Possible Experiments for Determination of Beta Interactions. I*

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Some possible experiments are proposed for deciding the relative strength of scalar and vector interactions and that of tensor and axial vector interactions in beta decay. For all the proposed experiments, it is necessary to measure the recoil of the nucleus directly or the nuclear resonance fluorescence caused by a gamma ray which follows the beta decay or K capture. Formulas are given for decays involving the beta-gamma cascades and K capture-gamma-gamma cascades. We describe possible experiments for decays involving beta-gamma-gamma cascades, but do not give explicit formulas. They will be presented in a subsequent paper. These experiments will also determine whether the beta interaction is the same for both signs of the electric charge.

1. INTRODUCTION

RECENT experiments verifying parity nonconservation in beta decay, namely on the beta-ray angular distribution from polarized nuclei, the longitudinal polarization of beta particles, and the beta-circularly polarized gamma angular correlation, have suggested the relations among the coupling constants in beta interactions to be $C_i=-C_i'$ for STP and $C_i=C_i'$ for VA.[1] Assuming these relative signs between the C_i's and C_i''s, the results of these experiments are in agreement with theory, whatever the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$ may be. In order to obtain information on these ratios, the most direct method is to measure the beta-recoil nucleus angular correlations. Data of this sort have been presented by many authors.[2] In this paper, we investigate other possible recoil experiments in beta decay and K capture which could give more information on the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$.

The reason for requiring a measurement of the recoil momentum of the nucleus is as follows. In all of the phenomena of beta decay without observation of the recoil, the theoretical transition probabilities are the same for the assumption of either STP with $C_i=-C_i'$ or VA with $C_i=C_i'$ (or a linear combination of these two interactions).[3] The measurement of the recoil momentum is extremely important, since it can distinguish between these possibilities. The measurement of the recoil momentum can be performed by direct observation or by the nuclear resonance fluorescence caused by a gamma ray following the beta decay or K capture.[4,5] In fact, such an experiment has been performed in the K capture-gamma decay of Eu^{152} by Goldhaber, Grodzins, and Sunyar.[6] Since our methods can be applied in both positron and electron decays, we can determine whether the beta interaction is the same for both signs of the electric charge.

In Sec. 2, possible experiments are presented for the cases of beta-gamma decay and K-capture gamma-gamma decay. We shall consider measurements of the nuclear resonance fluorescence (or the resonant scattering of nuclear gamma rays) and the circular polarization of gamma rays, but not of the polarization of beta rays. In Sec. 3, the calculation of the resonance probability is discussed. Explicit formulas are given for decays involving beta-gamma cascades in Sec. 4, and for decays involving K capture-gamma-gamma cascades in Sec. 5. In Sec. 6, some remarks concerning experimental applications are given. In Appendix I, angular

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[1] C. S. Wu, in *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. Lipkin (North-Holland Publishing Company, Amsterdam, the Netherlands, 1958), p. 346. The authors wish to thank Dr. Wu for showing this article to them before publication. See also M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958). Slight modifications are also possible. For example, $C_i=C_i'$ for all $STPVA$, where VA is dominant, may fit all experimental data within experimental errors.

[2] The experimental data on the beta-(recoil nucleus) angular correlation of the neutron [J. M. Robson, Phys. Rev. **100**, 933 (1955)]; He^6 [B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955); Szalay *et al.*, Padua Conference, 1957 (to be published)]; Ne^{19} [Maxson, Allen, and Jentschke, Phys. Rev. **97**, 109 (1955); M. L. Good and E. J. Lauer, Phys. Rev. **105**, 213 (1957); W. P. Alford and D. R. Hamilton, Phys. Rev. **105**, 673 (1957)]; Ne^{23} [R. W. Ridley, Nuclear Phys. **6**, 34 (1958)]; and A^{35} [Hermansfeld, Maxson, Stählin, and Allen, Phys. Rev. **107**, 641 (1957)] may all be compatible with the same values of coupling constant ratios if certain corrections to the experimental results on He^6 are taken into account. (B. M. Rustad and S. L. Ruby, postdeadline paper presented at the New York Meeting of the American Physical Society, 1958).

