Measurement of the Neutron Strength Function $\bar{\Gamma}_n^0/D$ for Nuclides with 45 to 75 Nucleons*

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Neutron transmission measurements have been made on many of the nuclides whose atomic weights lie between 45 and 75. Parameters of individual resonances were deduced for some of these nuclides and for all of them a value of the strength function $\bar{\Gamma}_n^0/D$ was obtained. The experimental values of the strength function indicate a maximum near $A=53$. This is consistent with the prediction of the theory of Feshbach, Porter and Weisskopf for a square-well potential with $V_0=44$ Mev, $\zeta=0.03$ and $R=(1.26A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm. The shape of the observed maximum is not in agreement with this simple theory, however, so a discussion is given which compares the present measurements with predictions based on more realistic models of the nucleus.

INTRODUCTION

THE Argonne fast-neutron velocity selector[1] has been used to measure the neutron transmission of 14 nuclides in the mass range from 45 to 75. The primary objective of these experiments was to obtain values of the neutron strength function $\bar{\Gamma}_n^0/D$. Many experimental studies have shown that D, the average spacing per spin state and $\bar{\Gamma}_n^0$, the average reduced neutron width, defined as the average of the neutron widths divided by the square root of the corresponding resonant energies, vary by several orders of magnitude from one nuclide to another. However, the ratio $\bar{\Gamma}_n^0/D$ shows little variation with nucleon number. The "strong absorption" theory of Feshbach, Peaslee, and Weisskopf[2] based on the so-called "black nucleus" model of the nucleus predicts no variation of this ratio with atomic mass, whereas the theoretical treatments based on the optical model, first introduced by Serber, Fernbach, and Taylor[3] and extended to low-energy neutron scattering by Feshbach, Porter, and Weisskopf,[4] do predict such a variation which, in fact, exhibits several maxima. Data have been presented which show the existence of such maxima near $A=55$[5,6] and $A=155$,[7] in qualitative agreement with the theory. The present work is a study of the resonance structure of elements which contribute to the maximum near $A=55$.

ANALYSIS

In the study of the resonant structure of nuclides in the mass region of interest to this work, two classes of structure were observed. The first consists of structure in which the resonances are closely spaced so that many lie in the energy region in which the resolution of the equipment is sufficient to allow a study of individual resonances. The second class is made up of the structure of the lighter nuclides, in which the level spacing is large. In these cases the majority of the resonances lie at high energies where the resolution of the equipment is not sufficient to permit individual resonances to be studied.

As a consequence of the existence of these two classes, two different methods of analysis are required in order to obtain values of the strength function. Data which belong to the first class were analyzed by the usual area techniques.[8,9] The strength function was then deduced from the measured individual widths.

For data of the second class, in which individual resonances could not be resolved, values of the strength function were deduced from the area above the measured transmission curves over a broad range of energy. Other investigators[10,11] have used the same kind of information for the heavier nuclides but, since their techniques rest on basic assumptions which are not valid for the light nuclides studied here, a different method was necessary.

To aid in relating the area above a transmission curve to $\bar{\Gamma}_n^0/D$, the following simplifying assumptions were made.

1. The resonances satisfy the single-level Breit-Wigner relations. For the present measurements we need consider only the case of target nuclei which interact with neutrons by radiative capture or by s-wave elastic-scattering processes, with which are associated capture and scattering cross sections σ_γ and σ_s. The total cross section is then given by

$$\sigma=\sigma_\gamma+\sigma_s,$$

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[1] Bollinger, Coté, Dahlberg, and Thomas, Phys. Rev. **105**, 661 (1957).
[2] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[3] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[4] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[5] R. Coté and L. M. Bollinger, Phys. Rev. **98**, 1162 (1955).
[6] H. Marshak and H. W. Newson, Phys. Rev. **106**, 110 (1957), hereafter referred to as MN.
[7] Carter, Harvey, Hughes, and Pilcher, Phys. Rev. **96**, 113 (1954).
[8] Melkonian, Havens, and Rainwater, Phys. Rev. **92**, 702 (1953).
[9] Seidl, Hughes, Palevsky, Levin, Kato, and Sjöstrand, Phys. Rev. **95**, 476 (1954).
[10] D. B. Gayther and K. P. Nicholson, Proc. Phys. Soc. (London) **A70**, 51 (1957).
[11] D. J. Hughes and V. E. Pilcher, Phys. Rev. **100**, 1249(A) (1955).

with

$$\sigma_\gamma=\sigma_0\frac{\Gamma_\gamma}{\Gamma}\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{1}{1+x^2}, \quad (1)$$

and

$$\sigma_s=4\pi R'^2+\sigma_0\frac{\Gamma_n}{\Gamma}\left[1+\left(\frac{2R'\Gamma}{\lambda\!\!\!^{-}_0\Gamma_n}\right)x\right]\frac{1}{1+x^2}, \quad (2)$$

where

$$\begin{aligned}&\Gamma=\Gamma_n+\Gamma_\gamma,\\ &x=(2/\Gamma)(E-E_0),\\ &\sigma_0=4\pi\lambda\!\!\!^{-}_0{}^2g\frac{\Gamma_n}{\Gamma}=\frac{2.6\times10^6}{E_0(\text{ev})}g\frac{\Gamma_n}{\Gamma}.\end{aligned} \quad (3)$$

In these relations E is the kinetic energy of the neutron; E_0, $2\pi\lambda\!\!\!^{-}_0$, and σ_0 are the neutron energy, wavelength, and cross section at exact resonance; and Γ, Γ_γ, and Γ_n are the total, γ ray, and neutron widths, respectively. The statistical factor g is given by

$$g=\tfrac{1}{2}[1\pm1/(2I+1)],$$

where I is the spin of the target nucleus; and R' is the nuclear radius as defined by $\sigma_p=4\pi R'^2$, where σ_p is the potential-scattering cross section.

