Photodisintegration of the Deuteron in the Medium Energy Range*

J. J. DE SWART AND R. E. MARSHAK
University of Rochester, Rochester, New York
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We have calculated the photodisintegration of the deuteron in the medium energy range using the Gartenhaus wave function for the deuteron and the Gartenhaus plus spin-orbit wave functions for the final states. The results are in good agreement with experiment. It is shown that the cross sections are sensitive to the triplet odd scattering phase shifts and the D-state probability, but rather insensitive to the detailed shapes of the wave functions. The results indicate that it is not necessary to relinquish Siegert's theorem in order to explain the large isotropic term in the angular distribution in this energy region, provided one assumes a rather high percentage of D state (7%) for the ground state of the deuteron.

1. INTRODUCTION

THE photodisintegration of the deuteron in the medium energy range (20 Mev$<E_\gamma<$100 Mev) is for the most part an electric dipole transition from the ground state of the deuteron to the continuum states of the n-p system. If only central forces are considered to operate between the neutron and the proton, the differential cross section is predicted to have the usual $\sin^2\theta$ form. The presence of a substantial isotropic component of the cross section in the energy region under consideration would seem to imply the operation of noncentral forces. Attempts to take into account the effects of noncentral forces on the n-p wave functions have not been too successful[1] and have led Wilson[2] to argue for the importance of virtual pion transitions and Austern[3] to question the validity of the Siegert theorem.[3a]

Recent work by Signell and Marshak[4] and by Gammel and Thaler[5] has led to two-nucleon potentials which fit very well all of the single and double scattering data up to 150 Mev.[6] We have used the SM potential to calculate the photodisintegration of the deuteron. We assume that the Siegert theorem holds and we simply insert the wave functions which were computed by Signell and Marshak into the expression for the transition probability. We use the Gartenhaus wave function[7] for the ground state of the deuteron and the modified Gartenhaus wave functions (which are derived from a combination of Gartenhaus and spin-orbit potentials[4]) for the final triplet odd-parity states of the n-p system. De Swart *et al.*[8] have shown that the spin-orbit force is essential for the triplet odd-parity states but can be omitted for the triplet even-parity states (e.g., the deuteron).

In Sec. 2 we write down the fundamental equations, in Sec. 3 we present the results of the calculation, in Sec. 4 we give an approximate calculation, and finally in Sec. 5 we discuss the results.

2. FUNDAMENTAL EQUATIONS

Assuming that the Siegert theorem holds, the cross section for the photodisintegration of the deuteron, when we take into account only the electric dipole transition, is given by

$$\frac{d\sigma}{d\Omega}=\frac{e^2}{\hbar c}\frac{M\omega k}{16\pi\hbar}\langle|D|^2\rangle, \tag{1}$$

where

$$D=\int\Psi_{sc}^{(-)*}\boldsymbol{\varepsilon}\cdot\mathbf{r}\Psi_d d\tau,$$

M = nucleon mass, $\boldsymbol{\varepsilon}$ = polarization direction of the incident γ ray, ω = angular frequency of the γ ray, k = wave number of the final n-p system, and $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ = relative coordinate. The angular brackets indicate that the sum is to be taken over the final states and the average over the initial states and polarizations.

The deuteron wave function is

$$\Psi_d^M=N\left\{\frac{u(r)}{r}\mathcal{Y}_{011}^M+\frac{w(r)}{r}\mathcal{Y}_{211}^M\right\}, \tag{2}$$

where $u(r)\rightarrow e^{-\alpha r}$ outside the potential. Here $\mathcal{Y}_{LSJ}^M$ are the normalized eigenfunctions for the total angular momentum J, orbital angular momentum L, total spin S, and total angular momentum component in z direction M. $R=1/\alpha$ = radius of the deuteron $=4.3154\times10^{-13}$ cm. We can relate N to the triplet effective range $r_{0t}=1.70\times10^{-13}$ cm by

$$N=\left(\frac{2}{R-r_{0t}}\right)^{\frac{1}{2}}=0.87\times(10^{-13}\text{ cm})^{-\frac{1}{2}}.$$

