Doublet Separation in Nuclei from Signell-Marshak Potential

B. P. NIGAM* AND M. K. SUNDARESAN*
Division of Pure Physics, National Research Council, Ottawa, Canada
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The spin-orbit splittings for the $l=1$ and $l=3$ levels of Ca^{41} and $l=2$ level of O^{17} have been calculated in the spirit of Brueckner's theory in the first Born approximation from the spin-orbit term that Marshak and Signell have added to the two-nucleon Gartenhaus potential. The results obtained are much larger than those obtained from the conventional tensor force in second order.

I. INTRODUCTION

RECENTLY, on the basis of Brueckner's theory, many calculations have been carried out to explain the doublet spin-orbit separation in nuclei as arising from the tensor force of the two-body interaction. The tensor force in general seems to give too small a contribution to the splitting.[1–3] Bell and Skyrme[4] have obtained the doublet splittings from the experimental two-body scattering data. Since the Signell-Marshak[5] potential gives good fit to the two-body scattering data, it is of interest to evaluate the doublet splittings it predicts in nuclei. We have attempted here to calculate this in the framework of Brueckner's[6] theory.

II. CALCULATION OF THE DOUBLET SPLITTING

Signell and Marshak[5] have added the following spin-orbit term to the Gartenhaus[7] potential[8]:

$$V_{SO}=\frac{V_0}{(r/r_0)}\frac{d}{d(r/r_0)}\left[\frac{e^{-r/r_0}}{(r/r_0)}\right]\mathbf{L}\cdot\mathbf{S}\quad\text{for}\quad r\geqslant r_c,$$
$$=V_{SO}(\mathbf{r})|_{r=r_c}\qquad\text{for}\quad r<r_c,\qquad(1)$$

where $V_0=30$ Mev, $r_0=1.07\times10^{-13}$ cm, $r_c=1/M=0.21\times10^{-13}$ cm, $\mathbf{S}=\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$, and $\mathbf{L}=(\mathbf{r}\times\mathbf{p})$. The interaction energy due to this spin-orbit term is

$$\Delta E=\sum_2\int d^3k_1d^3k_1'd^3k_2d^3k_2'\,\psi_1^*(\mathbf{k}_1)\psi_2^*(\mathbf{k}_2)\times[I_D(\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_1',\mathbf{k}_2')-I_X(\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_2',\mathbf{k}_1')]\psi_1(\mathbf{k}_1')\psi_2(k_2'),\quad(2)$$

where ψ_1 and ψ_2 are the wave functions of the extra core nucleon and the core nucleon, respectively, and the summation is over the core particles 2. I_D and I_X are the direct and exchange contributions, respectively. In the first Born approximation I_D is given by

$$I_D(\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_1',\mathbf{k}_2')=\frac{1}{(2\pi)^6}\int d^3x_1d^3x_2\,e^{i\mathbf{k}_1\cdot\mathbf{x}_1+i\mathbf{k}_2\cdot\mathbf{x}_2}\times V_{SO}(\mathbf{x}_1-\mathbf{x}_2)e^{-i\mathbf{k}_1'\cdot\mathbf{x}_1-i\mathbf{k}_2'\cdot\mathbf{x}_2}$$
$$=\frac{1}{(2\pi)^3}\delta(\mathbf{K}-\mathbf{K}')(\mathbf{S}\cdot[\mathbf{k}\times\mathbf{k}'])v_{SO}(q),\quad(3)$$

where

$$v_{SO}(q)=4\pi iV_0e^{-\alpha}r_0^5\left[\frac{\alpha^2(1+\alpha)}{q^2r_c^2}j_2(qr_c)-\frac{\alpha^2}{qr_c}j_1(qr_c)-\frac{1}{1+q^2r_0^2}\left(\frac{\alpha\sin qr_c}{qr_c}+\cos qr_c\right)\right],$$

and $\mathbf{q}=\mathbf{k}-\mathbf{k}'$, $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$, $\mathbf{k}=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$; $\mathbf{K}'=\mathbf{k}_1'+\mathbf{k}_2'$, $\mathbf{k}'=\frac{1}{2}(\mathbf{k}_1'-\mathbf{k}_2')$; $\alpha=r_c/r_0$; j_l is a spherical Bessel function. I_X is obtained from I_D by interchanging $\mathbf{k}_1'$ and $\mathbf{k}_2'$.