2. The radiation width Γ_γ is small compared with the total width Γ, so that one may write $\Gamma_n/\Gamma=1$. On the basis of those radiative capture widths which have been measured, any such widths may be expected to be less than 1 ev.[12]

3. The width resulting from the thermal motion of the target nuclei, i.e., Doppler broadening, is small compared with the total width, so that it may be neglected. In the calculation, $\Delta=2(mkTE_0/M)^{\frac{1}{2}}$ has been taken to be zero. Here m is the mass of the neutron, M is the atomic mass of the target, k is the Boltzmann constant, T is the absolute temperature, and E_0 is the energy of the incident neutron. For the cases studied, Δ may be as large as several ev, but the total widths are one hundred times this in most instances.

4. Interference between potential and resonant scattering can be neglected.

5. The resonances in the structures studied do not overlap, i.e., the transmission at any energy is approximately equal to that resulting only from the nearest resonance and the potential scattering.

On the basis of the above assumptions, it is easily shown (see Appendix I) that the strength function may be deduced from the measured transmissions by the relation

$$\frac{\bar{\Gamma}_n{}^0}{D}\equiv\frac{\sum_r(g\Gamma_n{}^0)_r}{\Delta E}\approx\frac{2(2.6\times10^6\pi n\bar{g})^{-\frac{1}{2}}}{\Delta E}\bar{g}\times\sum_j\frac{E_j}{\tau_j}\left(\frac{T_0-T_j}{T_0Y_j}\right)\Delta\tau. \quad (4)$$

[12] J. S. Levin and D. J. Hughes, Phys. Rev. **101**, 1328 (1956).

Here E_j, τ_j, and T_j are the incident neutron energy, time of flight, and transmission corresponding to the time channel j. The sum is over all channels contained in the energy interval ΔE. T_0 is the off-resonance transmission, a quantity determined by the effective potential-scattering cross section. Y is a tabulated correction factor which is introduced to take into account self-shielding effects at the resonances. The statistical factor $\bar{g}$ is unity for target nuclei which have zero spin. For nuclei of nonzero spin, the average of the two possible values of g, namely $\bar{g}=\frac{1}{2}$ was used. In defining the strength function as is done in Eq. (4), the customary assumption is made that the two possible spin states contribute equally to the cross section for compound nucleus formation.

Equation (4) depends on some of the unknown parameters to the extent that these parameters appear in Y and g. The question of the dependence on g has already been covered, but the dependence through Y may require some clarification. In order to obtain proper values of Y_j it is necessary to know a corresponding value of $n\sigma_0$. These were computed from Eq. (3) on the basis of assumption 2. This simply assumes that all the resonances have a peak height σ_0 that is determined by an average of the theoretical maxima for the allowed spin states. It is clear that only one value of Y should be associated with each resonance but, since the locations of the resonances are not always known, a value of Y_j is computed for each measured point.

Since the number of resonances included in the determination of many of the values of $\bar{\Gamma}_n{}^0/D$ was small, some care was taken in the evaluation of the errors assigned to each value. For ease in calculation, the assumption is made that both the reduced neutron widths and the level spacings are distributed exponentially; i.e.,

$$\rho(x)dx=\lambda e^{-\lambda x}dx,$$

where $\lambda=1/\bar{x}$. On this basis, the distribution of the average values of either of the above quantities is

$$\rho(z)dz=\frac{N^n}{(N-1)!}\frac{1}{\bar{x}}\left(\frac{z}{\bar{x}}\right)^{N-1}\exp(-Nz/\bar{x})dz,$$

where $z=(x_1+x_2+\cdots+x_i\cdots+x_N)/N$ and $\bar{x}$ is the true average value of the x_i. A joint density function can be formed from those for the spacings and reduced neutron widths. It is

$$\rho\left(\frac{x/y}{z/s}\right)=\frac{(zN-1)!}{[(N-1)!]^2}\frac{(\bar{\alpha}/\alpha)^{N-1}}{(1+\bar{\alpha}/\alpha)^{2N}},$$

where $\bar{x}$ and $\bar{y}$ are the actual averages, $s=(y_1+y_2+\cdots y_i+\cdots+y_N)/N$, $\alpha=z/s$ and $\bar{\alpha}=\bar{x}/\bar{y}$. There is equal probability that the measured ratio will be larger or smaller than the true ratio.