$\Psi_{sc}^{(-)}$ is the scattering solution corresponding to an

* Assisted in part by the International Business Machines Corporation.
[1] See J. Bernstein, Phys. Rev. **106**, 791 (1957).
[2] R. R. Wilson, Phys. Rev. **104**, 218 (1956).
[3] N. Austern, Phys. Rev. **108**, 973 (1957).
[3a] Compare A. J. F. Siegert, Phys. Rev. **52**, 787 (1937) and N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).
[4] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); **109**, 1229 (1958).
[5] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957).
[6] At 150 Mev, the recent Harwell triple scattering experiments which measure the D function give much better agreement with the SM than with the GT potential [A. E. Taylor (private communication)].
[7] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[8] de Swart, Signell, and Marshak, Nuovo cimento **6**, 1189 (1957).

incoming wave so that

$$\Psi_{sc}^{(-)M'}=\sum_{L,J}\{4\pi(2L+1)\}^{\frac{1}{2}}\exp[-i(\delta_{LJ}-\tfrac{1}{2}L\pi)]\times\frac{v_{LJ}(kr)}{kr}C_{0M'M'}{}^{L1J}\mathcal{Y}_{L1J}{}^{M'}, \quad (3)$$

where $v_{LJ}(kr)\rightarrow\sin[kr+\delta_{LJ}-L(\pi/2)]$ when $r\rightarrow\infty$.

Equation (1) can be rewritten in the form

$$d\sigma/d\Omega=a+b\sin^2\theta. \quad (4)$$

We define now the quantities[9]

$$L_i=\Lambda_i+c_i\Gamma_i,\quad (i=0,1,2), \quad (5a)$$

$$L_F=\tfrac{3}{5}2^{\frac{1}{2}}\alpha\int_0^\infty wv_{32}rdr, \quad (5b)$$

where

$$\Lambda_i=\alpha\int_0^\infty uv_{1i}rdr, \quad (6a)$$

$$\Gamma_i=\alpha\int_0^\infty wv_{1i}rdr, \quad (6b)$$

and

$$c_0=-2^{\frac{1}{2}},\quad c_1=2^{-\frac{1}{2}},\quad c_2=-2^{\frac{1}{2}}/10. \quad (7)$$

We get for the coefficients in the cross section (4)

$$a=[B(k)/36]\{4L_0^2-8L_0L_2\cos(\delta_0-\delta_2)+9L_1^2-18L_1L_2\cos(\delta_1-\delta_2)+13L_2^2+18L_F^2-12L_0L_F\cos(\delta_0-\delta_F)+18L_1L_F\cos(\delta_1-\delta_F)-6L_2L_F\cos(\delta_2-\delta_F)\}, \quad (8)$$

$$b=[B(k)/24]\{8L_0L_2\cos(\delta_0-\delta_2)+3L_1^2+18L_1L_2\cos(\delta_1-\delta_2)+7L_2^2+12L_F^2+12L_0L_F\cos(\delta_0-\delta_F)-18L_1L_F\cos(\delta_1-\delta_F)+6L_2L_F\cos(\delta_2-\delta_F)\}, \quad (9)$$

with

$$B(k)=\frac{1}{12}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}N^2R^2. \quad (10)$$

Here the subscripts 0, 1, 2 refer to the $^3P_{0,1,2}$ states and F refers to the $^3F_2$ state.

In the derivation we have neglected the mixing of the $^3P_2$ and $^3F_2$ states due to the tensor force. We can write the total cross section as

$$\sigma_T=[4\pi B(k)/9][L_0^2+3L_1^2+5L_2^2+(15/2)L_F^2]. \quad (11)$$

3. CALCULATIONS

The amplitudes L have been calculated numerically using the Gartenhaus wave function for the deuteron and the Signell-Marshak (SM) potential to calculate the functions v and the phase shifts. Actually, a core of 9 Bev up to $r=0.55\times10^{-13}$ cm ($\mu r=0.39$) has been inserted in the original SM potential[4] to eliminate a bound $^3P_2$ state. Moreover, the coupling term between the $^3P_2$ and $^3F_2$ state has been removed. It has been shown[4] that this has very little effect on the phase shifts (and therefore on the scattering cross section). But it is necessary to make these changes when the wave function itself is to be used.

