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Scattering of μ^- Mesons by Nuclei*

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Previous hand calculations for the scattering of μ^- mesons by nuclei have been extended by using a IBM-650 Computer, to include energies corresponding to $v/c=0.2$ ($E=2.1$ Mev) and 0.4 ($E=9.6$ Mev) for the nuclei $Z=48$ and 80, the nucleus being considered as a uniformly charged sphere of radius $R=1.2A^{\frac{1}{3}}\times10^{-13}$ cm.

INTRODUCTION

IN an earlier paper[1] we have discussed the interest in scattering polarized μ mesons from heavy nuclei. In that paper the results of Sherman[2] for the scattering of electrons by point nuclei were corrected by an exact phase-shift analysis for the scattering of 2.1-Mev ($v/c=0.2$) μ^- mesons by a uniformly charged nucleus of $Z=80$ and radius $R=1.2A^{\frac{1}{3}}\times10^{-13}$ cm. The method of reference 1 has been extended using an IBM-650 computer to include energies corresponding to $v/c=0.2$ ($E=2.1$ Mev) and 0.4 ($E=9.6$ Mev) for uniformly charged nuclei of $Z=48$ and 80, and radius $R=1.2A^{\frac{1}{3}}\times10^{-13}$ cm. To facilitate machine programing the calculation was modified from the previous one in two respects: (1) In reference 1 the logarithmic derivatives of the large components of the μ-meson wave function are joined at the nuclear surface to determine phase shifts. For the machine program, large and small components were joined simultaneously in a method described by Sliv and Volchok.[3] (2) The outside wave functions were calculated by power series solution of the radial Dirac equations as described by Yennie, Ravenhall, and Wilson.[4] In the previous paper they had been calculated from confluent hypergeometric functions.

RESULTS

In Tables I–IV are given the cross section, $\sigma(\theta)$, and left-right asymmetry parameters, $S(\theta)$, for the various cases. The ratio of the cross section to Sherman's point-nucleus results and the asymmetry parameter for a point nucleus are also listed for comparison. In terms of $S(\theta)$ and $\sigma(\theta)$ the differential cross section for a particle of polarization $\boldsymbol{\sigma}$ and momentum direction $\hat{p}_1$ scattering to momentum direction $\hat{p}_2$ is

$$\frac{d\sigma(\theta,\varphi)}{d\Omega}=\sigma(\theta)\left[1+S(\theta)\frac{\boldsymbol{\sigma}\cdot\hat{p}_1\times\hat{p}_2}{\sin\theta}\right].$$

A fuller discussion and formulas for obtaining $S(\theta)$ and $\sigma(\theta)$ are given in reference 1.

Results are not given for energies higher than 10 Mev because the 3-figure accuracy of Sherman's tables is not sufficient where there is cancellation due to finite-size effects. Furthermore, at higher energies the use of a uniform charge distribution would not be expected to yield realistic results, especially at large angles where in

TABLE I. The differential cross section and asymmetry parameter for the case $Z=80$, $v/c=0.2$. σ_0 and $S_0(\theta)$ refer to the point nucleus results of Sherman.

θ	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
σ (barns)	6200	370	87	38	17	6.8	2.8	1.3	0.75	0.64	0.67
σ/σ_0	1.0	0.92	1.02	1.37	1.35	0.99	0.57	0.30	0.19	0.16	0.17
$100\,S(\theta)$	0.3	0.0	1.0	0.2	−1	−3	−5	−11	−18	−17	−10
$100\,S_0(\theta)$	0.21	−0.19	−0.96	5.6	8.2	−3.6	−20	−28	−26	−19	−9.6

TABLE II. The differential cross section and asymmetry parameter for the case $Z=80$, $v/c=0.4$.

θ	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
σ (barns)	340	28	5.1	1.1	0.29	0.085	0.032	0.015	0.0088	0.0065	0.0058
σ/σ_0	1.00	1.27	1.10	0.66	0.32	0.15	0.082	0.049	0.034	0.028	0.027
$100\,S(\theta)$	0.2	0.0	−1	−2	−4	−5	−8	−15	−20	−22	−14
$100\,S_0(\theta)$	−0.04	1.5	3.9	0.21	−10.4	−23	−33	−37	−34	−26	−14

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[1] J. Franklin and B. Margolis, Phys. Rev. **109**, 525 (1958).
[2] N. Sherman, Phys. Rev. **103**, 1601 (1956).
[3] L. A. Sliv and B. A. Volchok, "Tables of Coulomb phases and amplitudes taking into account the finite nuclear size," Academy of Sciences of the U.S.S.R. Report, 1956 [translation: Atomic Energy Commission Report AEC-tr-2875 (Office of Technical Information Services, Washington, D. C., 1957)].
[4] Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954).