The probable errors for the average parameters reported in this paper are defined so that the probability

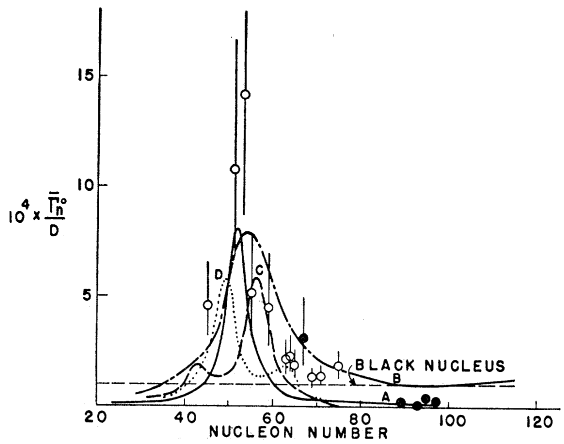

FIG. 1. Comparison of experimental values of the strength function $\bar{\Gamma}_n^0/D$ with the predictions of the optical and strong absorption models. The points marked by o are results reported in this paper and represent the values of $\bar{\Gamma}_n^0/D$ for Sc^{45}, V^{51}, Cr^{53}, Mn^{55}, Co^{59}, Cu^{63}, Zn^{64}, Cu^{65}, Ga^{69}, Ga^{71}, and As^{75}. The results of several other investigators have been added to increase the scope of the experimental data in order to demonstrate more clearly the areas of agreement and disagreement between the results of the various theoretical calculations and the experimental evidence. These points are for the following: Zn^{67}—Dahlberg and Bollinger[17]; Y^{89}—Good, Neiler, and Gibbons, Phys. Rev. **109**, 926 (1958); Nb^{93}—Saplakoglu, Bollinger, and Coté, Phys. Rev. **109**, 1258 (1958); Mo^{95} and Mo^{97}—Harvey, Hughes, Carter, and Pilcher, Phys. Rev. **99**, 10 (1955). Curve *A* was calculated on the basis of a square-well potential with $V_0=44$ Mev, $\zeta=0.03$, and $R=(1.26A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm. Curve *B* was taken from Weisskopf.[20] A diffuse-edge potential was assumed with $V_1=-V_0(1+\exp[(r-R)/d])^{-1}$ and $\zeta=0.08$, in which $V_0=42$ Mev, $R=1.35A^{\frac{1}{3}}\times10^{-13}$ cm, and $K_0d=1.65$. Curve *C* was taken from Vladimirskii.[22] The nucleus was assumed to be a prolate spheroid with (minor axis/major axis) = 0.9 and with a square-well potential for which $V_0=42$ Mev, $\zeta=0.03$, and $R=1.45A^{\frac{1}{3}}\times10^{-13}$ cm. Curve *D* was obtained from *C* and represents the case in which the nucleus is assumed to be an oblate spheroid.

that the true value lies between the indicated limits is 0.5. Although the value of the strength function can be determined without a knowledge of *N*, the number of resonances included in the average, the error associated with each measurement cannot. For those cases in which individual resonances were studied, *N* was obtained without question. For those cases in which individual resonances were not separated, an estimate based on the shape of the resonance structure and on any additional information regarding the location of energy levels was made.

RESULTS

Portions of the results for the elements studied have been reported previously in the form of curves of cross section *vs* energy,[13] so such a presentation has been omitted here. The values of $\bar{\Gamma}_n^0/D$ that were obtained from this series of experiments, along with the limits for each measurement, are listed in Table I and shown in Fig. 1.

[13] *Neutron Cross Sections*, compiled by D. J. Hughes and J. A. Harvey, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

SCANDIUM

The present measurement of the total cross section of scandium was restricted to the energy range from 0.7 kev to 40 kev. Since Pattenden's[14] data show no resonances between 0.0015 ev and 0.7 kev, this region was not studied. The scandium thicknesses in the Sc_2O_3 samples used were 0.712 g/cm^2 and 1.938 g/cm^2. Recently[6] MN published data on the total cross section of scandium which extend from 2 to 105 kev. The present data are in qualitative agreement with these except that two additional resonances were observed—at 3.36 and 6.8 kev. The value of $\bar{\Gamma}_n^0/D$ was obtained by the method of Eq. (4) and is listed in Table I.

VANADIUM

The total cross section of vanadium was studied over the range from 0.100 kev to about 30 kev. Normal vanadium samples with thicknesses of 1.03 and 8.25 g/cm^2 were used. An additional sample which was used had a thickness of 0.109 g/cm^2 and was enriched to 22.8% in V^{50}.

MN have reported resonances in vanadium at 4.1, 6.6, 11.5, 16.6, and 22.1 kev.[6] These results are in qualitative agreement with the present data in which peaks were observed at 4.18, 6.4, and 11.5 kev. The shape of the transmission curve above 11.5 kev is such as to indicate the presence of additional resonances consistent with the data of MN. Analysis of the resonance at 4.18 kev yields a value of $\Gamma=0.5\pm0.1$ kev, a result also in agreement with the data of MN.

Resonances were observed at 167 ev and 1430 ev which have been assigned to V^{50} on the basis of measurements on the 8.25-g/cm^2 sample of normal vanadium and the sample enriched in V^{50}. Upon the assumption that $\Gamma_\gamma=0.6$ ev and $g=\frac{1}{2}$, values of $\sigma_0=6200$ barns and $\Gamma=2.85$ ev were obtained for the 167-ev resonance. A value of $\Gamma=100\pm40$ ev was obtained for the 1430-ev resonance.

