PHYSICAL REVIEW VOLUME 111, NUMBER 1 JULY 1, 1958

Neutron Cross Sections and Scattering Lengths of Cu^{63} and Cu^{65}†

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(Received March 10, 1958)

It is of interest to know whether the thermal-neutron scattering lengths of Cu^{63} or Cu^{65} are significantly different from natural copper. From measurements of the intensity of the (111) reflections of natural and isotopically enriched samples, the scattering lengths of Cu^{63}, Cu^{65}, and natural Cu are found to be $(+0.672\pm0.015)\times10^{-12}$ cm, $(+1.109\pm0.019)\times10^{-12}$ cm, and $(+0.790\pm0.023)\times10^{-12}$ cm, respectively.

The incoherent and absorption cross sections for Cu^{63} and Cu^{65} are deduced from the values of the cross sections of Cu^{63} and natural copper. The (spin) incoherent cross sections of the two isotopes are zero within the accuracy of the data and the incoherent cross section of natural copper is 0.4004 ± 0.1416 barn. The absorption cross sections for Cu^{63}, Cu^{65}, and natural Cu are $(2.512\pm0.058)\lambda$, $(1.459\pm0.144)\lambda$, and $(2.1728\pm0.0182)\lambda$ barns, respectively, where λ is in angstroms.

INTRODUCTION

THE study of short-range order in the brasses is of fundamental interest but is made difficult by the similarity of the scattering powers of copper and zinc for both x-rays and neutrons. Therefore, the authors were motivated to determine whether the neutron scattering lengths of Cu^{63} and Cu^{65} were significantly different from natural copper.

SAMPLE PREPARATION

Two lots consisting of 99.2% Cu^{63}, 0.8% Cu^{65}, and 98.3% Cu^{65}, 1.7% Cu^{63} in the oxide form were obtained from the Stable Isotope Division of the Oak Ridge National Laboratory. The oxides were crushed to pass a 270-mesh sieve, and metallic briquettes in the form of sintered disks 0.85 in. in diameter by 0.20 in. long were made by reducing the oxide in a grade 60 porous graphite crucible at 400°C for 2 hours. Briquettes of natural copper were prepared in the same way from Bakers' and Adamson reagent grade CuO wire. A nickel powder standard was prepared by filling a nickel tube 0.875 in. in diameter (0.0006-in. wall) with Mathieson Company 200-mesh nickel powder.

A copper briquette was examined with x-rays for preferred orientation. The diffraction pattern of the briquette surface was in excellent agreement with that of a random powder sample. The briquette was broken and examined for completeness of reduction, and it was concluded that the method of sample preparation was satisfactory.

DETERMINATION OF SCATTERING LENGTHS

The neutron scattering lengths of Cu^{63}, Cu^{65}, and natural Cu were determined relative to Ni by comparing the (111) reflections according to the relation[1]:

$$P/I_0 \propto N_c^2 b^2 (\sin\theta \sin 2\theta)^{-1} M (V\rho'/\rho) A(\theta) e^{-2W}, \quad (1)$$

where P = number of neutrons diffracted into the counter per unit time, I_0 = number of neutrons in the incident beam crossing a unit area per unit of time, N_c = number of unit cells per cm³, b = scattering length, θ = Bragg angle, M = multiplicity, V = volume of sample in beam, ρ' = measured density, ρ = theoretical density, $A(\theta)$ = absorption factor, and $2W$ = Debye-Waller temperature factor.

Care was taken to insure that the same portion of the incident beam was intercepted by all samples. The briquettes were stacked on top of one another to form a cylindrical sample. The beam was wider than the diameters of the samples and its height was constant throughout the experiments and less than the height of the briquettes. The beam passed through a fission counter before falling upon the sample in order to determine a number proportional to I_0.

Values for the mass absorption coefficients were determined from the transmissions through a briquette of Cu^{63}, Cu^{65}, natural Cu, and the nickel standard. Appropriate corrections were made for half-wavelength contamination in the neutron beam. The mass absorption coefficients so determined were, respectively, 0.07390 ± 0.00627, 0.08516 ± 0.00653, and 0.19321 ± 0.00270 cm² g⁻¹ for Cu^{63}, natural Cu, and Ni for a wavelength of 1.076 A, and 0.21939 ± 0.00628 and 0.20714 ± 0.00194 cm² g⁻¹ for Cu^{65} and Ni for a wavelength of 1.071 A. Changes in instrumentation were made between the measurements on natural Cu and Cu^{63} and those on Cu^{65}.

