Pion Decay and Possible Nonlocal Effects in the Theory of Fermi Interactions*

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Possible nonlocal effects in the theory of Fermi interactions are studied phenomenologically. Under certain simple assumptions, an effective Hamiltonian is derived which reduces to the local $V-A$ interaction in the limit in which the momentum transfer between the two pairs of Fermi particles is very small. For larger values of the momentum transfer, corrective terms appear which may account phenomenologically for the deviations of the ρ value and the ratio $R=(\pi\rightarrow e+\nu)/(\pi\rightarrow\mu+\nu)$ from the predictions of the local $V-A$ theory. The pion and muon decays are analyzed in detail. Other weak processes are briefly discussed.

1. INTRODUCTION

RECENTLY, Feynman and Gell-Mann, and Sudarshan and Marshak have proposed a universal vector minus axial vector ($V-A$) interaction for weak processes involving four fermions.[1,2] This interaction can be expressed in the form

$$H_{\rm int}=(8)^{\frac{1}{2}}G(\bar{\psi}_A\gamma_\mu a\psi_B)(\bar{\psi}_C\gamma_\mu a\psi_D)+\text{H.c.},\qquad(1)$$

where $a=(1+\gamma_5)/2$, A, B, C, and D refer to the fermion fields under consideration and the conservation of leptonic charge is assumed.

Several invariance and symmetry properties of this Hamiltonian have recently been stressed. Sudarshan and Marshak have observed the chirality invariance of Eq. (1). Sakurai has emphasized the fact that this Hamiltonian may be derived from the requirement of invariance under a special class of mass reversal transformations for every one of the fields ψ_i *separately*, namely

$$\psi_i\rightarrow\eta\gamma_5\psi_i;\quad \bar{\psi}_i\rightarrow-\eta^*\bar{\psi}_i\gamma_5,\qquad(2)$$

where the phase η is assumed to be the *same* for all the fields involved.[3] Another interesting feature of Eq. (1) is that it satisfies the relations

$$(AB)(CD)=(CB)(AD)=(AD)(CB),\qquad(3a)$$

where

$$(AB)(CD)=\sum_\sigma[\bar{\psi}_A\Gamma_\sigma\psi_B]\times[\bar{\psi}_C\Gamma_\sigma(g_\sigma+g_\sigma'\gamma_5)\psi_D]+\text{H.c.}\qquad(3b)$$

is the general four-component Hamiltonian and the ψ's have been regarded as anticommuting fields. It is perhaps interesting to observe that the condition (3) imposed on the four-component Hamiltonian plus the requirements of the two-component theory of the neutrino [i.e., either $g_i=g_i'$ or $g_i=-g_i'$ ($i=1,2\cdots5$)] determine uniquely the $V-A$ interaction except for the relative sign of g_i and g_i'.[4]

As has been discussed in great detail in references 1 and 2, this theory has had a number of very striking successes. Since then, it has received additional support from an important experiment on K capture followed by a γ transition which indicates that the dominant G-T interaction is[5] A, from the new results on the asymmetry of electrons in neutron decay,[6] and from the experiments on the asymmetry of neutrons produced in the capture of polarized muons.[7] Moreover, a reexamination of the He^6 recoil experiment which seemed to be one of the major obstacles against the $V-A$ interaction, indicates that the results of this experiment are not conclusive.[8]

Thus, Eq. (1) is able to describe the low-energy β-decay phenomena and it can also account for the gross features of the muon decay. This interaction, however, seems to present two cases of disagreement with experiment:

(1) Ruderman and Finkelstein have found long ago that the ratio R of the transition probabilities of the $\pi\rightarrow e+\nu$ and $\pi\rightarrow\mu+\nu$ decays can be calculated for pure P and A couplings independently of the details of the strong interactions of the pion with the intermediate nucleon fields.[9] The theoretical value of R for the case

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1 R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

2 E. C. G. Sudarshan and R. E. Marshak, Padua-Venice International Conference on Mesons and Recently Discovered Particles, (1957) (to be published).