In order to obtain a value of $\bar{\Gamma}_n^0/D$ for V^{51} from the data, the method of Eq. (4) was applied.

TABLE I. Experimental values of $\bar{\Gamma}_n^0/D$ and their range of uncertainty.

Nuclide	$10^4\times(\bar{\Gamma}_n^0/D)$	Probable range of $10^4\times(\Gamma_n^0/D)$
Sc^{45}	4.6	3.2 – 6.6
V^{51}	10.8	6.9 –16.7
Cr^{53}	14.2	8.7 –22.8
Mn^{55}	5.1	3.2 – 7.8
Co^{59}	4.5	2.9 – 6.9
Cu^{63}	2.16	1.53– 3.02
Zn^{64}	2.30	1.61– 3.32
Cu^{65}	1.90	1.35– 2.66
Ga^{69}	1.17	0.87– 1.51
Ga^{71}	1.30	0.96– 1.74
As^{75}	1.83	1.30– 2.56

[14] N. J. Pattenden, Proc. Phys. Soc. (London) **A68**, 104 (1955).

CHROMIUM

Samples of normal chromium 10.6 g/cm² and 1.38 g/cm² thick were used as well as the separated isotopes listed in Table II. Figure 2 shows transmission as a function of time of flight for normal chromium and for samples enriched in Cr^{50} and Cr^{53}. No resonance structure was observed below 15 kev in Cr^{52} and Cr^{54}.

These results confirm the suggestion of Melkonian, Havens, and Rainwater,[8] with regard to their data on normal chromium, that the resonance structure at about 5 kev is due to Cr^{53} and Cr^{50}.

The method of Eq. (4) was applied to the data on Cr^{53} and the result is listed in Table I.

Since the shape of the resonance structure in Cr^{50} appears to be that of a single resonance, it has been analyzed as such and consequently no value of $\bar{\Gamma}_n{}^0/D$ is listed for this isotope. For this resonance at 5.5 kev, Doppler broadening is only a small fraction of the observed width and the resolution width of the apparatus is small enough that it has an appreciable effect only near the peak, so an attempt was made to fit a Breit-Wigner shape to the data. Since radiative capture widths are generally less than 1 ev, it was assumed that $\Gamma_n/\Gamma=1$. Interference between resonance and potential scattering was included in the analysis. The best fit, shown in Fig. 2, was obtained with the following parameters: $\sigma_0=473$ barns, $E_0=5500$ ev, $\Gamma=1500$ ev, and $R'=2.67\times10^{-13}$ cm. If the assumption is made that the entire thermal capture cross section of Cr^{50} is due to this resonance, a value of $\Gamma_\gamma=2.9$ ev is obtained. Although this is large compared with values of Γ_γ for other nuclei in this mass region, the ratio Γ_n/Γ is still close enough to unity that the original assumption is still a good one.

The value required for R' in order to fit the data is of some interest. As defined by Feshbach, Porter, and Weisskopf, R' is the nuclear radius to be used in calculating the potential-scattering cross section $\sigma_p=4R'^2$. The theories based on the optical model of the nucleus, which predict a variation of the strength function with nucleon number, also predict a variation of R'/R with A, where R is the nuclear radius given by one of the semiempirical relations between R and A. In the case of Cr^{50} a value of $R'/R=0.5$ was used for the curve, shown in Fig. 2, which provides the best fit to the data. As an example of the theoretical variation, the value predicted by the theory based on the optical model with a square-well potential with $V_0=44$ Mev, $\zeta=0.03$, and $R=(1.26A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm is 0.46.

TABLE II. Isotopic composition of chromium samples.

Sample	Cr^{50}	Cr^{52}	Cr^{53}	Cr^{54}
Normal	4.31	83.76	9.55	2.38
Cr^{50}	88.3	11.0	0.6	0.2
Cr^{52}	0.3	99.1	0.4	0.2
Cr^{53}	0.3	16.2	82.4	1.1
Cr^{54}	1.2	11.4	4.3	83.1

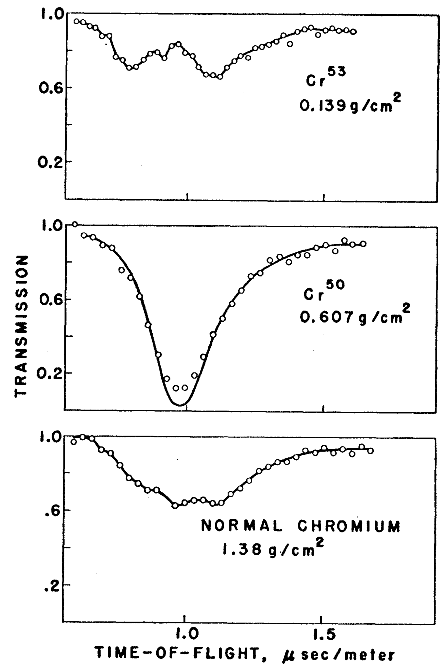

FIG. 2. The neutron transmission of normal chromium and two of its isotopes, Cr^{53} and Cr^{50}. The solid curve plotted with the Cr^{50} data is a theoretical curve based on the resonance parameters $\sigma_0=473$ barns, $\Gamma=1500$ ev, and $R'=2.67\times10^{-13}$ cm.