[9] We use Austern's notation, reference 3.

TABLE I. Photodisintegration parameters.[a]

Photon energy	80 Mev	53.5 Mev	22.4 Mev
δ_0	11.57	13.00	7.24
δ_1	−19.16	−14.98	−6.68
δ_2	10.8	9.52	4.68
δ_F	−1.38	−0.80	−0.10
Λ_0	0.128	0.240	0.704
Λ_1	0.243	0.349	0.731
Λ_2	0.154	0.264	0.711
Γ_0	0.069	0.109	0.206
Γ_1	0.099	0.128	0.192
Γ_2	0.078	0.115	0.204
L_0	0.030	0.086	0.412
L_1	0.313	0.439	0.866
L_2	0.143	0.248	0.682
L_F	0.123	0.138	0.148
L_H	0.177	0.290	0.735

[a] The phase shifts are in degrees, the amplitudes in units of 10^{-13} cm.

Calculations have been performed for energies corresponding to n-p scattering in the laboratory at 150 Mev, 100 Mev, and 40 Mev and hence for γ-ray energies of 80 Mev, 53.5 Mev, and 22.4 Mev, respectively. The results are given in Table I.

To show the importance of the various transitions, the coefficients a and b and the total cross sections σ_T have also been calculated neglecting certain of the transitions. The results are given in Table II together with the experimental results.[10]

For comparison we give the values predicted on the basis of the theory of Schiff, Marshall, and Guth[11] using the Hulthén wave function for the deuteron and no interactions in the final states.

It is evident that the agreement of our theory with experiment is excellent and it is clear that the interference of the $^3F_2$ with the ^3P states is essential to achieve the good agreement.[3] Without the $^3F_2$ transition, the isotropic part of the cross section would be too small (and the $\sin^2\theta$ part would be too large).

4. APPROXIMATE CALCULATION

It seems worth while to examine in greater detail how the large isotropic contribution to the differential cross section is achieved. To do this we look at an approximate expression for a. If we set the cosines of the phase-

[10] The experimental results are taken from the review article of L. Hulthén and M. Sugawara, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 39, p. 129; Lew Allen, Jr., Phys. Rev. **98**, 705 (1955); Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956); J. Halpern and E. V. Weinstock, Phys. Rev. **91**, 934 (1953).

[11] L. I. Schiff, Phys. Rev. **78**, 733 (1950); J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950).

TABLE II. Photodisintegration cross sections.[a]

Photon energy	22.4 Mev a	b	σ_T	53.5 Mev a	b	σ_T	80 Mev a	b	σ_T
Exp.	4.8±0.8	47±8	465±50	5.1±0.9	9.5±1.5	140±15	4.5±0.8	3.9±0.6	88±10
$^3S_1+^3D_1\rightarrow ^3P_{0,1,2}+^3F_2$	4.1	50	470	4.7	10.5	147	4.1	4.3	88
S.M.G.[b]	0	55	460	0	12.7	107	0	5.8	48
$^3S_1\rightarrow ^3P_{0,1,2}$	0.4	52	442	0.2	12.8	110	0.56	5.6	54
$^3S_1+^3D_1\rightarrow ^3P_{0,1,2}$	1.5	52	453	1.7	12.5	126	1.4	6.0	68

[a] Cross sections in μb.
[b] From the theory of reference 11.

shift differences equal to 1, we get for a and b:

$$a=[B(k)/36]\{4(L_2-L_0+\tfrac{3}{2}L_F)^2+9(L_1-L_2+L_F)^2\}, \quad (12a)$$

$$b=\tfrac{1}{6}B(k)\{L_0^2+3L_1^2+5L_2^2+(15/2)L_F^2\}-\tfrac{3}{2}a. \quad (12b)$$

We see that large a can be achieved if L_2-L_0 and L_1-L_2 are sufficiently large and positive.