3 J. J. Sakurai (to be published). In the more general case in which the phases η_i are not restricted *a priori* to be the same for all the fields, invariance under the transformation of Eq. (2) implies that the interaction is either a linear combination of S, T, and P (with $|g_S|=|g_P|$) or a linear combination of V and A (with $|g_V|=|g_A|$). In both cases the two component condition $g_i'=g_i$ ($i=1,2\cdots5$) or $g_i'=-g_i$ ($i=1,2\cdots5$) follows. Of course, the relative sign of g_i' and g_i must be determined from experiments.

4 This statement can be easily proved by a simple extension to the parity nonconserving case of the methods discussed in a paper by R. H. Good [Revs. Modern Phys. **27**, 187 (1955), see Sec. VIII]. Essentially the same result and its connection with a particular formulation of the universal theory has been emphasized by R. E. Behrends, Phys. Rev. **109**, 2217 (1958).

5 Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958).

6 These results have been communicated to the author by Professor Lederman.

7 Coffin, Sachs, and Tycko, Bull. Am. Phys. Soc. Ser. II, **3**, 52 (1958).

8 B. M. Rustad and S. L. Ruby, post-deadline paper presented at the New York Meeting of the American Physical Society, 1958.

9 M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).

of the A interaction is

$$R=\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2\sim 1.3\times 10^{-4}. \quad (4a)$$

Experimentally no $\pi\to e+\nu$ have been found and[10]

$$R_{\exp}=(-0.04\pm 0.9)\times 10^{-5}. \quad (4b)$$

(2) Recent measurements indicate that the experimental value of the Michel parameter ρ is somewhat lower than the theoretical value $\rho=\frac{3}{4}$ given by the two-component theory of the neutrino.[11,12]

Although at first sight the difficulties (1) and (2) seem to be of a very different nature, they have in common the fact that in both processes the momentum transfer is quite considerable in comparison with that involved in ordinary β decay. This suggests the possibility that corrections to the Hamiltonian of Eq. (1) may arise when the momentum transfer involved becomes comparable with a certain unit of mass, characteristic of the weak interactions.

Recently, Lee and Yang have suggested the idea that the deviations of the ρ parameter from the predictions of the two-component theory of the neutrino may be due to the fact that the interaction is not precisely local.[13] Thus, the usual Hamiltonian of Fermi processes in which the fermion fields are assumed to interact at the same point of space-time through momentum independent interactions may be a first approximation of the complete theory. Lee and Yang have restricted themselves to the consideration of a special class of nonlocal Lagrangians whose mathematical structure is suggested by the Lagrangian of processes in which a virtual boson is propagated between pairs of real fermions via local momentum independent interactions. In this case the nonlocal effects appear phenomenologically as momentum dependent corrections to the coupling constants of the uncorrected theory.

In another recent paper, Bludman and Klein have discussed phenomenologically the deviations of the ρ value in muon decay on the basis of a wider class of nonlocalities in which, for example, the derivatives of the fields may appear linearly and, hence, the transformation properties in spin space of the perturbative terms may be different from those of the uncorrected interaction[14] (for an illustration of this point see, for example, Sec. II of the present paper). These nonlocal interactions have a mathematical structure similar to that of a Hamiltonian describing processes in which intermediate fields, *including fermions*, are "virtually propagated" via local momentum independent interactions.

We would like now to emphasize the fact that this wide class of nonlocalities may not only explain the deviations of the ρ value and other parameters which appear in the theory of muon decay, but may also account for the failure of the $V-A$ interaction in the prediction of the ratio R. In fact, although the Ruderman-Finkelstein calculation of R treats the strong interaction of the pion with the intermediate fields with great generality, *it assumes that the leptons interact locally with the pion black box through momentum independent interactions.* If this assumption is removed and use is made of a sufficiently wide class of nonlocal interactions, the failure of the uncorrected theory may be explained, at least phenomenologically.

In Sec. II we proceed to construct an effective Hamiltonian for the Fermi processes which may describe phenomenologically some of the possible nonlocal effects discussed above. In Sec. III we apply this Hamiltonian to the study of various weak processes.

