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# Calculation of the $O^{16}(d,p)O^{17}$ Angular Distribution*

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Results of calculations based on the Tobocman theory are presented for the angular distribution of the outgoing protons produced by the $O^{16}(d,p)O^{17}$ stripping reaction in which the residual nucleus is left either in the ground state or in the first excited state, and for incident energies well below the Coulomb barrier. A variety of nuclear models are examined, including the optical model, and the theoretical distributions compared with experimental ones.

## I. INTRODUCTION

The angular distribution of the outgoing particle for the $(d,p)$ and $(d,n)$ stripping reactions has been the subject of numerous theoretical investigations[1] based on varying approaches and different simplifying assumptions. The original treatment of Butler[2] neglected the effects of the nuclear Coulomb potential and the interaction of the outgoing particle or of the deuteron with the nucleus. Subsequent investigations by Yoccoz,[3] Grant,[4] and Tobocman[5] have dealt with one or both of these effects. Tobocman,[5] in particular, has developed a theory taking the Coulomb field into account, and containing two parameters related to the specific nuclear model chosen specifying the type of nuclear interaction. Calculations based on this theory have been presented by Tobocman and Kalos[6] for a number of cases of interest.

It is the purpose of the present investigation to consider in some detail, from the point of view of the Tobocman theory, the $O^{16}(d,p)O^{17}$ stripping reaction for a variety of models, including the optical model of Feshbach, Weisskopf, and Porter,[7] and, in particular, for incident deuteron energies well below the Coulomb barrier of the oxygen nucleus. Since it is just in this energy region that the pure-stripping theory of Butler may be expected to breakdown, it is of interest to determine the extent to which the inclusion of the Coulomb and secondary nuclear effects will enable agreement to be attained. The experimental data selected for comparison are those of Grosskreutz,[8] who has obtained angular distributions for a wide range of energies near or below the Coulomb barrier.

## II. CALCULATIONS

The Tobocman expression for the $(d,p)$ cross section has the form[5]

$$\sigma_L(\theta_p)=A\sum_{m=-L}^{L}\left|\sum_{l=0}^{\infty}P_l^{|m|}(\cos\theta_p)\sum_{\lambda}a_{l\lambda}^{m}g_{l\lambda}^{L}\right|^2, \quad (1)$$

where

$$g_{l\lambda}^{L}=\int_{R(1+M_N/M_I)}^{\infty}dr\, h_L^{(1)}(i|K_N|r)\times[F_\lambda(\eta_D,K_Dr)-\alpha_\lambda H_\lambda(\eta_D,K_Dr)]\times[F_l(\eta_P,K_Pr')-\beta_l H_l(\eta_P,K_Pr')], \quad (2)$$

where $a_{l\lambda}^{m}$ are quantities dependent on Coulomb phase factors and on Clebsch-Gordan coefficients, $A$ is a constant for a particular bombarding energy, and $r'=r(1+M_N/M_I)^{-1}$. The parameters $\alpha_\lambda$ and $\beta_l$ are the partial scattering amplitudes for deuterons and protons, respectively, expressible in terms of the appropriate logarithmic derivatives at the surface of the nucleus, while $L$, $l$, and $\lambda$ are angular momentum quantum numbers for the neutron, proton, and deuteron, respectively. The notation for the remaining quantities follows that of Tobocman and Kalos.[6]

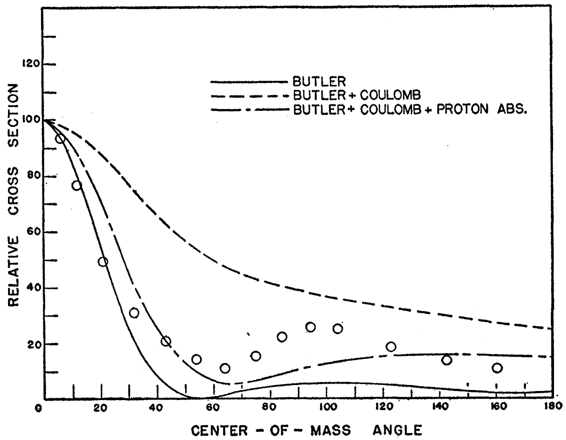

FIG. 1. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.00$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Curves are presented for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

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* This research was supported in part by the U. S. Air Force, through the Office of Scientific Research of the Air Research and Development Command.