[3] This is true in the following cases: (1) the beta decay is an nth forbidden transition ($n=0$ means an allowed transition) with $\Delta J=\pm(n+1)$; or (2) the beta decay is an nth forbidden transition with $\Delta J=\pm n$ and the beta-ray spectrum has an allowed shape. The proof is on p. 438 of M. Yamada and M. Morita, Progr. Theoret. Phys. (Japan) **8**, 431 (1952); on p. 438 of M. Morita, Progr. Theoret. Phys. (Japan) **9**, 345 (1953); and on p. 2051 of M. Morita and R. S. Morita, reference 1.

[4] K. G. Marmfors, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, the Netherlands, 1955), p. 521.

[5] F. Metzger, Phys. Rev. **101**, 286 (1956), and **103**, 983 (1956).

[6] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958). L. A. Page has also considered a similar experiment [Nuovo cimento **7**, 727 (1958)].

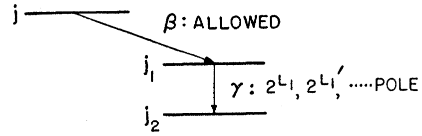

FIG. 1. Beta-gamma decay.

integrals, taking into account the resonance condition, are given. Appendix II contains a list of the possible experiments for the case of the beta-gamma-gamma decay.

2. POSSIBLE EXPERIMENTS FOR DETERMINING THE RELATIVE MAGNITUDES OF THE COUPLING CONSTANTS IN BETA INTERACTIONS

As is well known, the simplest experiment for determining the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$ in beta interactions is a measurement of the beta-neutrino angular correlation.[2,7] The beta-neutrino-gamma angular correlation in a successive beta and gamma decay is also useful.[8] This correlation can be obtained by the following measurements:

1. The circular polarization of a cascade gamma ray in coincidence with the beta ray and recoil nucleus.
2. The beta-(nuclear recoil)-gamma directional correlation.
3. The beta-ray spectrum in coincidence with the gamma-ray and recoil nucleus.

Two other possible experiments, the nuclear recoil-gamma directional correlation and the measurement of circular polarization of the gamma ray in coincidence with the recoil nucleus, have been proposed by Frauenfelder, Jackson, and Wyld,[9] and by Treiman.[10]

If we use the method of nuclear resonance fluorescence to obtain information about the recoil of the nucleus, the following experiments are available in the cases of beta-gamma decay and (K-capture)-gamma-gamma decay. The experiments must determine one of the following:

TABLE I. g_i's in Eq. (3).

Inequality involving p, q, and K	$p \geq q+K$	$p+q \geq K \geq p \sim q$	$q \geq p+K$	$K > p+q$
g_1	Eq^2	$\frac{1}{2}Eq(p+q-K)$	pEq	0
g_2	$-Kq^2$	$\frac{1}{4}q[-p^2+(K-q)^2]$	0	0
g_3	0	$\frac{1}{4}E[-q^2+(K-p)^2]$	$-KpE$	0
g_4	$-\frac{1}{3}q^3$	$\frac{1}{12}[-2p^3-2q^3-K^3+3K(p^2+q^2)]$	$-\frac{1}{3}p^3$	0

4. the absolute intensity of resonant gamma rays and the nuclear level width (in the case of beta decay);
5. the directional correlation between the beta and resonant gamma rays;
6. the beta-ray spectrum in coincidence with a resonant gamma ray;
7. the circular polarization of the resonant gamma ray in coincidence with the beta ray;
8. the circular polarization of the resonant gamma ray in the cases of both beta decay and K capture;
9. the circular polarization of the first (second) gamma ray in coincidence with the second (first) gamma ray and the recoil nucleus (in the case of K capture);
10. the circular polarization of the first gamma ray in coincidence with the resonant second gamma ray (in the case of K capture).