COBALT

Maxima in the resonance structure were observed at 4.89 and 8.05.‡ MN list resonances at 4.7, 7.8, and 28.3 kev. A sample,[15] 99.8% pure and 0.448 g/cm² thick, was used in the measurements. In order to compute $\bar{\Gamma}_n{}^0/D$ for cobalt, a summation as indicated by Eq. (A2) was performed and to it was added the value of $g\Gamma_n{}^0$ for the resonance at 132 ev, the final sum being divided by ΔE. The reduced neutron width of the 132-ev resonance was derived from the data of Seidl *et al.*[9]

MANGANESE, ZINC, AND GALLIUM

Values of $\Gamma_n{}^0$ from Bollinger *et al.*[16] were used to obtain the sum of the reduced neutron widths for manganese up to 15 kev. The result of this summation is 15.5 $(ev)^{-\frac{1}{2}}$ when the correct value of 1.14 $(ev)^{-\frac{1}{2}}$ is used for the reduced neutron width of the 337-ev

‡ *Note added in proof.*—In a survey run made with much improved resolution, resonances were observed in cobalt at 4.3, 5.0, 8.0, 10.4, 16.6, 21.5, 24, and 30 kev.

[15] The value for cobalt of $\bar{\Gamma}_n{}^0/D=2.38\times10^{-4}$ reported earlier[13] was based on measurements made with a sample which was later found to have a large iron impurity. The values reported by us for the total cross section of cobalt in BNL-325 suffer from the same fault. Since some of the resonance structure observed with the impure sample was not seen with the pure sample, but was observed in earlier work by other investigators, it seems likely that their work may have suffered from this same difficulty. The authors are indebted to Dr. Carl T. Hibdon for pointing out that the sample used in the early measurements on cobalt might be impure.

[16] Bollinger, Dahlberg, Palmer, and Thomas, Phys. Rev. **100**, 126 (1955).

resonance. In order to be consistent with the data of the present paper, the sum of $g\Gamma_n^0$ has been divided by ΔE, with $\Delta E=15$ kev, to obtain $\bar{\Gamma}_n^0/D$.

The data on zinc, which include a value of $\bar{\Gamma}_n^0/D$ for Zn^{67}, have been presented by Dahlberg and Bollinger.[17] In addition to the value for Zn^{67}, a value of $\bar{\Gamma}_n^0/D$ can be obtained from their data on Zn^{64} by the method described in the present paper. The thickness of the sample used was 1.03 g/cm^2 and its isotopic constitution was 93.1% Zn^{64} and 6.3% Zn^{66}. A value of $g\Gamma_n^0$ for the 2750-ev resonance, derived from the data of Dahlberg and Bollinger under the assumption that $\Gamma_\gamma=0.35$, was added to $\sum g\Gamma_n^0$ obtained by the method of Eq. (A1) for the energy region between this resonance and 15 kev. Division of this new sum by ΔE yields the value of the strength function for Zn^{64}, the only even-even nucleus studied.

Data on gallium were presented by Palmer and Bollinger.[18] Since the value of $\bar{\Gamma}_n^0/D$ calculated from their data on Ga^{71} appeared to be high, because of the atypically great widths of the resonances at 290 and 380 ev, transmission measurements above 380 ev were made again in order to apply the method of Eq. (A1) and hence improve the statistical accuracy of the measurement by including more resonances. Similar measurements were also made on Ga^{69}. The sample thicknesses for Ga^{69} and Ga^{71} were 1.48 g/cm^2 and 0.841 g/cm^2, respectively.

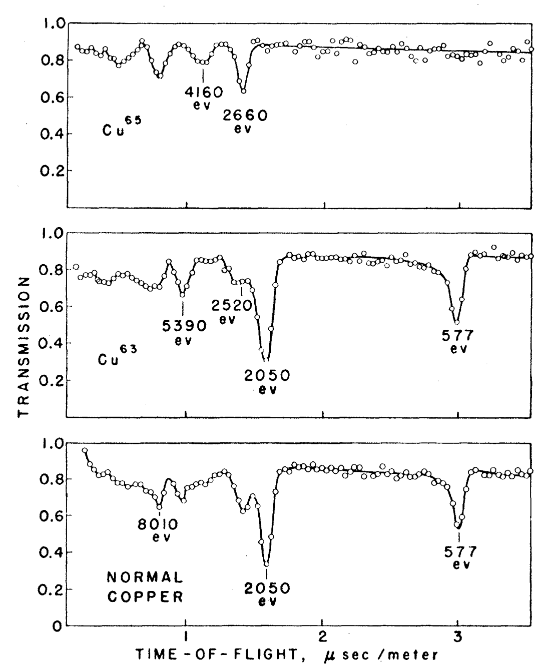

FIG. 3. The neutron transmission of normal copper and its isotopes.

[17] D. A. Dahlberg and L. M. Bollinger, Phys. Rev. **104**, 1006 (1956).

[18] R. R. Palmer and L. M. Bollinger, Phys. Rev. **102**, 228 (1956).