2. THE "EFFECTIVE" HAMILTONIAN

In this section we proceed to construct an "effective" Hamiltonian for the weak interactions which may express some of the ideas explained in the Introduction. In doing so, we will make some assumptions in order to keep the structure of the Hamiltonian as simple as possible.

We consider the Hamiltonian density

$$H_{\text{int}}=\sum_i g_i\left[\bar{\psi}_A\Gamma_i\left(\frac{p_\lambda^A}{M},\frac{p_\lambda^B}{M}\right)\psi_B\right]\times\left[\bar{\psi}_C\Gamma_i\left(\frac{p_\lambda^C}{M},\frac{p_\lambda^D}{M}\right)\psi_D\right]+\text{H.c.}, \quad (5a)$$

where the operators $\Gamma_i(p_\lambda/M)$ are functions of the four momenta p_λ of the particles involved and M is a certain mass, characteristic of the weak interaction. In the limit $p_\lambda/M\to 0$ the operators $\Gamma_i(p_\lambda/M)$ reduce to the ordinary Dirac interactions Γ_i ($i=S, V, T, A, P$) of the local theory. We adopt the convention of coupling in the same covarient the field operators of the particles which lie at the same vertex of Puppi's triangle [i.e., $(e\nu)$, $(\mu\nu)$, (NP) and possibly (ΛP), (ΣN), etc.]. We will regard this as the natural order in the discussion of the universal Fermi interaction.

(1) We assume that the particles which belong to the same pair interact essentially at the same point of space time. Under this assumption, it is clear that the operator connecting the fermions A and B will depend only on the four momentum $P_\lambda=p_\lambda^B-p_\lambda^A$ which is transferred to the pair of particles A and B by the interaction with the other pair, i.e.,

$$\Gamma\left(\frac{p_\lambda^A}{M},\frac{p_\lambda^B}{M}\right)\equiv\Gamma\left(\frac{P_\lambda}{M}\right). \quad (5b)$$

[10] H. L. Anderson and C. Lattes, Nuovo cimento **6**, 1356 (1957).

[11] K. Crowe, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1957). Crowe's value is $=0.68\pm 0.02$.

[12] L. R. Rosenson, Phys. Rev. **109**, 958 (1958). Rosenson's value is $=0.67\pm 0.05$.

[13] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957).

[14] S. Bludman and A. Klein, Phys. Rev. **109**, 550 (1958).

(2) We now postulate that in the limit of zero momentum transfer the local $V-A$ interaction is exactly valid. This imples that

$$g_i=0 \quad (i=S,\,T,\,P),$$

and

$$\lim_{P_\lambda/M\to 0} \Gamma_V\left(\frac{P_\lambda}{M}\right)=\gamma_\lambda a. \tag{5c}$$

(3) If we further assume that the condition of the two-component theory of the neutrino (i.e., $g_i'=g_i$) is valid in the nonlocal theory, Eq. (5c) and the requirement of Lorentz invariance tell us that the most general form of the covariant (AB) is

$$(AB)_\lambda=\bar{\psi}_A\left[f\gamma_\lambda+\frac{h_1}{M}P_\lambda+\frac{h_2}{M}\boldsymbol{P}\gamma_\lambda+\frac{h_3}{M^2}\boldsymbol{P}P_\lambda\right]a\psi_B, \tag{5d}$$

where, in general, f and the h_i $(i=1,\,2,\,3)$ are functions of P^2/M^2. (4) If the nonlocal interaction is strictly universal, then f and the h_i are universal functions of P^2/M^2, i.e., their dependence on P^2/M^2 would be the same in all the Fermi processes. In that case, we arrive at the following Hamiltonian density for the weak interactions

$$\begin{aligned}H_{\text{int}}&=(AB)_\lambda(CD)_\lambda\\&=\left\{\bar{\psi}_A\left[f\gamma_\lambda+\frac{h_1}{M}P_\lambda+\frac{h_2}{M}\boldsymbol{P}\gamma_\lambda+\frac{h_3}{M^2}\boldsymbol{P}P_\lambda\right]a\psi_B\right\}\\&\quad\times\left\{\bar{\psi}_C\left[f\gamma_\lambda-\frac{h_1}{M}P_\lambda-\frac{h_2}{M}\boldsymbol{P}\gamma_\lambda\right.\right.\\&\qquad\left.\left.+\frac{h_3}{M^2}\boldsymbol{P}P_\lambda\right]a\psi_D\right\}+\text{H.c.}\end{aligned} \tag{6}$$