In the case of more complicated decay schemes, there are many possible experiments for determining the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$ in beta interactions. A list of the possible experiments is given in Appendix II for the case of the beta-gamma-gamma decay.

3. THE RESONANCE CONDITION

Since the mechanism of nuclear resonance fluorescence, or the resonant scattering of gamma rays, has been discussed in many papers,[4,5] we do not enter into the details of it here. We focus attention especially upon the resonance scattering by Doppler shift due to the nuclear recoil from preceding radiations, namely the beta particle and K capture and gamma rays. First, we shall neglect such factors as the natural width of the nuclear level, the thermal motion of the nucleus, and the slowing down of the recoil nucleus before gamma emission. Then the probability of finding a resonant gamma ray is proportional to the probability that the component of nuclear recoil momentum K (before emission of the resonant gamma ray) in the direction of emission of the gamma ray lies between K_0 and K_0+dK_0, where K_0 is the gamma-ray momentum. Thus the probability can be expressed in the form $P(K_0)dK_0d\Omega_\gamma$, including K_0 as a parameter in P. These probabilities are given in Secs. 4 and 5 after appropriate angular integrations over the momenta of the emitted particles.

If we include such factors as the natural width of the nuclear level, the thermal motion of the nucleus, and the slowing down of the recoil nucleus before gamma emission, then the probability of finding resonant gamma rays can be written as $[\int I(K)P(K)dK]d\Omega_\gamma$ where $I(K)$ is an appropriate distribution function and $P(K)$ has the same functional form as $P(K_0)$. For simplicity, we drop the subscript zero of K_0, hereafter.

[7] For example, M. Morita, Phys. Rev. **90**, 1005 (1953); Progr. Theoret. Phys. (Japan) **9**, 345 (1953), and **10**, 364 (1953). The electron-neutrino angular correlation has been given up to the first forbidden transition, taking into account the nuclear charge and all interferences among $STPVA$.

[8] M. Morita, Nuclear Phys. **6**, 132 (1958).

[9] Frauenfelder, Jackson, and Wyld, Phys. Rev. **110**, 451 (1958).

[10] S. B. Treiman, Phys. Rev. **110**, 448 (1958).

4. BETA-GAMMA DECAY

We assume the decay scheme to be $j \xrightarrow{\beta} j_1 \xrightarrow{\gamma} j_2$, where j, j_1, and j_2 are spins in the initial, intermediate, and final nuclear states, respectively. The beta decay is an allowed transition and the gamma ray has a mixture of the 2^{L_1}-, $2^{L_1'}$-, $\cdots$ pole radiations (see Fig. 1).[11] The beta-resonant gamma angular correlation is obtained from the beta-neutrino-gamma angular correlation[8] and Eqs. (A1) and (A2) as follows:

$$P=0 \quad \text{for} \quad q<|p\cos\theta+K|,$$

$$\begin{aligned}
P(\theta,\tau,E,K)dEdKd\Omega_e d\Omega_\gamma = F(Z,E)dEdKd\Omega_e d\Omega_\gamma \xi\Bigg\{ &\left(1-a\frac{1}{Eq}p\cos\theta(p\cos\theta+K)+b\frac{1}{E}\right)\left[\sum_{L_1}(j_2\|L_1\|j_1)^2\right] \\
&+\Bigg[\left(A_1\frac{1}{E}p\cos\theta-B_1\frac{1}{q}(p\cos\theta+K)\right)[12j_1(j_1+1)]^{-\frac{1}{2}}[2+j_1(j_1+1)-j(j+1)] \\
&+\left(-A_2\frac{1}{E}p\cos\theta+B_2\frac{1}{q}(p\cos\theta+K)\right)\delta_{jj_1}(3)^{-\frac{1}{2}}\Bigg] \\
&\times\tau\left[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_1(L_1L_1'j_2j_1)\right] \\
&+c\frac{1}{Eq}p\cos\theta(p\cos\theta+K)(-)^{j_1-j}2[2(2j_1+1)/3]^{\frac{1}{2}}W(j_1j_111;2j) \\
&\qquad\times\left[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_2(L_1L_1'j_2j_1)\right]\Bigg\} \quad \text{for} \quad q\geq|p\cos\theta+K|. \qquad (1)
\end{aligned}$$