[1] For references dealing with the earlier work in this field see R. Huby, *Progress in Nuclear Physics* (Butterworths-Springer, London, 1953), Vol. 3, p. 177.

[2] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[3] J. Yoccoz, Proc. Phys. Soc. (London) **A67**, 813 (1954).

[4] I. P. Grant, Proc. Phys. Soc. (London) **A67**, 981 (1954); **A68**, 244 (1955).

[5] W. Tobocman, Phys. Rev. **94**, 1655 (1954).

[6] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).

[7] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[8] J. C. Grosskreutz, Phys. Rev. **101**, 706 (1956).

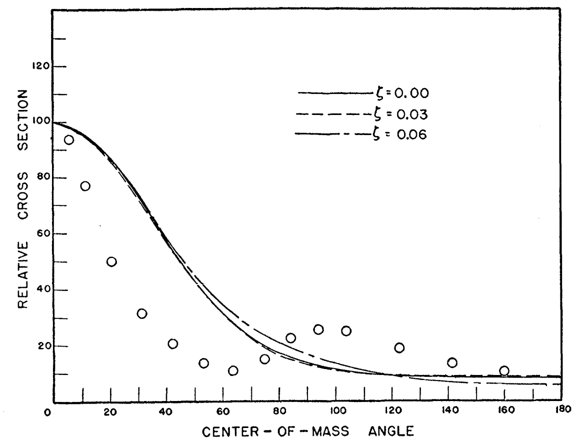

FIG. 2. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.00$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=32$ Mev and for $\zeta=0$, 0.03, and 0.06.

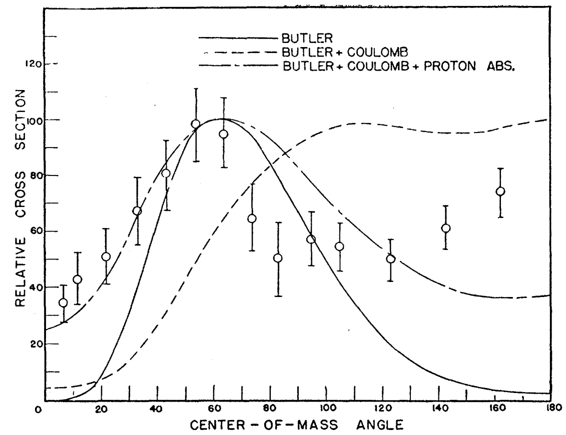

FIG. 4. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.00$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Curves are presented for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

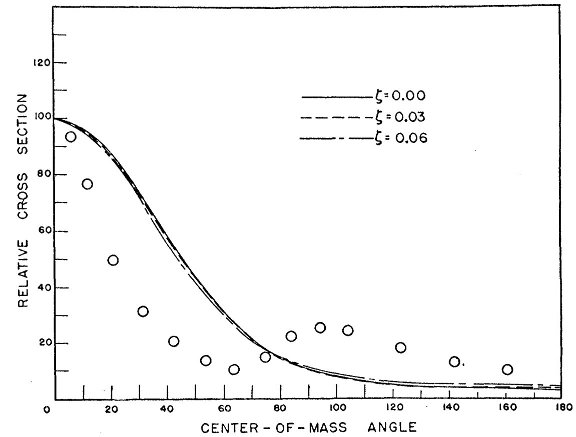

FIG. 3. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.00$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=42$ Mev and for $\zeta=0$, 0.03, and 0.06.

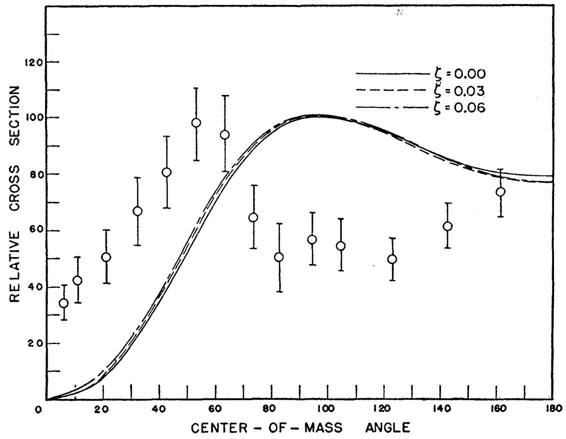

FIG. 5. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.00$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=32$ Mev and for $\zeta=0$, 0.03, and 0.06.