Gallium represents the worst case for application of the method of Eq. (A1) since the assumptions that Γ_γ and Δ are small compared with Γ are poor ones for the isotopes of this element. Values of Γ_γ have not been measured for gallium, but the reasonable assumption that $\Gamma_\gamma\sim0.3$ ev requires a correction of about 6% in the measured value of $\bar{\Gamma}_n^0/D$. The neglect of the Doppler broadening requires a correction of about 6% in the same direction. In the course of this work an error in the data of Palmer and Bollinger was noted. Upon its correction the resonance parameters for the resonance listed by them at 770 ev becomes $E_0=707$ ev, $\Gamma=1.15\pm0.20$ ev and $\sigma_0=1350\pm50$ barns. The corrected results of the present measurements, combined with the data of Palmer and Bollinger for the lower energy resonances, yield the values of $\bar{\Gamma}_n^0/D$ listed in Table I.

COPPER

The transmission as a function of time-of-flight is shown in Fig. 3 for a sample of normal copper and for those enriched in Cu^{63} and Cu^{65}. The separated isotopes were in the form of CuO with sample thicknesses of 2.88 g/cm^2 of Cu^{63} and 2.37 g/cm^2 of Cu^{65}. The sample of Cu^{63} was 99.85% pure, the Cu^{65} 98.15% pure. In addition to the structure shown, there is a resonance at 230 ev which has been identified as belonging to Cu^{65}.

TABLE III. Resonance parameters for copper. In order to obtain values of σ_0 and Γ, the assumptions were made that $\Gamma_\gamma=0.4$ ev and $g=\frac{1}{2}$.

E_0 (ev)	Isotopic assignment	σ_0 (barns)	Γ (ev)
230	65	250	0.42
577	63	1550	1.30
2050	63	626	32.4
2520	63	...	...
2660	65	486	19.5
4160	65	308	33.8
5390	63	240	76.0

Application of standard area analysis to the resonance structure of copper under the assumptions that $\Gamma_\gamma=0.4$ and $g=\frac{1}{2}$ yields the results listed in Table III.§

Values of $\bar{\Gamma}_n^0/D$ were obtained in the same fashion as those for gallium.

ARSENIC

Samples 8.95, 1.10, 0.332, and 0.0925 g/cm^2 thick were used in this measurement of the total cross section of arsenic. Standard area methods were used to obtain the resonance parameters. Thick and thin samples were available for the first three resonances, so that values of Γ_γ could be obtained. These, along with the other resonance parameters for arsenic, are listed in Table IV.

§ *Note added in proof.*—In a survey run made with much improved resolution, resonances were observed in copper at 2.04, 2.52, 2.66, 3.91, 4.44, 4.84, 5.34, 5.90, 6.4, and 7.9 kev; many resonant peaks were observed above this energy, but most likely they are not due to single resonances.

A weighted average value of the radiative-capture widths for the first three resonances was used in order to find the parameters for the next five resonances. The average values of the level spacing D, the reduced neutron width, and $\bar{\Gamma}_n{}^0/D$ are 100 ev, 0.0365 ev, and 1.83×10^{-4}, respectively.

DISCUSSION

Preliminary results for some of the nuclides for which the final values are listed herein were reported previously.[5] Since then, MN have published results giving values of $\bar{\Gamma}_n{}^0/D$ for nuclides in the same mass region and in fact for many of the same nuclei. Some comparison of the two sets of data is therefore in order.

The results of MN were taken over the energy range from a few kev to about 100 kev, while the present results were obtained from the energy region from 0 to 15 kev. The wider energy range allowed MN to include more resonances in their averages and hence to achieve greater statistical accuracy. However, the present data suffer appreciably from poorer statistical accuracy only near the peak in $\bar{\Gamma}_n{}^0/D$. In view of this, one would expect agreement between the two sets of results everywhere except, perhaps, near the peak. This is in fact the case, for the agreement is within the experimental errors for all nuclei off the peak, except arsenic. MN have discussed the result for arsenic, since their value appeared to be quite high compared with their values for neighboring nuclei. They indicated that a different choice of potential-scattering cross section, namely a value based on the total cross section in the electron-volt energy region, would lower the value of $\bar{\Gamma}_n{}^0/D$ very substantially. This case points up the rather large dependence of some of the results of MN on values of the potential-scattering cross section. This difficulty is not important in our determinations of $\bar{\Gamma}_n{}^0/D$ because the off-resonance cross section is much smaller than the resonance contribution in the low-energy region that is of interest to us.

The two highest values reported here, those for V^{51} and that for Cr^{53}, have very large statistical errors because so few resonances were included in the averages. MN have not made measurements on Cr^{53} so no comparison is possible. They have discussed the case of V^{51}, however, and attribute the high value in the ANL work reported in this paper to poor statistical sampling. The large errors placed on the present value indicate that the statistical sample is indeed rather poor, and thus it is certainly conceivable that it may not be a representative sample of the wider energy range. A comparison of the two values for V^{51}, keeping in mind the meaning of the errors, leads to the conclusion that no major disagreement exists even though the values differ by a factor of two. The over-all conclusion to be drawn from this comparison is that there is rather good agreement between the two sets of results; but it should be noted that, although the values of the strength function for V^{51} and Cr^{53} are based individually on results which are statistically rather poor, the combined evidence, which includes these results and fragmentary results for V^{50} and Cr^{50}, seems to indicate that the high values obtained may be correct. In the case of V^{50} this fragmentary evidence is a value of $\bar{\Gamma}_n{}^0/D$ based on only two resonances and for Cr^{50} it is the very large value of the reduced neutron width. On the basis of this evidence, the suggestion is made that the high values obtained may not be the result of poor statistical sampling, but rather are a measure of the strength function near zero energy. This suggests, of course, a variation of the strength function with energy which is not predicted by any theory.