Here

$$F_n(L_1L_1'j_2j_1)=(-)^{j_2-j_1+1}\{(2j_1+1)(2L_1+1)(2L_1'+1)\}^{\frac{1}{2}}(L_1L_1'1-1|n0)W(j_1j_1L_1L_1';nj_2),$$

and θ is the angle between the directions of the beta and gamma rays. p, q, and K are the momenta of the electron, neutrino, and gamma ray, respectively, in units of mc. E is the electron energy in units of mc^2. $F(Z,E)$ is the Fermi function. ξ, a, b, and c are defined by Jackson *et al.*,[12] $\Lambda_{J'J}$ being replaced by unity in our case. $A_{1,2}$ and $B_{1,2}$ are defined as follows:

$$\begin{aligned}
\xi A_1 &= [\pm 2\,\mathrm{Re}(C_TC_T'^*-C_AC_A'^*)+(\alpha Z/p)2\,\mathrm{Im}(C_TC_A'^*+C_T'C_A^*)]M_{\mathrm{GT}}^2. \\
\xi A_2 &= [2\,\mathrm{Re}(C_SC_T'^*+C_S'C_T^*-C_VC_A'^*-C_V'C_A^*) \\
&\qquad \pm(\alpha Z/p)2\,\mathrm{Im}(C_SC_A'^*+C_S'C_A^*-C_VC_T'^*-C_V'C_T^*)]M_{\mathrm{F}}\cdot M_{\mathrm{GT}}. \\
\xi B_1 &= [\pm 2\,\mathrm{Re}(C_TC_T'^*+C_AC_A'^*)+(\gamma/E)2\,\mathrm{Re}(C_TC_A'^*+C_T'C_A^*)]M_{\mathrm{GT}}^2. \\
\xi B_2 &= [-2\,\mathrm{Re}(C_SC_T'^*+C_S'C_T^*+C_VC_A'^*+C_V'C_A^*) \\
&\qquad \mp(\gamma/E)2\,\mathrm{Re}(C_SC_A'^*+C_S'C_A^*+C_VC_T'^*+C_V'C_T^*)]M_{\mathrm{F}}\cdot M_{\mathrm{GT}}.
\end{aligned} \qquad (2)$$

Here, the upper (lower) sign refers to the electron (positron) decay. $\tau=1$ (-1) for a right- (left-) hand circularly polarized gamma ray.[13] $(j_1j_2m_1m_2|jm)$ and $W(a,b,c,d;e,f)$ are the Clebsch-Gordan and Racah coefficients, respectively. $(j_2\|L_1\|j_1)$ is the nuclear matrix element of the gamma transition. Equation (1) is to be used in analysis of experiments 5, 6, and 7.

The relative intensity of the resonant gamma rays is obtained from the beta-neutrino-gamma angular correlation[8]

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[11] For a gamma decay, the parity condition is always taken into account by the relation $L+L'+\delta+\delta'=\text{even}$, where $\delta=0(+1)$ for a magnetic (electric) radiation. For simplicity, we do not write it in this paper.

[12] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957), and Nuclear Phys. 4, 206 (1957). We use the symbols M_{F}^2, M_{GT}^2 and $M_{\mathrm{F}}\cdot M_{\mathrm{GT}}$ instead of their $|M_{\mathrm{F}}|^2$, $|M_{\mathrm{GT}}|^2$, and $|M_{\mathrm{F}}|\cdot|M_{\mathrm{GT}}|$, respectively. Our M_{F} and M_{GT} are real and they may be positive or negative.