A somewhat more general expression is

$$H_{\text{int}}=(AB)_\lambda\left(\delta_{\lambda\mu}-k\frac{P_\lambda P_\mu}{M^2}\right)(CD)_\mu, \tag{6a}$$

where k is a function of P^2/M^2. The effect of the term in $P_\lambda P_\mu$ is negligible in the problems discussed in this paper, and will not be considered in the next sections.

In the construction of Eq. (6) we have restricted ourselves to the consideration of a "nonlocal Hamiltonian" which has a structure similar to that of a diagram describing a process in which intermediate fields (*including fermions*) are "propagated" via local momentum independent interactions and such that it reduces to the local $V-A$ theory in the limit $P_\lambda/M\to 0$. (As an illustration of a Feynman diagram that may bring about terms of the form indicated in Eq. (5d), see, for example, Fig. 1.)

The assumptions (1) and (4) do not play a significant role in the present description of the failure of the local theory in the prediction of the ratio R. They are additional assumptions used to simplify the structure of the Hamiltonian and reduce the number of parameters of the nonlocal universal theory. The assumption (1) has the desirable consequence that the dynamical effects of the nonlocality vanish automatically in the case of processes involving very low momentum transfer, such as beta decay. The assumption (4) is, of course, of interest if one wishes to study possible nonlocal effects in all the Fermi interactions. If one interprets the nonlocal effects as an interaction propagated by intermediate fields, then assumption (4) is equivalent to postulating that the interaction between the leptons and the intermediate fields occurs through the same Feynman diagrams for the various Fermi processes.

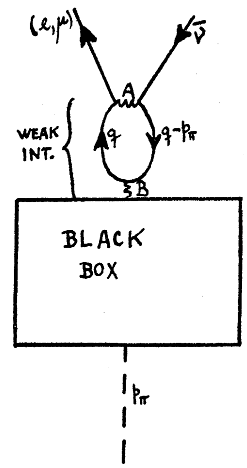

FIG. 1. A Feynman diagram for pion decay whose mathematical structure (for the $e\nu$ part of the matrix element) is similar to that indicated in Eq. (5c), even when the local interactions are momentum independent. The interactions between four fermions at A and B could be strictly local or "propagated" by intermediate fields (short wavy lines) of very large mass. The quantities q and p_π are the four-momenta of the virtual fermion and pion, respectively.

We now notice the following properties of Eq. (6):

(a) The fact that the ρ value is not very different from the prediction of the local $V-A$ theory suggests that the terms in h_i and the momentum dependence of f in Eq. (6) may be considered as small perturbations for a momentum transfer of the order of those involved in muon decay.

(b) The term in h_3 plays no important role in the discussion of the next sections and has been included only for reasons of generality.[15] As we will see in the next section, the terms in h_1 and h_2 may be very important in discussing the pion decay. The existence of at least one of them is necessary, from the point of view of this paper, to explain any large modifications in the predictions of the ratio R.†

(c) It is important to notice that Eq. (6) does no

[15] For any particular Fermi interaction, the term in h_3 can, of course, be reduced to the form of the term in h_1. We have preferred to write the Hamiltonian in the form indicated in Eq. (6) in order not to destroy the universality of the function h_i.

† *Note added in proof.*—Of course, the deviations of R may be described alternatively by adding a small pseudoscalar term to the local interaction. In the present approach the terms in h_i may be conceived as dynamical byproducts of the fundamental interactions or as small nonlocal effects, an idea which may perhaps render more plausible the smallness of these terms and which is also able to describe deviations of the ρ value. Moreover, the interaction has now a definite chirality in the limit $P_\lambda\to 0$.

longer satisfy exactly any of the invariance and symmetry properties briefly described in the introduction. From the point of view of this paper, *these invariance and symmetry requirements may be regarded as limit theorems which are exactly valid in the case of zero momentum transfer.*