To see how the differences between the amplitudes L arise, we consider the transitions separately.

$^3S_1\rightarrow ^3P_{0,1,2}$ Transitions

Outside the potential, the final-state wave functions are given by

$$v_{LJ}(kr)=kr\{\cos\delta_{LJ}j_L(kr)-\sin\delta_{LJ}n_L(kr)\}. \quad (13)$$

We replace "outside the potential" by

"if $r\geqslant\mu$",

where $\mu=1.41\times10^{-13}$ cm (=Compton wavelength of π meson). We choose for $r<\mu$

$$v_{LJ}(kr)=krj_L(kr)\cos\delta_{LJ}. \quad (14)$$

This is, of course, an arbitrary choice. In particular, the wave function inside the potential is not correct, but the contributions to the integrals Λ from the inside are rather small, so that it is a reasonable approximation for the amplitudes under consideration.

The expression for Λ_i becomes

$$\Lambda_i=A\cos\delta_i-B\sin\delta_i, \quad (15)$$

where

$$A=\alpha k\int_0^\infty uj_1(kr)r^2dr, \quad (16a)$$

$$B=\alpha k\int_\mu^\infty un_1(kr)r^2dr. \quad (16b)$$

The integrals A and B are solved using for the S-state wave function of the deuteron the Gartenhaus wave function[7] and the Hulthén wave function

$$u_H=e^{-\alpha r}-e^{-\beta r}. \quad (17)$$

Here β is slightly dependent on the D-state probability and we take the value $\beta=1.340\times10^{+13}$ cm. Our expression for Λ reduces to the amplitude L_H given by Schiff, Marshall, and Guth[11] if the phase shifts reduce to zero and we use (17) to calculate A.

Substituting the approximate values in (15) and using our phase shifts, we get the results listed in Table III. We see that the agreement is extremely good. The differences at high energies between these approximate values and those calculated with the Gartenhaus wave function and the SM potential are presumably due chiefly to the effects of the repulsive spin-orbit potential in the $^3P_0$ state. From (15) we see too, that a large positive $\Lambda_1-\Lambda_2$ and not large negative $\Lambda_2-\Lambda_0$ can be reached only if δ_0 and δ_2 are positive and δ_1 is negative. This requires a long-range repulsive tensor potential. Use of the Hulthén wave function for the deuteron instead of the Gartenhaus wave function proves that the isotropic part of the cross section is not very sensitive to the S-wave part of the deuteron wave function. At

TABLE III. Approximate parameters.[a]

Photon energy	22.4 Mev I	II	III	53.5 Mev I	II	III	80 Mev I	II	III
A	0.735	0.722		0.290	0.298		0.177	0.191	
B	0.097	0.089		0.216	0.203		0.201	0.200	
Λ_0	0.717	0.705	0.704	0.234	0.245	0.240	0.133	0.146	0.128
Λ_1	0.741	0.728	0.731	0.336	0.340	0.349	0.233	0.246	0.243
Λ_2	0.724	0.713	0.711	0.250	0.260	0.264	0.136	0.150	0.154
$\Lambda_2-\Lambda_0$	0.007	0.008	0.007	0.016	0.015	0.024	0.003	0.004	0.026
$\Lambda_1-\Lambda_2$	0.017	0.015	0.020	0.086	0.080	0.085	0.097	0.096	0.089

[a] The amplitudes are in units of 10^{-13} cm. Values I are calculated using the Hulthén wave function for the deuteron. Values II are calculated using the Gartenhaus wave function for the deuteron. Values III are the corresponding values from Table I.