TABLE IV. Resonance parameters for arsenic. $\bar{\Gamma}_\gamma=0.287\pm0.03$ is the weighted average of values of Γ_γ for the first three resonances and is used in the analysis of the five resonances at higher energies. The assumption that $g=\frac{1}{2}$ was made for all resonances.

E_0 (ev)	σ_0 (barns)	Γ (ev)	Γ_γ (ev)	Γ_n (ev)
47	3100±470	0.336±0.032	0.296±0.035	0.0392±0.0024
92	653±210	0.286±0.089	0.269±0.057	0.0155±0.002
254	1160±106	0.30 ±0.15	0.23 ±0.18	0.07 ±0.03
322	3200± 90	1.42 ±0.08	0.287±0.03	1.13 ±0.08
457	725±325	0.385±0.07	0.287±0.03	0.10 ±0.06
535	2214± 25	3.0 ±0.15	0.287±0.03	2.71 ±0.15
666	870±220	0.52 ±0.10	0.287±0.03	0.23 ±0.10
746	1565± 25	2.67 ±0.22	0.287±0.03	2.38 ±0.23

Since many calculations based on the optical model have been made which lead to results which are relevant to the present experimental results, brief mention will be made of the essential features of each, and of their agreement with the present measurements.

The original work of Feshbach, Porter, and Weisskopf[4] was done with a square-well potential of the form $V=-V_0(1+i\zeta)$. A curve based on such a potential with $\zeta=0.03$, $V_0=44$ Mev and $R=(1.26A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm is shown as A in Fig. 1. For a potential of this form, a value of $\zeta=0.05$ provides the compromise necessary to fit the experimental results both in this mass region and for $A>130$. The peak is lower for this value than for a smaller value such as 0.03 and consequently the agreement with the present results is worse. It is clear that this curve represents the experimental results only to the extent that it shows a maximum at the appropriate A for parameters which are consistent with those for the peak near $A=155$.

Attempts at refinement of this model have lead to a potential with a diffuse boundary.[19] Weisskopf[20] has discussed this calculation and a curve taken from his paper is shown as B in Fig. 1. Except for the fact that the particular choice of parameters is such that the peak does not coincide with the experimental peak, this curve appears to fit the data better than any of the others. The major discrepancy lies in the fact that the curve

[19] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954); Morrison, Muirhead, and Murdoch, Phil. Mag. **46**, 795 (1955).
[20] Victor F. Weisskopf, Revs. Modern Phys. **29**, 174 (1957).

passes well above the well-determined points for Y^{89}, Nb^{93}, Mo^{95}, and Mo^{97}.

The fact that nuclei are known to be nonspherical has led to modification of the above calculations. Margolis and Troubetzkoy[21] and Vladimirskii, Ilyina, Panov, Radkevich, and Sokoloviskii[22] have made calculations, based on a square-well potential on the interaction of neutrons with spheroidal nuclei. Though different approaches were made to the problem, the results are in excellent agreement. Only those of Vladimirskii *et al.* were carried out for the mass region of interest here. The results of these calculations for spheroidal nuclei with semiaxes a and $b=0.9a$ are shown as B in Fig. 1. Curve C is the result for oblate spheroidal nuclei with semiaxes $a=0.9b$ and b. Although neither of these curves describe the experimental results well, D would clearly be the better choice of the two.

Chase, Wilets, and Edmonds[23] have considered the scattering of neutrons by a rotating, deformed, even-even nucleus with a diffuse-surfaced complex potential. Unfortunately, these calculations have been carried out only for $A>130$, so no representation of them is shown in Fig. 1. In their most refined calculation this group has used a deformation which varies in accordance with the deformations deduced from $E2$ transition probabilities.[24] The result of this calculation is in very good agreement with the experimental results. The slope of the strength function curve near $A=130$ is such that it appears as though it might pass through the low values of the strength function in the vicinity of $A=120$. However, nothing more positive than this can be said about the fit to the data for the lighter nuclides.

In view of the work outlined above, it is clear that none of the simple models discussed lead to results which are in agreement with the experiment. So many factors regarding the interaction of the nucleus with neutrons have been neglected in the formulation of the optical model that it is pointless to try to obtain a perfect fit to the experimental data. However, it does seem proper to question the fit in regions where the results for many nuclides differ markedly from the theoretical predictions. The inclusion of details pertaining to the individual nuclides appears to bring the experimental and theoretical values into much better agreement for $A>130$. Let us therefore inquire whether a similar approach is likely to be helpful for the mass region $40<A<130$.