TABLE III. The differential cross section and asymmetry parameter for the case $Z=48$, $v/c=0.2$.

θ	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
σ (barns)	2200	150	35	12	4.6	2.0	0.90	0.48	0.31	0.24	0.22
σ/σ_0	0.98	1.04	1.18	1.14	0.93	0.69	0.48	0.35	0.28	0.25	0.24
$100\,S(\theta)$	0.0	0.3	0.3	−0.5	−2	−4	−7	−10	−11	−9	−5
$100\,S_0(\theta)$	−0.0	0.4	1.2	0.08	−3.7	−8	−11	−12	−11	−8.2	−4.4

TABLE IV. The differential cross section and asymmetry parameter for the case $Z=48$, $v/c=0.4$.

θ	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
σ (barns)	130	8.6	1.5	0.37	0.10	0.031	0.010	0.0044	0.0027	0.0023	0.0022
σ/σ_0	1.05	1.04	0.83	0.56	0.34	0.18	0.098	0.059	0.047	0.048	0.052
$100\,S(\theta)$	0.06	−0.2	−0.9	−1.6	−2	−3	−5	−9	−14	−13	−8
$100\,S_0(\theta)$	0.2	0.2	−1.2	−4.3	−8.3	−12	−15	−16	−15	−12	−6.3

fact polarization effects are expected to be large. The cross section falls off sharply with increasing energy, particularly for a finite-size nucleus so that measurements of μ-meson scattering above 10 Mev would be extremely difficult.

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Scattering of 220-Mev Polarized Protons by Complex Nuclei*

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Extending earlier measurements, we have studied yields and asymmetries in the scattering of a highly polarized proton beam by Be^9, C^{12}, Al^{27}, and Ca^{40}. In the work on Be^9 and Al^{27}, the beam was monochromatized through the use of a regenerative deflector; its mean energy on striking the targets was 219.6 Mev, and its standard deviation in energy (including the effect of short time fluctuations) was less than 1.1 Mev. With this technique, together with a refined procedure for the analysis of the distribution in range of scattered protons, we have been able to separate elastic from inelastic scattering in Be^9 at angles from 8° to 37.5°, and to estimate the inelastic scattering involving the excited state at 2.4 Mev. The separation of elastic scattering in Ca^{40} could also be made, although the regenerator was not used. The results are compared with approximate calculations based on the optical model with $\boldsymbol{\sigma}\cdot\mathbf{L}$ coupling; a potential is found such that, with variation of the nuclear radius alone, good fits are obtained to measurements on the four nuclei.

INTRODUCTION

POLARIZATION effects in the elastic scattering of high-energy protons have been studied intensively at this laboratory[1] and elsewhere[2] over an energy interval extending from 60 Mev to 660 Mev. It has become clear from the experiments that the polarization can be very large, and can in fact approach 100% under certain conditions. The first strong maximum in the function $P(\theta)$ is usually found when $2kR\,\sin(\theta/2)\simeq 2.2$, k and θ being the wave number and scattering angle in the center-of-mass system, and R the nuclear radius. Some experiments[1,3] have also revealed subsequent minima and maxima in $P(\theta)$ at large angles, but only after careful elimination of inelastic events that can compete strongly with elastic scattering in the angular region beyond the first maximum in the polarization. Measurements in which such separation has been accomplished have given data on polarization effects involving one or more of the lowest excited states of the target nuclei.

Most theoretical accounts[2] of the qualitative features of the elastic polarization have been achieved through the addition of a spin-orbit term to the central potential of the usual optical model. It was the purpose of the present work to extend our original measurements at 220-Mev to a variety of nuclei, and then to investigate

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[1] Chesnut, Hafner, and Roberts, Phys. Rev. **104**, 449 (1956); referred to in the text of this paper as CHR.

[2] An excellent bibliography of most of the experimental and theoretical work in this field is found in the review article by L. Wolfenstein, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1956), Vol. 6, p. 43.

[3] Alphonce, Johannson, and Tibell, Nuclear Phys. **3**, 185 (1957).