TABLE IV. Approximate parameters.[a]

Photon energy	22.4 Mev			53.5 Mev			80 Mev		
	I	II	III	I	II	III	I	II	III
C	0.168	0.200		0.077	0.121		0.046	0.088	
D	−0.016	−0.064		0.047	0.035		0.049	0.055	
Γ_0	0.168	0.207	0.206	0.064	0.110	0.109	0.044	0.075	0.069
Γ_1	0.165	0.191	0.192	0.086	0.126	0.128	0.059	0.101	0.099
Γ_2	0.169	0.205	0.204	0.068	0.114	0.115	0.044	0.076	0.078
L_F	0.153	0.148	0.148	0.123	0.139	0.138	0.103	0.123	0.123

[a] The amplitudes are in units of 10^{-13} cm. Values I are calculated using the Feshbach-Schwinger D-state wave function corresponding to 3.7% D state for the deuteron. Values II are calculated using the Gartenhaus D-state wave function corresponding to 6.7% D state for the deuteron. Values III are the corresponding values from Table I.

higher energies, however, use of the Hulthén wave function will reduce the total cross section.

$^3D_1 \to {}^3P_{0,1,2}$ Transitions

At first sight, one should expect these amplitudes to depend rather strongly on the D-state probability. This, however, is not true for the low-energy region and is only partly true for the higher energies. A large contribution to these amplitudes comes from the region outside the potential (for small energies more than for high energies) and hence from the region where the asymptotic behavior of the D-state wave function is determined largely by the quadrupole moment. The smaller the D-state probability, the larger the outside wave function has to be. This tends to cancel the reduction in the amplitude in the same way as it cancels the reduction in the quadrupole moment if one lowers the D-state probability. We find that $-d\Gamma/dE$ is the larger, the smaller the D-state probability.

We can also write for these transitions,

$$\Gamma_i = C\cos\delta_i - D\sin\delta_i, \tag{18}$$

where

$$C = \alpha k \int_0^\infty w j_1(kr) r^2 dr, \tag{19a}$$

$$D = \alpha k \int_\mu^\infty w n_1(kr) r^2 dr. \tag{19b}$$

These integrals have been evaluated using the Gartenhaus wave function (corresponding to 6.7% D-state probability) and the best of the Feshbach-Schwinger[12] wave functions (corresponding to about 3.7% D-state probability). The results are given in Table IV. We see that the agreement with the "exact" values is again good for the Gartenhaus wave function, but that the values obtained by using the Feshbach-Schwinger wave function are smaller especially at the higher energies.

$^3D_1 \to {}^3F_2$ Transition

This transition amplitude is less dependent on the D-state probability. This amplitude has been calculated using the same D-state wave functions as before. Because the phase shift is very small in this case, we can take

$$L_F = \tfrac{3}{5} 2^{\frac{1}{2}} \alpha k \int_0^\infty w j_3(kr) r^2 dr. \tag{20}$$

The results are also given in Table IV. We see that this approximation is excellent for the Gartenhaus wave function.

We can now calculate the values for a, b, and σ_T with our approximate expressions for Λ_i, Γ_i, and L_F. It is interesting to inquire how we obtain the large isotropic cross section. From our formula (12a), we see that two quantities are important, i.e.,

$$L_2 - L_0 + \tfrac{3}{2} L_F \quad \text{and} \quad L_1 - L_2 + L_F.$$

Because δ_0 and δ_2 have the same sign and order of magnitude, we see at once that $\Lambda_2 - \Lambda_0$ is small. The value $L_2 - L_0$ is almost entirely due to the $^3D_1 \to {}^3P_0$ transition:

$$L_2 - L_0 = \Lambda_2 - \Lambda_0 + 2^{\frac{1}{2}}(\Gamma_0 - 0.1\Gamma_2).$$

We see that the first term in (12a) gets its contribution practically only through the interference between the transitions from the $^3D_1$ state to the $^3P_0$ and $^3F_2$ states. This term is especially important at low energies.