For ease of discussion, let us consider the effects of deformation and diffuseness separately. All quadrupole moments of odd-A nuclides between calcium (closed proton shell, $Z=20$) and copper ($A=65$) may be expected to be negative,[25] corresponding to an oblate spheroidal shape. On the basis of the expected size of these moments, the corresponding deformations $\beta=1.06\Delta R/R_0$ may be computed; here R_0 is the mean nuclear radius and ΔR is the difference between the major and minor semiaxes of the spheroid. The values of β for the nuclides under consideration should lie in the range from 0 to 0.1, a range which may be compared with values of 0.25 to 0.47 for nuclides which form the maximum in $\bar{\Gamma}_n^0/D$ near $A=155$. Curves C and D of Fig. 1 correspond to $\beta=0.079$ and therefore are representative of the effects of the maximum deformations expected for nuclides in this mass range. It is clear from these curves and from the complete investigation of Vladimirskii *et al.*[22] that deformations of the size expected for the nuclei between $A=70$ and $A=130$ cannot alone be responsible for the observed values of the strength function in this region.

Consider now the case in which diffuseness alone is included in the calculations. We have seen that models with a constant diffuseness provide the best fit to the experimental results treated in this paper, but that these fail for the region $90<A<130$. Any constant value of the diffuseness consistent with the low values required for the strength function in this region of A cannot be used in calculations without destroying the agreement for nearly all other regions of A.[26] Agreement could be obtained throughout the mass region by a variation of the diffuseness, but such a procedure seems to be unjustified since the results of the electron scattering experiments at Stanford[27] indicate that the diffuseness parameter does not vary as a function of A.

The conclusion reached on the basis of the above discussion is that the experimental results are not in satisfactory agreement with the results of using either a variable deformation or a variable diffuseness in optical model calculations, at least if these parameters are varied in a manner consistent with other known nuclear properties.

APPENDIX

On the basis of the assumptions listed in the text, the direct results of the transmission measurements, uncorrected for resolution, can be related to $\bar{\Gamma}_n^0/D$. From the definition in Eq. (4), it is clear that the problem is simply that of calculating $\sum_r(g\Gamma_n^0)_r$.

The area above a single transmission dip is given by

$$A_E=\int_0^\infty\left(\frac{T_0-T}{T_0}\right)dE\approx\frac{2E_0}{\tau_0}\int_0^\infty\left(\frac{T_0-T}{T_0}\right)d\tau,$$

[21] B. Margolis and E. S. Troubetzkoy, Phys. Rev. **106**, 105 (1957).

[22] Vladimirskii *et al.*, Columbia Conference on Neutron Interactions, New York, September, 1957 (unpublished).

[23] Chase, Wilets, and Edmonds, Phys. Rev. **110**, 1080 (1958).

[24] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

[25] Townes, Foley, and Low, Phys. Rev. **76**, 1415 (1957).

[26] H. Feshbach, Proceedings of the Conference on Neutron Physics by Time-of-Flight, Gatlinburg, Tennessee, November, 1956; Oak Ridge National Laboratory Report ORNL-2309 (unpublished).

[27] Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956); Hahn, Hofstadter, and Ravenhall, Phys. Rev. **105**, 1353 (1957); Downs, Ravenhall, and Yennie, Phys. Rev. **106**, 1285 (1957).

where E_0 is the energy of the neutron at exact resonance, τ_0 is the time of flight in μsec/m for neutrons of energy E_0, and T_0 is the off-resonance transmission. For any value of $n\sigma_0$ this area and the resonance parameters can be related in a general expression through the function $Y(n\sigma_0)$, which can be calculated from curves available in the literature.[8] The relation between these is

$$A_E{}^2 = Y^2(n\sigma_0)\pi n\sigma_0\Gamma^2,$$

where $\pi n\sigma_0\Gamma^2$ is, as a matter of fact, $A_E{}^2$ for a thick sample. On the basis of this last equation and assumptions 1 and 2, the following expression can be obtained for $g\Gamma_n{}^0$:

$$g\Gamma_n{}^0 = g(2.6\times10^6\pi ng)^{-\frac{1}{2}}A_E/Y.$$

The sum of the $g\Gamma_n{}^0$ of N resonances in an energy interval defined by an upper limit E_U and a lower limit E_L is given by

$$\sum_{r=1}^{N}(g\Gamma_n{}^0)_r = \sum_{r=1}^{N}\left[(2.6\times10^6\pi ng)^{-\frac{1}{2}}g\frac{2E_0}{\tau_0}\times\int_{\tau_L}^{\tau_U}\left(\frac{T_0-T}{T_0}\right)d\tau\right]_r. \quad \text{(A1)}$$

On the basis of assumption 5, one can write

$$\sum_{r=1}^{N}\frac{T_0-T_r}{T_0}\approx\frac{T_0-T_\phi}{T_0},$$

where T_ϕ is the transmission at a particular energy due to all N resonances included in the sum. When $\bar{g}$, as defined in the text, and the measured transmissions T_j are substituted in Eq. (A1), it becomes

$$\sum_{r=1}^{N}(g\Gamma_n{}^0)_r = 2\bar{g}(2.6\times10^6\pi n\bar{g})^{-\frac{1}{2}}\times\sum_{j}\frac{E_j}{\tau_j}\left(\frac{T_0-T_j}{T_0Y_j}\right)\Delta\tau, \quad \text{(A2)}$$

in which the sum on the right side is taken over all the channels in the energy range E_U to E_L. This range is determined in an arbitrary fashion so that any prejudicial selection of the limits based on the apparent form of the resonance structure can be avoided. Because of certain properties of the velocity selector, the upper limit is chosen to be the nearest channel to 15 kev; zero is taken as the lower limit. The final expression for the strength function can then be written as in Eq. (4) of the text.