3. ANALYSIS OF VARIOUS WEAK PROCESSES

In this section we proceed to analyze weak processes on the basis of Eq. (6). In the analysis of the pion and muon decay, we shall replace h_i by the first term h_i^0 in their expansion in powers of P^2/M^2. For reasons of generality we shall further write

$$f\left(\frac{P^2}{M^2}\right)=f^0+f^1\frac{P^2}{M^2}+\cdots, \tag{7a}$$

where

$$f_0^2/\sqrt{8}=G \tag{7b}$$

is the β-decay coupling constant. In this approximation, and neglecting the term in h_3 which plays no important role in the following discussions we may write Eq. (6) in the form

$$(AB)(CD)=(8)^{\frac{1}{2}}G \times\left\{\bar{\psi}_A\left[\left(1+\beta\frac{P^2}{M^2}\right)\gamma_\lambda+\epsilon_1\frac{P_\lambda}{M}+\frac{\epsilon_2}{M}\boldsymbol{P}\gamma_\lambda\right]a\psi_B\right\} \times\left\{\bar{\psi}_C\left[\left(1+\beta\frac{P^2}{M^2}\right)\gamma_\lambda-\frac{\epsilon_1}{M}P_\lambda-\frac{\epsilon_2}{M}\boldsymbol{P}\gamma_\lambda\right]a\psi_D\right\}, \tag{8a}$$

where

$$\beta=f^1/f^0 \quad\text{and}\quad \epsilon_i=h_i^0/f^0. \tag{8b}$$

(*1*) *Pion decay.*—Setting $(AB)=(e\nu)$ and $(CD)=(\eta\eta)$ (where η stands for the nucleon field in the black box) in Eq. (8a), we immediately obtain for R the result

$$R=\left[\frac{m_e(1+\beta m_\pi^2/M^2)+(\epsilon_1+\epsilon_2)m_\pi^2/M}{m_\mu(1+\beta m_\pi^2/M^2)+(\epsilon_1+\epsilon_2)m_\pi^2/M}\right]^2 \times\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2. \tag{9a}$$

Remembering that the contributions of β are relatively small in pion decay (this is best seen in the analysis of muon decay) and comparing Eq. (9a) with Eq. (4b), we get

$$\frac{(\epsilon_1+\epsilon_2)}{M}m_\pi^2\sim -m_e. \tag{9b}$$

The fact that the small corrections indicated in Eqs. (8a) and (9b) may have such a drastic effect in pion decay can be understood physically as follows. The local $V-A$ interaction almost forbids the $\pi\to e+\bar{\nu}$ decay. In fact, this interaction tends to emit the electron and antineutrino with opposite helicity. Because the $\pi\to e+\bar{\nu}$ is a two-body decay, the conservation of angular momentum requires, however, that the e and $\bar{\nu}$ should have exactly the same helicity. Thus, the conservation of angular momentum acts as a selection rule that decreases enormously the contribution of the local $V-A$ interaction and, therefore, small perturbations in the Hamiltonian may change radically the predictions of the local theory.[16]

It is well known that in the local $V-A$ interaction the theoretical value for $R_\gamma=(\pi\to e+\bar{\nu}+\gamma)/(\pi\to\mu+\bar{\nu})$ is less than the experimental upper limit.[17,18] It is clear that the inclusion of terms in ϵ_1 and ϵ_2 of the order of magnitude indicated in Eq. (9b) does not alter this satisfactory situation.

(*2*) *Muon decay.*—We shall regard the terms in β and ϵ_i as small perturbations for the momentum transfers involved in muon decay. Setting $H_{\text{int}}=(\mu\nu)^\dagger(e\nu)$ and neglecting terms of relative order m_e/p (p=electron momentum), and higher powers of β and ϵ_i, we get the following expression for the energy-angle distribution of electrons from polarized muons[19]:

$$dN(x,\theta)=\frac{m_\mu^5G^2}{3\times 2^7\pi^4}x^2dxd\Omega\{3-2x+\bar{\beta}x(2-x)+\bar{\epsilon}_2x +\cos\theta[1-2x-\bar{\beta}x^2-\bar{\epsilon}_2x]\}, \tag{10a}$$

where

$$\bar{\beta}=2\beta m_\mu^2/M^2;\quad \bar{\epsilon}_2=2\epsilon_2m_\mu/M. \tag{10b}$$