Upon introducing the Fourier transforms of the core wave functions, the result of the summation over the core particles can be replaced by a mixed density function $\rho(r_1,r_2)$. Kisslinger[1] has shown that $\rho(r_1,r_2)$ can be approximately separated in terms of $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ and $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ in the form $\rho(R)\exp(-\gamma r^2)$ with $\gamma=4.61/R_0^2$, $R_0=1.18\times10^{-13}A^{\frac{1}{3}}$ cm. We shall make use of this approximation. Then we obtain

$$\Delta E=\frac{4\pi i}{(2\pi)^6}\int d^3k_1d^3k_1'd^3R\,\psi_1^*(\mathbf{k}_1)\rho(R)\int d^3k_2d^3k_2'd^3r\times\exp(-\gamma r^2)e^{i(\mathbf{k}_2-\mathbf{k}_2')\cdot\mathbf{R}+\frac{1}{2}i(\mathbf{k}_2+\mathbf{k}_2')\cdot\mathbf{r}}\times(\tfrac{1}{2}\boldsymbol{\sigma}_1\cdot[\mathbf{k}\times\mathbf{k}'])\{v_{SO}(|\mathbf{k}-\mathbf{k}'|)+v_{SO}(|\mathbf{k}+\mathbf{k}'|)\}\times\psi_1(\mathbf{k}_1')\delta(\mathbf{K}-\mathbf{K}').\quad(4)$$

Now an integration over one of the variables, say $\mathbf{k}_2'$, can be carried out because of the δ function. Then

$$\Delta E=\frac{-8\pi i}{(2\pi)^6}\int d^3k_1d^3k_1'd^3R\,\psi_1^*(\mathbf{k}_1)\rho(R)\times e^{i\mathbf{p}\cdot\mathbf{R}}\{D(\mathbf{k}_1,\mathbf{k}_1')+X(\mathbf{k}_1,\mathbf{k}_1')\}\psi_1(\mathbf{k}_1'),\quad(5)$$

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* National Research Laboratories Postdoctorate Fellows.
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[8] We use units $\hbar=c=1$.

where

$$D(\mathbf{k}_1,\mathbf{k}_1')=\int_{|\mathbf{k}|\leqslant k_F} d^3k d^3r \times\exp[-\gamma r^2+i(\mathbf{k}_1-\tfrac{1}{2}\mathbf{p}-2\mathbf{k})\cdot\mathbf{r}] \times(\tfrac{1}{2}\boldsymbol{\sigma}_1\cdot[\mathbf{k}\times\mathbf{p}])v_{SO}(p), \tag{6}$$

$$X(\mathbf{k}_1,\mathbf{k}_1')=\int_{|\mathbf{k}|\leqslant k_F} d^3k d^3r \times\exp[-\gamma r^2+i(\mathbf{k}_1-\tfrac{1}{2}\mathbf{p}-2\mathbf{k})\cdot\mathbf{r}] \times(\tfrac{1}{2}\boldsymbol{\sigma}_1\cdot[\mathbf{k}\times\mathbf{p}])v_{SO}(|2\mathbf{k}+\mathbf{p}|),$$

and $\mathbf{p}=\mathbf{k}_1'-\mathbf{k}_1$. Here k_F is the Fermi momentum equal to 1.29×10^{13} cm^{-1}. Since the maximum contribution to ΔE comes from where $\mathbf{p}=0$, we evaluate D and X at this point. This gives the leading term of ΔE proportional to $d\rho/dR$. The higher terms in the Taylor expansion of D and X will give terms involving higher derivatives of ρ. Then we have

$$\begin{aligned} D(\mathbf{k}_1,\mathbf{k}_1')&=\pi^2(\tfrac{1}{2}\boldsymbol{\sigma}_1\cdot[\mathbf{k}_1\times\mathbf{k}_1'])F_D(k_1),\\ X(\mathbf{k}_1,\mathbf{k}_1')&=\pi^2(\tfrac{1}{2}\boldsymbol{\sigma}_1\cdot[\mathbf{k}_1\times\mathbf{k}_1'])F_X(k_1), \end{aligned} \tag{7}$$

where

$$\begin{aligned} F_D(k_1)&=8v_{SO}(0)\left(\frac{k_F}{k_1}\right)^3 \times\int_0^\infty dy\, y\exp\left[-\frac{\gamma}{k_1^2}y^2\right]j_1(y)j_2\left(2\frac{k_F}{k_1}y\right),\\ F_X(k_1)&=8\int_0^\infty dy\, 2y^2\exp\left[-\frac{\gamma}{k_1^2}y^2\right]j_1(y) \times\int_0^{k_F/k_1} dx\, x^3 v_{SO}(2k_1x)j_1(2xy), \end{aligned} \tag{8}$$