Calculations based on the above expressions have been made for bombarding energies of 2 Mev and 2.51 Mev for both the ground state and the first excited state of the residual nucleus $O^{17}$. These energies are well below the Coulomb barrier of oxygen, which is about 3 Mev or 3.5 Mev, depending on the value chosen for the nuclear radius. The $Q$ energies used in the calculations have been taken from the article by Ajzenberg and Lauritsen,[9] as have the $L$ values of 2 and 0 for the ground and first excited states of $O^{17}$, respectively.

The most laborious quantities to compute are the integrals $g_{l\lambda}{}^{L}$, whose integrands depend on both the regular and the irregular Coulomb wave functions, as well as on the spherical Hankel functions. Since the existent tables of the Coulomb functions[10] were found insufficiently comprehensive for the present purpose, it was necessary to calculate the required values directly. For this purpose the power series expansions given by Fröberg[11] appear to be satisfactory for the energies used. The integrals $g_{l\lambda}{}^{L}$ themselves were evaluated by numerical integration. The sums over $l$ and $\lambda$ in Eq. (1) were broken off at 7 (except for the less accurate results of Figs. 13 and 14, for which the sums were broken off at 5, and for which the Coulomb wave functions of Bloch *et al.*[10] were used), since higher values were found to contribute only negligibly. The Clebsch-Gordon coefficients and the Coulomb phase factors used in the calcu-

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[9] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

[10] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951); *Tables of Coulomb Wave Functions*, U. S. National Bureau of Standards Applied Mathematics Series (U. S. Government Printing Office, Washington, D. C., 1952), Vol. I.

[11] C. E. Fröberg, Revs. Modern Phys. **27**, 399 (1955).

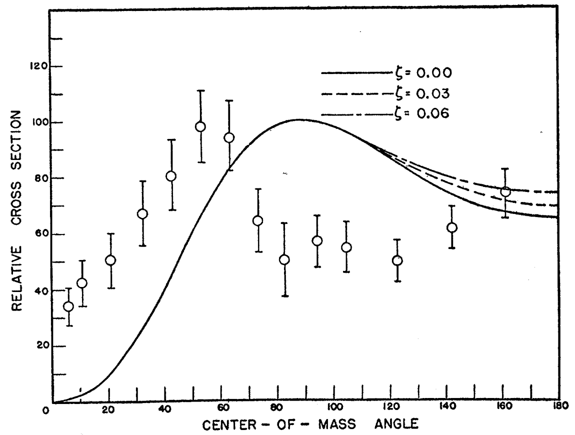

FIG. 6. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.00$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=42$ Mev and for $\zeta=0$, 0.03, and 0.06.

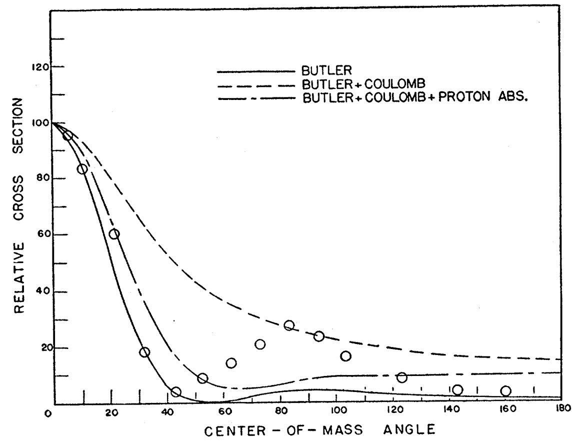

FIG. 7. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.51$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Curves are presented for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