[13] This τ is the same symbol used by several authors recently to indicate the state of the circular polarization of the gamma ray. Its sign is equal to that of the helicity of the gamma ray. In all of our previous work (e.g., reference 1), we adopted $p=1(-1)$ for left (right) circular polarization of a gamma ray. Thus p and τ are related by $p=-\tau$. On the other hand, the definition in optics is sometimes taken opposite to ours. In this case, the right-hand circularly polarized gamma ray has negative helicity, while the gamma ray with the same label used by us has positive helicity.

and Eqs. (A3)–(A8) as follows:

$$P(\tau,E_0,K)dKd\Omega_\gamma=dKd\Omega_\gamma\int F(Z,E)\xi\Big([g_1+ag_4+b(g_1/E)][\sum_{L_1}(j_2\|L_1\|j_1)^2]+\{(A_1g_2+B_1g_3)[12j_1(j_1+1)]^{-\frac{1}{2}}$$

$$\times[2+j_1(j_1+1)-j(j+1)]-(A_2g_2+B_2g_3)\delta_{jj_1}(3)^{-\frac{1}{2}}\}\tau[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_1(L_1L_1'j_2j_1)]$$

$$+cg_4(-)^{j_1-j+1}2[2(2j_1+1)/3]^{\frac{1}{2}}W(j_1j_111;2j)[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_2(L_1L_1'j_2j_1)]\Big)dE. \quad (3)$$

The forms of the g_i's are shown in Table I. They have values depending on inequalities involving p, q, and K. The integration over the electron energy may be performed numerically[14] (see also discussion in Sec. 6).

5. K-CAPTURE GAMMA-GAMMA DECAY

We write down here the recoil nucleus (of the j_1 state)-gamma-gamma angular correlation in the cascade, $j\text{-}^{K\text{ capture}}\!\!\rightarrow j_1\text{-}^{\gamma_1}\!\!\rightarrow j_2\text{-}^{\gamma_2}\!\!\rightarrow j_3$ (see Fig. 2). The direction of the recoil from the neutrino emission is chosen as the z axis. The first and second gamma rays are assumed to be emitted in the directions with polar angles θ_1, $\varphi_1\equiv 0$, and θ_2, φ_2, respectively. We neglect the Fierz term, b, hereafter. The result is

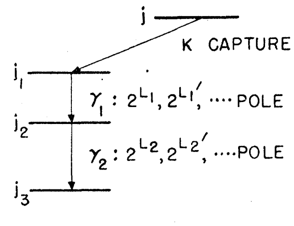

FIG. 2. K capture-gamma-gamma decay.

$$W(\theta_1,\theta_2,\varphi_2,\tau_1,\tau_2)d\Omega_{\text{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}$$

$$=\sum_{L_1,L_1'}\sum_{L_2,L_2'}\sum_{n,n_1,n_2}c^{(n)}(-)^{1+L_1+n_1+n_2}[(2L_1+1)(2L_1'+1)(2n_1+1)(2j_1+1)(2j_2+1)]^{\frac{1}{2}}$$

$$\times\tau_1^{n_1}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)(L_1L_1'1-1|n_10)X\begin{Bmatrix} j_1 & j_2 & L_1 \\ j_1 & j_2 & L_1' \\ n & n_2 & n_1 \end{Bmatrix}$$

$$\times[\textstyle\sum_\mu(-)^\mu(nn_10\mu|n_2\mu)D_{-\mu,0}^{(n_1)}(0,\theta_1,0)D_{\mu,0}^{(n_2)}(\varphi_2,\theta_2,0)]$$

$$\times\tau_2^{n_2}(j_3\|L_2\|j_2)(j_3\|L_2'\|j_2)F_{n_2}(L_2L_2'j_3j_2)d\Omega_{\text{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}, \quad (4),$$

with $n+n_1+n_2=$ even.[15]