with $y=k_1r$ and $x=k/k_1$. Let us consider the extra core nucleon to be at the top of the Fermi distribution. Then we can put

$$\begin{aligned} \psi_1(\mathbf{k}_1)&=(2\pi)^{\frac{3}{2}}\delta(k_1-k_F)Y_l^0(\hat{k}_1),\\ \psi_1(\mathbf{k}_1')&=(2\pi)^{\frac{3}{2}}\delta(k_1'-k_F)Y_l^0(\hat{k}_1'), \end{aligned} \tag{9}$$

where Y_l^0 is the usual spherical harmonic and $\hat{k}_1$ is a unit vector along $\mathbf{k}_1$. Substituting (7) and (9) in Eq. (5), the integrals over $\mathbf{k}_1$ and $\mathbf{k}_1'$ can be carried out. We obtain

$$\Delta E=-[F_D(k_F)+F_X(k_F)]\begin{Bmatrix}\tfrac{1}{2}l\\ -\tfrac{1}{2}(l+1)\end{Bmatrix} \times\int_0^\infty j_l^2(k_FR)\frac{1}{R}\frac{d\rho}{dR}R^2dR \Big/ \int_0^\infty j_l^2(k_FR)R^2dR, \tag{10}$$

TABLE I. Splittings in Mev.

	Observed	Calculated $a=1.0$	Calculated $a=1.5$
$Ca^{41}(l=1)$	0.5	0.13	0.36
$Ca^{41}(l=3)$	>2.0	1.31	1.06
$O^{17}(l=2)$	5.0	1.02	0.78

where the upper line holds for $j=l+\frac{1}{2}$ and the lower line for $j=l-\frac{1}{2}$. $F_D(k_F)$ and $F_X(k_F)$ were calculated numerically from Eq. (8). We found that $[F_D(k_F)+F_X(k_F)]$ has the value $[53.7+11.6]\times10^{-65}$ Mev cm^5 for Ca^{41} and the value $[38.2+6.7]\times10^{-65}$ Mev cm^5 for O^{17}. The doublet separation is obtained from Eq. (10) by forming $\Delta(\Delta E)=\Delta E(j=l+\frac{1}{2})-\Delta E(j=l-\frac{1}{2})$. The result obtained for Ca^{41} is

$$\Delta(\Delta E)=-\tfrac{3}{4}[65.3\times10^{-65}\text{ Mev cm}^5]\frac{2l+1}{2} \times\int_0^\infty j_l^2(k_FR)\frac{1}{R}\frac{d\rho}{dR}R^2dR \Big/ \int_0^\infty j_l^2(k_FR)R^2dR. \tag{11}$$

The factor $\frac{3}{4}$ appears in Eq. (11) because this interaction is nonzero only in the triplet spin states of the two-nucleon system. An extra factor $\frac{3}{4}$ would appear in Eq. (11) if the Marshak term were nonzero only in triplet isotopic-spin states. The magnitude of the splittings are presented in Table I with this latter factor included.

For $\rho(R)$ we have assumed[1,3] a region of uniform distribution bounded by a region where it falls off from its uniform value to zero linearly over a shell of thickness $a\lambda\!\!\!^{-}_\pi$ of the nuclear radius R_0. The splittings presented in Table I are calculated for the two values, $a=1$ and $a=1.5$.

III. DISCUSSION

The results obtained here are much larger than those obtained from a tensor force in second order. Considering the fact that the higher order Born approximations could lead to an increase of the magnitude of the splittings by about 50%,[3] we can expect better agreement with experimental values. However, for O^{17}, we see from Table I that the discrepancy is larger. This could be attributed to the approximation of separating the mixed density function $\rho(\mathbf{r}_1,\mathbf{r}_2)$ in terms of $\rho(R)\exp(-\gamma r^2)$ with $\gamma=4.61/R_0^2$. Since γ is larger for lighter nuclei, the presence of $\exp(-\gamma r^2)$ in the r integration reduces the value of $[F_D+F_X]$ for light nuclei. A more appropriate choice for the dependence of this factor on A is probably also necessary to obtain results agreeing with experiments.

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