$A(\theta)$, the absorption factor for cylindrical specimens,[2] depends upon the linear absorption coefficient, μ_l, and the radius of the specimen R. The densities of the briquettes and the nickel powder were obtained from their weight and physical dimensions. For each briquette and the nickel powder, the values of $\mu_l R$, $A(\theta)$, and the product $V\rho' A(\theta)$ were obtained. $V\rho' A(\theta)$ was summed for the briquettes and a summary of these quantities is given in Table I.

The neutron diffractometer scanned continuously at $\frac{1}{2}$° in 2θ per hour. The counter and monitor printed out

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† Work performed under the auspices of the U. S. Atomic Energy Commission.

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[1] G. E. Bacon, *Neutron Diffraction* (Oxford University Press, London, 1955), pp. 89–98.

[2] A. J. Bradley, Proc. Phys. Soc. (London) **47**, 879 (1935).

TABLE I. Summary of significant quantities in calculating $V\rho' A(\theta)$.

Sample	$\mu_l R$	$A(\theta)$	$V\rho' A(\theta)$
	$\lambda = 1.076$ A		
Cu^{63} No. 1	0.32941	0.5763	3.5259
Cu^{63} No. 2	0.34471	0.5621	4.6089
Cu^{63} No. 3	0.32795	0.5775	4.6106
Cu^{63} No. 4	0.30882	0.5964	4.2918
		Total =	17.037 ±0.630
Cu No. 2	0.43137	0.4898	1.2325
Cu No. 3	0.40943	0.5069	3.9602
Cu No. 4	0.44336	0.4804	3.7779
Cu No. 5	0.42693	0.4931	3.8946
Cu No. 6	0.43501	0.4868	3.0041
		Total =	15.869 ±0.841
Ni	1.05923	0.2040	7.2225±0.146
	$\lambda = 1.071$ A		
Cu^{65} No. 1	0.70507	0.3180	0.8681
Cu^{65} No. 2	0.74076	0.3008	2.2776
Cu^{65} No. 3	0.72176	0.3098	1.5378
Cu^{65} No. 4	0.61153	0.3680	2.2805
		Total =	6.964 ±0.1862
Ni	1.18226	0.1595	5.9961±0.1159

TABLE II. Values of P/I_0 for Cu^{63}, Cu^{65}, natural Cu, and Ni.

Sample	$P(111)/I_0$	$P(200)/I_0$
	$\lambda = 1.076$ A	
Cu^{63}	1.0956	0.61936
Cu^{63}	1.1026	0.58885
Cu^{63}	1.1076	...
Mean	1.1019±0.0040	0.60410
Cu	1.3761	0.75393
Cu	1.4122	
Mean	1.3941±0.0055	0.75393
Ni	1.2107	0.69807
Ni	1.2229	0.70796
Mean	1.2168±0.0074	0.70302
	$\lambda = 1.071$ A	
Cu^{65}	1.7692	
Cu^{65}	1.7917	
Mean	1.7805±0.00637	
Ni	1.1462	
Ni	1.1192	
Mean	1.1327±0.00614	

at 0.1° in 2θ, and no counts were lost. More than one run was taken for each sample and in several runs the (200) reflection was included. The values P/I_0 summarized in Table II are the integrated reflections appropriately normalized by the fission-counter monitor.

From the values of $P(111)/I_0$, $V\rho' A(\theta)$, the Debye temperatures (Cu = 320°K, Ni = 400°K),[3] the scattering length of nickel ($b_{\mathrm{Ni}} = 1.03\times10^{-12}$ cm),[4] and Eq. (1), the scattering lengths of the Cu^{63}, Cu^{65}, and natural Cu samples were found. Solving the set of equations

$$\begin{aligned} 0.992b(\mathrm{Cu}^{63})+0.008b(\mathrm{Cu}^{65}) &= (0.6751\pm0.0144)\times10^{-12}\ \text{cm}, \qquad (2)\\ 0.017b(\mathrm{Cu}^{63})+0.983b(\mathrm{Cu}^{65}) &= (1.1015\pm0.0185)\times10^{-12}\ \text{cm}, \end{aligned}$$

we find $b(\mathrm{Cu}^{63}) = (+0.672\pm0.015)\times10^{-12}$ cm, $b(\mathrm{Cu}^{65}) = (+1.109\pm0.019)\times10^{-12}$ cm, and from direct measurement $b(\mathrm{Cu}) = (+0.790\pm0.023)\times10^{-12}$ cm. The principal sources of error were in the absorption factors, densities, and counting statistics, in that order. The standard deviations quoted include the estimate of these errors only, and not the possible error in the scattering length of Ni or other physical constants.

