $\sqrt{2}$, as Eq. (11) indicates. We have, then,

$$(ft)_{O^{14}}/ft = \lambda^2\mathfrak{M}^2/2.\qquad(24)$$

Using (23), (24), and the definition (17) of a, we get

$$|a| = \frac{1}{\omega}\left[\frac{3}{4}\left(\frac{137\Gamma_\gamma}{\omega}\right)\frac{ft}{(ft)_{O^{14}}}\right]^{\frac{1}{2}}.\qquad(25)$$

Before making use of experimental data on an individual transition, let us make a rough theoretical estimate of a. If we treat the transition magnetic moment as if it were due entirely to the intrinsic moments of the nucleons, ignoring orbital moments and more complicated effects such as exchange currents, then the transition moment μ is simply proportional to $\int\boldsymbol{\sigma}$, as is the Gamow-Teller amplitude $\lambda\mathfrak{M}$ in nonrelativistic approximation. The properties of the nucleus thus cancel out in the determination of a and we find

$$a \approx \frac{\mu_p-\mu_n}{2\lambda M} \approx \frac{4.70}{2.28M} \approx +\frac{2}{M}.\qquad(26)$$

This estimate should give the sign of a correctly and the magnitude of a roughly unless we are dealing with a transition in which $\int\boldsymbol{\sigma}$ is unusually small (as in the cases of C^{14} and P^{32}), when a may be much larger, but the test of the theory much poorer.

Perhaps the best example for our purposes is provided by the β^- and β^+ decays of B^{12} and N^{12}, respectively, to the ground state of C^{12}. The analogous γ ray is emitted by the 15.11-Mev level in C^{12}. The γ-ray width has been measured by Fuller and Hayward[10] with the result

$$\Gamma_\gamma = (0.69\pm0.07)(79\pm16 \text{ ev})(0.96) = 53\pm11 \text{ ev}.\qquad(27)$$

[10] E. Hayward and E. G. Fuller, Phys. Rev. **106**, 991 (1957).

The factor 0.96 is the fraction of γ rays to the ground state as determined by Waddell.[11] The ft value of B^{12} is quoted experimentally[12] as

$$\ln ft = 4.1\pm0.05 \quad\text{or}\quad ft = 12\,000\pm600.\qquad(28)$$

For O^{14} we have the experimental value[13]

$$ft = 3088\pm56.\qquad(29)$$

Combining these and substituting into formula (25) for a, we obtain

$$|a| = (0.039\pm0.004)/\omega = (2.34\pm0.25)/M.\qquad(30)$$

This agrees so well with the estimate of Eq. (26) that the sign of a is virtually certain to be plus. The total effect in the ratio of the B^{12} to the N^{12} spectrum is seen from (22) to be a variation of amount $(16/3)ak_0 \approx (16/3)a\omega \approx 20\%$ over the spectrum. This should be large enough to measure.

In the actual comparison of the spectra, the difference in end-point energies should, of course, be allowed for by factoring out the Fermi spectrum in each case and then taking the ratio as a function of electron energy. This ratio should be proportional to $1+(16/3)aE$.

We have restricted ourselves to corrections of first order in the lepton momenta. For a high-energy transition, however, second-order effects may also be important. These arise in the ordinary way from the retardation expansion of the Gamow-Teller coupling. It is significant, however, that they do not involve interference between the vector and the axial vector couplings and therefore they do not change sign as we go from β^- to β^+. In a comparison of the spectra of B^{12} and N^{12}, second-order effects must cancel out. The same applies to the small b term of Eq. (21).

According to the theory, then, the ratio of B^{12} and N^{12} spectra is changed by weak magnetism by an appreciable and calculable amount. It can be seen, however, from Eq. (21) that the ratio of ft values is negligibly affected.

ACKNOWLEDGMENTS

[11] C. N. Waddell, Ph.D. dissertation (1957), University of California, Berkeley.

[12] Weighted mean including results of E. Norbeck, Jr., Bull. Am. Phys. Soc. Ser. II, **1**, 329 (1956), quoted by Cook, Fowler, Lauritsen, and Lauritsen, Phys. Rev. **107**, 508 (1957).

[13] Bromley, Almquist, Gove, Litherland, Paul, and Ferguson, Phys. Rev. **105**, 957 (1957).