$$c^{(0)}=1, \quad (5)$$

$$c^{(1)}=-[(j_1+1)/j_1]^{\frac{1}{2}}B, \quad (6)$$

[14] Neglecting the effect of the nuclear charge [namely $F(Z,E)=1$], integration over the beta energy can be performed analytically. The results are

$$\int g_1dE=(A/96)\{(5E_0^2-K^2-5)-8E_0^2(E_0^2-K^2)^{-1}+2(E_0^2+K^2)(E_0^2-K^2)^{-2}\}+(1/16)\ln(E_0^2-K^2),$$

$$\int g_2dE=\int g_3dE=-(A^3E_0K/24)(E_0^2-K^2)^{-2},$$

$$\int g_4dE=(A/96)\{(-E_0^2+5K^2+5)-8K^2(E_0^2-K^2)^{-1}+2(E_0^2+K^2)(E_0^2-K^2)^{-2}\}-(1/16)\ln(E_0^2-K^2), \quad \text{for} \quad (E_0^2-1)^{\frac{1}{2}}\geqslant K\geqslant 0.$$

$$\int g_idE=0, \quad i=1-4, \qquad \text{for} \quad K>(E_0^2-1)^{\frac{1}{2}}.$$

Here $A=E_0^2-K^2-1$. It is a remarkable fact that these results are the same for both cases of $(E_0^2-1)^{\frac{1}{2}}\geqslant K\geqslant E_0-1$, and $E_0-1>K\geqslant 0$. Although the approximation $Z=0$ is not too good (except in the region of the lightest nuclei), it is useful for qualitative estimate os energy integrals. For example, if K is very close to $(E_0^2-1)^{\frac{1}{2}}$, the intensity of the resonant gamma rays becomes very small and if proportional to $(E_0^2-K^2-1)^3$. Actually, we may observe no resonant gamma rays in this case.

[15] Therefore, the recoil-gamma-gamma angular correlation may be applied as a test for invariance of the strong interactions under time reversal, but not for the weak interactions. The same is true for the case of beta-gamma-gamma correlation [M. Morita and R. S. Morita, Phys. Rev. **110**, 461 (1958)].

with

$$\xi B=2\,\mathrm{Re}\{\lambda_{jj_1}(C_TC_T'^*+C_AC_A'^*+C_TC_A'^*+C_T'C_A^*)M_{\mathrm{GT}}^2-\delta_{jj_1}[j_1/(j_1+1)]^{\frac{1}{2}}(C_SC_T'^*+C_S'C_T^*+C_VC_A'^*+C_V'C_A^* +C_SC_A'^*+C_S'C_A^*+C_VC_T'^*+C_V'C_T^*)M_{\mathrm{F}}\cdot M_{\mathrm{GT}}\},$$

and

$$\begin{aligned}\lambda_{jj_1}&=1 && \text{for } j\rightarrow j_1=j+1\\ &=1/(j_1+1) && \text{for } j\rightarrow j_1=j\\ &=-j_1/(j_1+1) && \text{for } j\rightarrow j_1=j-1.\end{aligned}$$

$$X\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & k\end{bmatrix}=\sum_{\zeta}(2\zeta+1)W(abkf;c\zeta)W(dfhb;e\zeta)W(adkh;g\zeta). \tag{7}$$

$$D_{m,0}^{(l)}(\varphi,\theta,0)=e^{-im\varphi}D_{m,0}^{(l)}(0,\theta,0)=\left(\frac{4\pi}{2l+1}\right)^{\frac{1}{2}}Y_{l,m}^*(\theta,\varphi). \tag{8}$$

τ_1 and τ_2 indicate the states of circular polarization of the first and second gamma rays, respectively. In Eq. (4), terms with $n+n_1+n_2=$odd would appear if the strong interactions were noninvariant under time-reversal.[15] Equation (4) is to be used in the analysis of experiment 9. For experiments 8 and 10, the angular integrals taking into account the resonance condition are necessary. They are given in Appendix I.