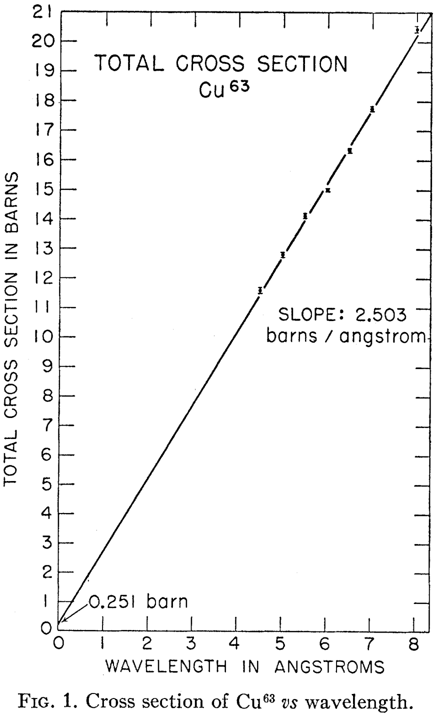

FIG. 1. Cross section of Cu^{63} *vs* wavelength.

DETERMINATION OF THE TOTAL CROSS SECTION FOR SLOW NEUTRONS

The total cross section of Cu^{63} as a function of neutron wavelength was measured at seven selected points between 4.5 and 8.0 angstroms on a nuetron spectrometer. The four Cu^{63} briquettes were placed one behind the other, the beam traversing them axially, and the transmission of the four was $1/e$ near the middle of the region investigated. The half-wavelength contamination passed by the magnetite monochromator was held to a minimum by suitable filter arrangements, and at no time did it exceed 0.8%. Precautions were taken to insure that the counter intercepted the full beam leaving the sample and that no decrease in transmission resulted from small-angle scattering. In order to minimize the effect of fluctuations in pile flux, measurements of transmitted and incident beams were made

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[3] R. W. James, *The Optical Principles of the Diffraction of X-rays* (G. Bell and Sons Ltd., London, 1954), p. 221.

[4] C. G. Shull and E. O. Wollan, Phys. Rev. **81**, 527 (1951).

with the sample periodically in and out of the beam. Approximately 100 000 counts were recorded for each wavelength and the cross sections were calculated from

$$\sigma_T = -\frac{1}{Nt}\ln\left(\frac{I_s - B_s}{I_0 - B_0}\right), \tag{3}$$

where I_s and I_0 are the intensities with the sample in and out of the beam, and B_s and B_0 are the corresponding background intensities with the monochromator turned out of its reflecting position. The product Nt, the density of atoms per cm², was $(0.8074 \pm 0.0074) \times 10^{23}$ cm^{-2} for the four Cu^{63} briquettes.

Neglecting inelastic scattering, the cross section in the wavelength region beyond the Bragg cutoff is the sum of the incoherent cross section which is independent of wavelength, and the absorption cross section which depends linearly upon the wavelength.[5] Figure 1 shows our results for the total cross section of Cu^{63}, and Fig. 2 shows the total cross section of natural copper obtained with the Brookhaven slow chopper.[6,7] A least-squares analysis[8] of these data, excluding the last four points in Fig. 2, gives $\sigma_{\text{inc}}(\text{Cu}) = 0.4004 \pm 0.1416$ barn and $\sigma_{\text{abs}}(\text{Cu}) = (2.1728 \pm 0.0182)\lambda$ barns, where λ is in angstroms. Using the natural abundances[9] of Cu^{63} and Cu^{65} and solving

$$\begin{aligned} 0.6909\sigma_{\text{abs}}(\text{Cu}^{63}) + 0.3091\sigma_{\text{abs}}(\text{Cu}^{65}) &= 2.173 \pm 0.018, \\ 0.998\sigma_{\text{abs}}(\text{Cu}^{63}) + 0.008\sigma_{\text{abs}}(\text{Cu}^{65}) &= 2.503 \pm 0.057, \end{aligned} \tag{4}$$

we find $\sigma_{\text{abs}}(\text{Cu}^{63}) = (2.512 \pm 0.058)\lambda$ and $\sigma_{\text{abs}}(\text{Cu}^{65}) = (1.459 \pm 0.144)\lambda$ barns. These values compare favorably with Pomerance's values,[10] when corrected for the latest gold value,[11] of $(2.487 \pm 0.199)\lambda$ and $(1.220 \pm 0.097)\lambda$ barns.