The quantity $x=p/p_{\max}$ and θ is the angle between the momenta of the electron and muon. With this definition of θ, Eq. (10a) is valid for the decays of μ^- and μ^+. The corrections proportional to $\bar{\beta}$ are the same as those studied in case II of reference 13. The corrections proportional to $\bar{\epsilon}_2$ are of the general type indicated in reference 14. It is interesting to observe that the terms proportional to ϵ_1 in Eq. (8a) contribute terms of order ϵ_1m_e/p in muon decay and are thus negligible.[20]

In the approximation explained above, the electrons in muon decay are completely polarized with negative helicity at all angles and energies. Thus, this important

[16] The $V-A$ interaction would exactly forbid the pion decay if the electron had zero mass.

[17] S. B. Treiman and H. W. Wyld, Phys. Rev. **101**, 1552 (1956).

[18] E. C. G. Sudarshan and R. E. Marshak, Bull. Am. Phys. Soc. Ser. II, **3**, 20 (1958).

[19] To describe the absorption of a muon and the creation of a neutrino, we use the covariant $(\mu\nu)^\dagger$ where $(\mu\nu)$ is defined by setting $A\equiv\mu$ and $B\equiv\nu$ in Eq. (5d).

[20] In our opinion, the analysis of the muon decay in reference 14 has been overcomplicated to a certain extent. In fact, the contribution to the matrix element of muon decay of the third term of Eq. (3) of that paper is a linear combination of the contributions of the first two terms. This observation is necessary in order to understand why we get a smaller number of independent parameters arising from those terms of the Hamiltonian which are linear in the derivatives of the fields. Moreover, our approach to the terms which are quadratic in the derivatives of the fields is different from that of reference 14 and is indicated by Eq. (8a) with the understanding that higher powers of the ϵ_i and β are neglected in muon decay.

prediction of the local $V-A$ theory is not modified by the first-order nonlocal effects discussed here.[21]

If $\bar{\beta}\neq 0$, the expression between curly brackets in Eq. (10a) is no longer a linear function of x and the momentum dependence cannot be described exactly by a single parameter. However, if $\bar{\beta}$ is small the spectrum of Eq. (10a) can be described to a good approximation by the Michel parameter ρ. We now require that the ratio of the values of the spectral distribution of Eq. (10a) at $x=\frac{1}{2}$ and at $x=1$ should be equal to the same ratio calculated from the local four-component theory with the experimental value of ρ. In this manner we get the "ρ condition":

$$\frac{2+\frac{3}{4}\bar{\beta}+\frac{1}{2}\bar{\epsilon}_2}{1+\bar{\beta}+\bar{\epsilon}_2}=\frac{9}{4\rho}-1. \quad (11a)$$

Setting $\rho=0.68$, Eq. (10c) reduces to

$$1.56\bar{\beta}+1.81\,\bar{\epsilon}_2=-0.31. \quad (11b)$$

The asymmetry parameter ξ is given by

$$\xi=\frac{1+(6/5)\bar{\beta}+\frac{3}{2}\bar{\epsilon}_2}{1+\frac{3}{5}\bar{\beta}+\frac{1}{2}\bar{\epsilon}_2}. \quad (12a)$$

The experimental value is[22]

$$\xi_{\text{exp}}=0.87\pm 0.12. \quad (12b)$$

The $V-A$ interaction predicts $\xi=1$.