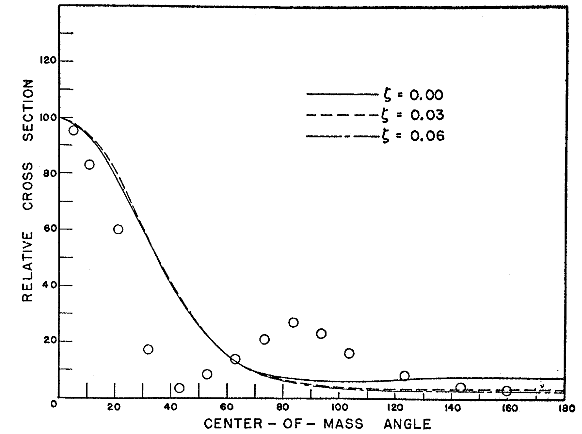

FIG. 8. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.51$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=32$ Mev and for $\zeta=0$, 0.03, and 0.06.

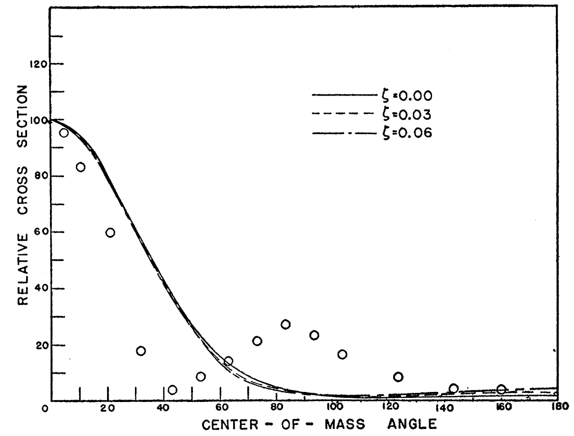

FIG. 9. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.51$ Mev, $Q=1.04$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=42$ Mev and for $\zeta=0$, 0.03, and 0.06.

lations were obtained from the tables of Simon[12] and those of Stanley and Wilkes,[13] respectively. The actual calculations were performed with the aid of an IBM Card-Programed Calculator.

## III. RESULTS

The results for the relative differential cross sections (taking into account the small corrections due to the finite mass of the oxygen nucleus) are presented in Figs. 1–14. Curves for bombarding energies of 2 Mev and 2.51 Mev are shown in Figs. 1–6, 13, and 14, and in Figs. 7–12, respectively. Figures 1–3, 7–9, and 13 refer to the case in which the residual nucleus is left in the first excited state, while Figs. 4–6, 10–12, and 14 correspond to formation in the ground state. A nuclear radius of $4.71\times10^{-13}$ cm has been adopted for all the calculations, except those for Figs. 13 and 14 which are for a radius of $3.76\times10^{-13}$ cm. Circles indicate the experimental points as given by Grosskreutz.[8] The curves have been normalized so that the maximum values are 100 on an arbitrary scale.

Calculations have been performed for the following nuclear models: (1) Butler, (2) Butler+Coulomb, (3) Butler+Coulomb+proton absorption for $l\leq1$, and (4) the optical model with the complex potential $V=-V_0(1+\zeta i)$ with $V_0=32$ Mev and 42 Mev, and $\zeta=0$, 0.03, and 0.06. The case for proton absorption for $l\leq2$ has also been considered, but since the results were found, in general, to be much poorer than for $l\leq1$, the curves are not shown.

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[12] A. Simon, Oak Ridge National Laboratory Report ORNL-1718, 1954 (unpublished).

[13] J. P. Stanley and M. V. Wilkes, *Tables of the Reciprocal of the Gamma Function for Complex Argument* (Computation Centre, University of Toronto, 1950).