The second term is

$$L_1 - L_2 + L_F = \Lambda_1 - \Lambda_2 + 2^{-\frac{1}{2}}(\Gamma_1 - \tfrac{1}{5}\Gamma_2) + L_F.$$

Because of the phase-shift differences we get a term from the interference between the transitions from the $^3S_1$ and $^3D_1$ states to the $^3P_{1,2}$ and $^3F_2$ states, which is important at high energies. Moreover, we have a term due to the interference between the transitions from the $^3D_1$ to the $^3P_1$ and $^3F_2$ states, which is important at the lower energies.

Using these approximate values for the transition amplitudes and the phase shifts from the SM potential, the values for a, b, and σ_T have been calculated. The results are given in Table V. We see clearly that the values using the Feshbach-Schwinger D-state wave function corresponding to 3.7% D state are too small everywhere for a, and in the high-energy region for σ_T. It should be noted that even virtual pion processes will not appreciably improve the agreement with σ_T (if the

[12] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951); see also R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952). We wish to thank Professor Feshbach for sending us the wave functions.

TABLE V. Approximate photodisintegration cross sections,[a] in μb.

E_γ	22.4 Mev			53.5 Mev			80 Mev		
	a	b	σ_T	a	b	σ_T	a	b	σ_T
Exp.	4.9	47	465	5.2	9.5	140	4.5	3.9	88
"Exact"	4.1	50	470	4.7	10.5	147	4.1	4.3	88
Approx. I (6.7%)	4.1	50	470	4.6	10.1	142	4.1	4.3	88
Approx. II (6.7%)							4.1	3.5	80
Approx. III (3.7%)	3.6	51	469	3.2	11	132	2.7	4.8	75

[a] For approx I: Gartenhaus S- and D-state wave functions are used for the deuteron. For approx II: Hulthén S-state and Gartenhaus D-state wave functions are used for the deuteron. For approx III: Gartenhaus S-state and Feshbach-Schwinger D-state wave functions are used for the deuteron. The "exact" values are the values taken from Table II.

D-state probability is low) since, according to Austern,[3] the L_0 amplitude will be reduced by such effects.

5. CONCLUSIONS

Our calculations indicate that it is possible to achieve a detailed understanding of the photodisintegration of the deuteron in the medium energy region without renouncing Siegert's theorem or introducing virtual pion effects which are not contained in Siegert's theorem. However, it must be emphasized that the Gartenhaus wave function which we have used involves a larger percentage of D-state probability ($\approx 7\%$) than normally assumed.[13] This is not in contradiction with the measured electric quadrupole and magnetic dipole moments of the deuteron, in view of the still unknown relativistic and pion exchange effects and the possibility that there may actually be a positive contribution to the magnetic moment of the deuteron from a spin-orbit potential in the isotopic-spin zero state of the n-p system.

In evaluating the significance of the good agreement between our predictions and experiment, it should be recalled that we have neglected the tensor coupling between the $^3P_2$ and $^3F_2$ final states in addition to restricting ourselves to the electric dipole transition. The effect of the tensor coupling can be shown to actually improve the agreement with experiment. The restriction to the electric dipole transition leads to an underestimate of a at the lowest energy considered.[14] In view of the latest Pennsylvania experiment[15] which has just been completed and the other experiments on deuteron photodisintegration which are under way at higher energies (at other laboratories), calculations of the magnetic dipole and electric quadrupole transitions on the basis of the SM potential have been started and will be reported at a later date.

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[13] L. Hulthén and M. Sugawara, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 39, p. 74.

[14] Compare Iwadare, Otsuki, Sano, Takagi, and Watari, Progr. Theoret. Phys. **16**, 658 (1956) and S. H. Hsieh, Progr. Theoret. Phys. **18**, 637 (1957).

[15] The recent Pennsylvania experiment at 20–23 Mev gives $a/b=0.133\pm0.02$, $\sigma_T=(580\pm80)$ μb [A. L. Whetstone and J. Halpern, Phys. Rev. **109**, 2072 (1958)].