The parameters $\bar{\beta}$ and $\bar{\epsilon}_2$ are very sensitive to the value of ξ. Because of the large error in Eq. (12b) it is impossible to determine uniquely these parameters at the present time. However, one can make the following observations about the parameter ξ:

(a) If $\bar{\epsilon}_2$ is of the same order of magnitude as the quantity $\epsilon_1+\epsilon_2$ which was determined in Eq. (9b), its effect on muon decay is negligible. From the "ρ condition" [Eq. (11b)] we get

$$\bar{\beta}=-0.20;\quad \bar{\epsilon}_2\ll\bar{\beta}. \quad (13)$$

From Eq. (12a) we can calculate ξ and we obtain, then, $\xi=0.87$ in good agreement with Eq. (12b).[23]

(b) Even if $\bar{\epsilon}_2$ is not negligible in muon decay, it is plausible to assume that $\bar{\beta}$ and $\bar{\epsilon}_2$ are relatively small in comparison with unity. In fact, if $\bar{\beta}$ were relatively large, the deviations from linearity in the expression between curly brackets in Eq. (10a) could be detected experimentally. If we arbitrarily assume that $|\bar{\beta}|<0.2$, Eqs. (11b) and (12a) tell us that $\bar{\epsilon}_2\leq 0$ and that $\xi\leq 0.87$. If we further require that $|\bar{\epsilon}_2|\leq 0.2$ we get the result that $0.80\leq\xi\leq 0.87$.

Thus, if the ρ value is indeed near 0.68, the present analysis suggests that ξ should be less than 1, the correction being of the order of 10% to 20%. It would be very interesting to decrease the error in the experimental determination of ξ to verify, at least, if its value is actually less than 1.

Another effect of the nonlocal interactions discussed here is to make more rapid the increase of the integrated asymmetry $a(x)$ with x, although the deviations from the predictions of the local theory are small if the study of $a(x)$ is limited to the upper half of the spectrum. In this connection it would be desirable to have an accurate determination of the parameter δ which measures the momentum dependence of the asymmetry.[24]

On the basis of Eq. (10a), we find that the lifetime of the muon is given by

$$\tau_{\text{n.l.}}=\frac{\tau_1}{1+\frac{3}{5}\bar{\beta}+\frac{1}{2}\bar{\epsilon}_2}, \quad (14a)$$

where τ_1 is the result of the local theory. Assuming that the vector part of the coupling constant in β decay is not affected by the pion cloud, Feynman and Gell-Mann have found

$$\tau_1=(2.26\pm 0.04)\times 10^{-6}\text{ sec}, \quad (14b)$$

which is in very good agreement with the experimental value

$$\tau_{\text{exp}}=(2.22\pm 0.02)\times 10^{-6}\text{ sec}. \quad (14c)$$

However, if the ρ value is approximately equal to 0.68, any reasonable combination of the coefficients $\bar{\beta}$ and $\bar{\epsilon}_2$ which can explain the difference with $\rho=0.75$, would increase the value of τ by about 10% to 15% so that the accurate agreement with experiment would not hold any longer. The difference might be accounted for by considering the strong π coupling effects in β decay. As the muon lifetime calculated from Eqs. (14a) and (14b) would be about 10% to 15% larger than the experimental value of Eq. (14c), this suggests that the pion cloud in β decay may decrease the effective value of $|g_V|^2$ by about 10% to 15%, a fact which could partly account also for the experimental ratio $\Sigma|g_{\text{GT}}|^2/\Sigma|g_F|^2=1.3\pm 0.1$.[25]

Several of the conclusions mentioned above are based on an experimental result $\rho\sim 0.68$. Should this value change significantly, our conclusions must be altered accordingly. However, the discussion on the pion decay given above would not be significantly affected by a modification in the value of ρ.

(3) K-Meson Decay.—In the spirit of this paper, we cannot predict the ratio $R_K=(K\rightarrow e+\nu)/(K\rightarrow\mu+\nu)$

[21] On the other hand, a theorem relating the longitudinal polarization of the electrons to the asymmetry parameter ξ [T. Kinoshita and A. Sirlin, Phys. Rev. **106**, 1110 (1957)] is no longer exactly valid when the nonlocal effects are included.

[22] D. H. Wilkinson Nuovo cimento **6**, 516 (1957).

[23] As was mentioned in Sec. II of reference 13, the negative sign of $\bar{\beta}$ in Eq. (13) indicates that the nonlocality cannot be interpreted as an interaction between the $(e\nu)$ and $(\nu\mu)$ pairs "propagated" by a single intermediate boson via a local momentum independent Lagrangian. It is interesting to remark that such a model would not only predict $\rho>\frac{3}{4}$ but also $\xi>1$ in the framework of the $V-A$ interaction.