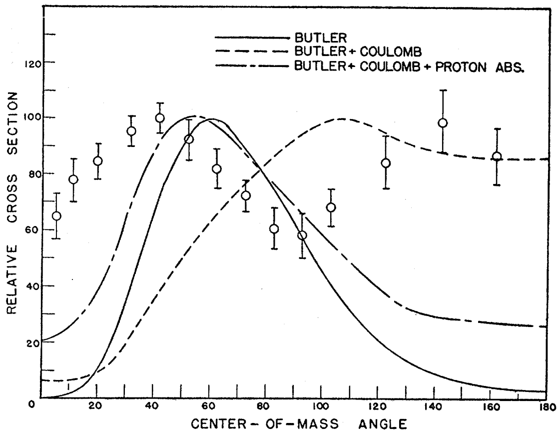

FIG. 10. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.51$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Curves are presented for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

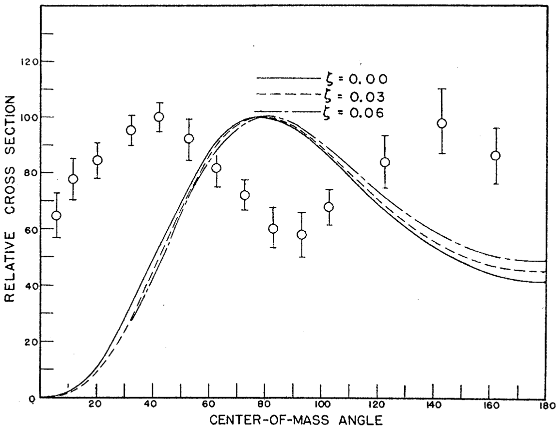

FIG. 12. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.51$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=42$ Mev and for $\zeta=0$, 0.03, and 0.06.

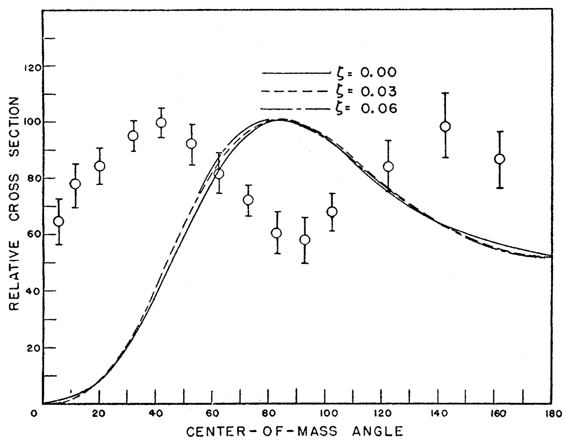

FIG. 11. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.51$ Mev, $Q=1.92$ Mev, and $R=4.71\times10^{-13}$ cm. Calculations are for the optical model with $V_0=32$ Mev and for $\zeta=0$, 0.03, and 0.06.

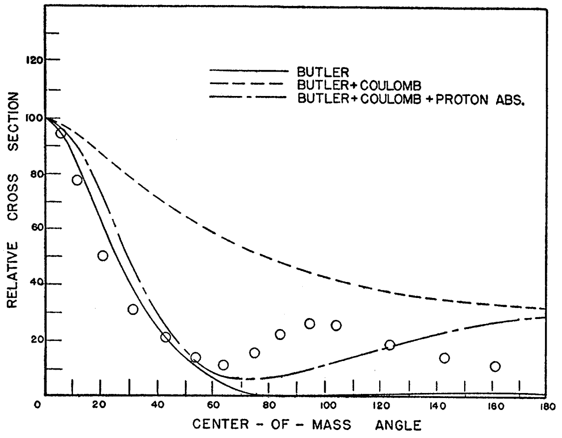

FIG. 13. The angular distribution of protons from the $O^{16}(d,p)O^{17*}$ reaction for $E_d=2.00$ Mev and $R=3.76\times10^{-13}$ cm for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

## IV. DISCUSSION OF RESULTS

An examination of the results presented reveals that the effect of the pure Coulomb field for both the ground and first excited states is to broaden the maxima, shifting them toward larger angles, and to raise the minima. This is in agreement with the conclusions of Tobocman and Kalos.[6] It is clear that insofar as the angular distribution is concerned, the Coulomb effect for the low bombarding energies chosen predominates over that of pure stripping for a wide range of angles, particularly for scattering in the backward direction.

The introduction of proton absorption for protons with $l\leq1$ is seen by comparison with the Butler+Coulomb case to narrow the maxima, moving them in toward smaller angles, and to reduce the backward yield. Thus, this interaction counteracts the effect of the Coulomb field, so that the resultant curves here turn out to be not so very different from the pure-stripping ones of Butler. This behavior would explain the well-known success of the Butler theory in the low-energy region, where one would not expect the theory to be particularly applicable.