The incoherent cross section for natural copper can be expressed in terms of the spin incoherent scattering of each isotope, the isotopic abundances, and the scattering length of each isotope[12]:

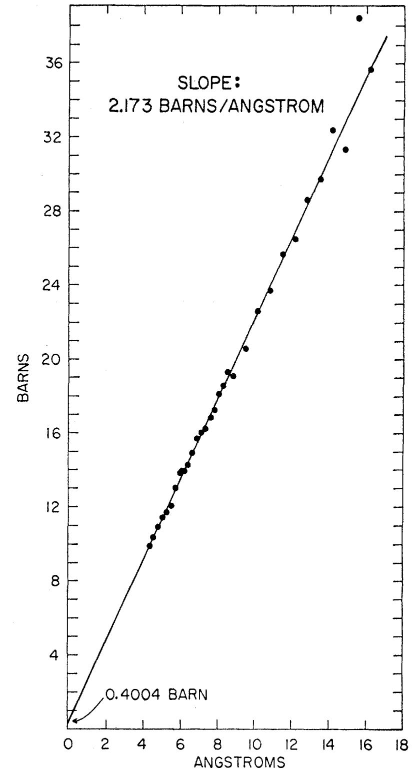

FIG. 2. Cross section of natural copper as obtained on the Brookhaven slow chopper.

$$\sigma_{\text{inc}}(\text{Cu}) = 0.6909\sigma_{\text{inc}}^{\text{spin}}(\text{Cu}^{63}) + 0.3091\sigma_{\text{inc}}^{\text{spin}}(\text{Cu}^{65}) + 4\pi(0.6909)(0.3091)[b(\text{Cu}^{63}) - b(\text{Cu}^{65})]^2. \tag{5}$$

From this expression an upper limit can be placed on the magnitude of the spin incoherence in natural copper and its isotopes. By computing the last term in Eq. (5) we find

$$0.6909\sigma_{\text{inc}}^{\text{spin}}(\text{Cu}^{63}) + 0.3091\sigma_{\text{inc}}^{\text{spin}}(\text{Cu}^{65}) = \sigma_{\text{inc}}^{\text{spin}}(\text{Cu}) = -0.1118 \pm 0.1527 \text{ barn}.$$

Within the standard errors of the measurements the spin incoherence of copper does not exceed 0.041 barn. The measured incoherent cross section of Cu^{63} was found to be 0.251 ± 0.30 barn. Thus within the accuracy of the measurements the incoherent cross sections of Cu^{63} and Cu^{65} can be taken to be zero.

ACKNOWLEDGMENTS

The authors are grateful to Dr. Norman Elliot for his suggestions in reducing the oxide and to Dr. Julius Hastings and Dr. Lester Corliss for their suggestions and cooperation in taking the diffraction data. They would like to acknowledge Mr. John Sondericker's valuable assistance in making the transmission measurements.

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[5] R. J. Bendt and I. W. Ruderman, Phys. Rev. 77, 575 (1950).

[6] D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (U. S. Government Printing Office, Washington, D. C., 1955), p. 143.

[7] The original data in reference 6 were supplied by H. Palevsky and R. R. Smith (private communication).

[8] B. C. Brookes and W. F. L. Dick, *Introduction to Statistical Method* (William Heinemann Ltd., London, 1951), pp. 184–199.

[9] *American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957), Table 7b-3.

[10] H. Pomerance, Phys. Rev. 88, 412 (1952).

[11] Gould, Taylor, Rustad, Melkonian, and Havens, Bull. Am. Phys. Soc. Ser. II, 2, 42 (1957).

[12] Let m_p be the fraction of atoms with scattering length b_p, and M_A and M_B be the isotopic abundances. Then

$$\begin{aligned} \sigma_{\text{inc}} = 4\pi\{\textstyle\sum_p m_p b_p^2 - (\sum_p m_p b_p)^2\} &= 4\pi\{M_A\langle b_A^2\rangle + M_B\langle b_B^2\rangle \\ &\quad - M_A^2\langle b_A\rangle^2 - M_B^2\langle b_B\rangle^2 - 2M_AM_B\langle b_A\rangle\langle b_B\rangle\} \\ &= 4\pi M_A[\langle b_A^2\rangle - \langle b_A\rangle^2] + 4\pi M_B[\langle b_B^2\rangle - \langle b_B\rangle^2] \\ &\quad + 4\pi M_AM_B[\langle b_A\rangle - \langle b_B\rangle]^2. \end{aligned}$$