[24] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957).

[25] A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. (to be published).

from the value of $\epsilon_1+\epsilon_2$ inferred from π decay, because the momentum transferred in K decay is considerably larger and higher powers of P^2/M^2 in the expansion of the functions h_i of Eq. (6) may become important. However, there exists the possibility that the nonlocal effects discussed here may increase the probability of the electron mode of decay. As an illustration of this point, if we make the naive assumption that the functions h_i are independent of P^2/M^2 and we use the values of $\epsilon_1+\epsilon_2$ determined from π decay and a reasonable value for $\bar{\beta}$ [we use the value of Eq. (13)] we get $1.3\times10^{-3}\leq R_K\leq 3.6\times10^{-3}$ which is smaller than the present upper limit $R_{K\text{ exp}}\leq 0.02$,[26] but considerably larger than the prediction of the local theory.

The nonlocal effects discussed here do not significantly affect the ratio $(K\rightarrow e+\pi+\nu)/(K\rightarrow\mu+\pi+\nu)$ because in these three-body decays the contribution of the local theory is relatively large [see discussion after Eq. (9b)].

In conclusion, we would like to mention that these nonlocal effects may change somewhat the predictions of local theories on the lifetime of such processes as $\Sigma\rightarrow p+e+\nu$. For example, if the function $f(P^2/M^2)$ can still be represented by the two terms of Eq. (7a) for the range of energies involved in Σ decay and the value of Eq. (13) for $\bar{\beta}$ is used, then it is easily seen that the prediction for the lifetime, as compared to the lifetime obtained in calculations which treat the weak interaction locally, is increased by a factor $\approx\frac{5}{2}$.

[26] M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Physics* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 407.

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Quantization Process for Massless Particles*

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This paper gives the quantization of a recently proposed theory for particles of arbitrary spin and zero mass. An interesting result is that there is a connection between the spin and the statistics of the particles. It is found that the spinor components of a boson/fermion field with integral/half-integral spin commute/anticommute off of the light cone whereas the spinor components of a boson/fermion field with half-integral/integral spin do not. As in the unquantized theory, the two-component neutrino and the photon are special cases.

I. INTRODUCTION

Recently a wave equation for massless particles was proposed[1] in which the Hamiltonian is[2]

$$H=(c/s)\mathbf{p}\cdot\mathbf{s} \tag{1}$$

($\mathbf{p}$ being the operator $-i\hbar\nabla$ and $\mathbf{s}$ being the angular momentum matrices for arbitrary spin s), and in which the wave function ϕ is related to the spin or components ψ of the field by

$$\psi=|H/c|^{s-\frac{1}{2}}\phi. \tag{2}$$

Also, as an auxiliary condition, only solutions with spin parallel or antiparallel to the momentum are retained. The purpose of this paper is to give the quantization of the theory. Since a uniform treatment of all spins is made, it is of interest to see how the spin and statistics are related and to find operator assignments for the number of particles, energy, momentum, and angular momentum.

The quantization process can be carried out in a straightforward way, using the coefficients of an expansion in plane waves. It is found that the spinor components of a boson/fermion field with integral/half-integral spin commute/anticommute off of the light cone, whereas the spinor components of a boson/fermion field with half-integral/integral spin do not. Also the different statistics lead to different expressions for the operators when they are written in terms of the wave function. For example, with Fermi-Dirac statistics the quantized Hamiltonian corresponds to the expectation value of the unquantized Hamiltonian, whereas with Bose-Einstein statistics it corresponds to the expectation value of the unquantized energy operator.

The equations for the spinor components ψ are a special case of the general Dirac-Pauli-Fierz theory,[3]

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[1] C. L. Hammer and R. H. Good, Jr., Phys. Rev. **108**, 882 (1957).

[2] The notation throughout the paper is the same as in reference 1.

[3] See, for example, H. Umezawa, *Quantum Field Theory* (Interscience Publishers, Inc., New York, 1956), Chap. IV, Sec. 3.