A secondary peak for the first excited state appears in the curves calculated with the inclusion of proton absorption in agreement with experiment, but at angles which are too large. Changing the nuclear radius $R$ will, of course change the position of this peak, or, in general, of any peak not centered at $\theta=0°$. However, as a comparison of Fig. 4 with Fig. 14 shows, the optimum value of the nuclear radius—as is the case for pure stripping, and in spite of the inclusion of the Coulomb and proton absorption effects—is still too high. Of course, the approximate character of this particular model should be kept in mind. Even though a semiclassical calculation yields a value for $l$ not too different from 1, quantum mechanically a certain amount of

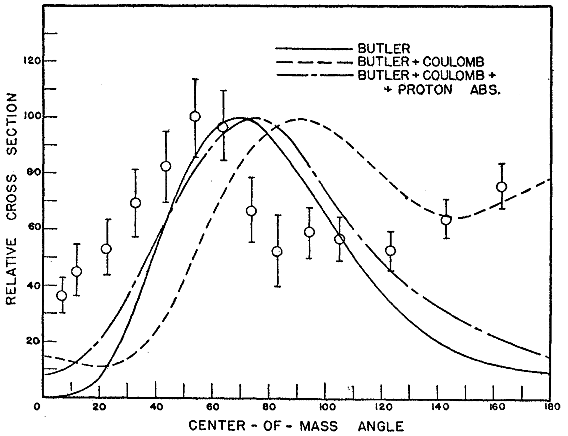

FIG. 14. The angular distribution of protons from the $O^{16}(d,p)O^{17}$ reaction for $E_d=2.00$ Mev and $R=3.76\times10^{-13}$ cm for the (a) Butler, (b) Butler+Coulomb, and (c) Butler+Coulomb+proton absorption for $l\leq1$ interactions.

proton absorption would be expected for the higher angular momenta, and the inclusion of these could readily change the nature of the angular distribution.

As an examination of the experimental curves reveals, the compound nucleus effects are here rather large, especially for the case in which the residual nucleus is formed in its ground state. In fact, as can be seen from Fig. 10, the experimental distribution for this case is very nearly symmetrical about 90° for an incident energy of 2.51 Mev. The excitation curve for the $O^{16}(d,p)O^{17}$ reaction actually indicates a resonance somewhere near this energy.[8] The Tobocman theory does not specifically take into account the possibility of the emission of protons by a compound nucleus and so, as might be expected, is inherently incapable of yielding a nonisotropic distribution having this symmetry. Indeed, it is rather remarkable that under the present unfavorable circumstances there should be any sort of agreement at all between the stripping theory and experiment, even in the forward direction where the stripping effects might be expected to be strongest.

For the optical model, curves are presented for various cases in Figs. 2, 3, 5, 6, 8, 9, 11, and 12. Both the proton and deuteron are assumed to move in the same potential.[14] Two conclusions emerge from a consideration of the results: (1) the agreement with experiment is quite poor, and (2) the curves corresponding to a given final nuclear level are almost identical. Examination shows that the closeness of the curves to each other is a consequence of the large values for the logarithmic derivatives obtained. This tends to make $\alpha_\lambda$ and $\beta_l$—and, as a result, the differential cross section—more or less independent of the potential parameters. As a matter of fact, the logarithmic derivative for the deuteron vanishes for a value of $V_0$ somewhere between 32 Mev and 42 Mev, indicating the existence of a nuclear resonance in this region. Likewise, there appears to be a proton resonance for a potential not too far above 42 Mev.

A pronounced change in the angular distributions would be expected near these resonances due to the rapid variation of $\alpha_\lambda$ and $\beta_l$. The possibility, therefore, seems to exist that a much better agreement with experiment might be obtained for the proper choice of potential parameters than is evidenced by the present results. It is hoped that a more thorough investigation of the optical model for a number of different stripping reactions can be made in the future.

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[14] As has been pointed out by W. Tobocman (private communication), this is somewhat unrealistic. As will be seen below, however, a more refined calculation should not affect the general conclusions.