Since Eq. (4) is very complicated, we present an example of a particular decay scheme: $\frac{5}{2}$(K capture)$\frac{5}{2}$(dipole)-$\frac{3}{2}$(dipole)$\frac{3}{2}$. For this decay scheme we give the formulas to be used in analysis of experiments 8, 9, and 10.

(a) Polarization correlation of two gamma rays and the nuclear recoil of the j_1 state (experiment 9):

$$\begin{aligned}W(\mathbf{p}_r,\mathbf{k}_1,\mathbf{k}_2,\tau_1,\tau_2)d\Omega_{\mathrm{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}=[&1-(1/50)\{3(\mathbf{k}_1\cdot\mathbf{k}_2)^2-1\}+\tau_1\tau_2\tfrac{3}{10}(\mathbf{k}_1\cdot\mathbf{k}_2)\\ &+\tau_1B\{-(7/10)(\mathbf{p}_r\cdot\mathbf{k}_1)+(21/125)((\mathbf{p}_r\cdot\mathbf{k}_2)(\mathbf{k}_1\cdot\mathbf{k}_2)-\tfrac{1}{3}(\mathbf{p}_r\cdot\mathbf{k}_1))\}\\ &+\tau_2B\{-(7/25)(\mathbf{p}_r\cdot\mathbf{k}_2)-(21/250)((\mathbf{p}_r\cdot\mathbf{k}_1)(\mathbf{k}_1\cdot\mathbf{k}_2)-\tfrac{1}{3}(\mathbf{p}_r\cdot\mathbf{k}_2))\}]d\Omega_{\mathrm{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}.\end{aligned} \tag{9}$$

Here, $\mathbf{p}_r$, $\mathbf{k}_1$, and $\mathbf{k}_2$ are unit momentum vectors of the recoil nucleus and the first and second gamma rays, respectively.

(b) (Circularly polarized gamma)-(resonant gamma) angular correlation (experiment 10):

$$\begin{aligned}P&=0 \quad \text{for} \quad P_r<|K_1\cos\theta+K_2|,\\ P(\theta,\tau_1,P_r,K_1,K_2)dK_1dK_2d\Omega_{\gamma1}d\Omega_{\gamma2}&=[1-(1/50)(3\cos^2\theta-1)\\ &\quad-(147/250)\tau_1B\cos\theta(K_1\cos\theta+K_2)(1/P_r)]dK_1dK_2d\Omega_{\gamma1}d\Omega_{\gamma2}\\ &\qquad \text{for} \quad P_r\geqslant|K_1\cos\theta+K_2|.\end{aligned} \tag{10}$$

Here, P_r, K_1, and K_2 are the magnitudes of the momenta of the neutrino and the first and second gamma rays, respectively. θ is the angle between the two gamma rays.

(c) The relative intensity of the resonant gamma ray (experiment 8):

$$P(\tau_2,P_r,K_1,K_2)dK_2d\Omega_{\gamma2}=[1-(1/50)(3h_5-1)-(21/250)\tau_2B(3h_3+h_6)]dK_2d\Omega_{\gamma2}, \tag{11}$$

with $h_i=I_i/I_1$, $i=3$, 5, and 6. The I_i ($i=1$, 3, 5, and 6) are given in Appendix I. Equations (9), (10), and (11) are to be used in the analysis of experiments 9, 10, and 8, respectively. It should be noticed that these three equations are the same for the magnetic and electric radiations, if we assume pure multipolarities for the first and second gamma rays.

If the gamma-gamma angular correlation and also the higher correlation among recoil and gamma rays are very small, we can neglect all terms with $n_1+n_2\geq 2$. In this case, the formulas to be used in the analysis of experiments 8, 9, and 10 are simplified as follows.

(d) Polarization correlation of two gamma rays and the nuclear recoil of the j_1 state:

$$W(\mathbf{p}_r,\mathbf{k}_1,\mathbf{k}_2,\tau_1,\tau_2)d\Omega_{\text{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}=\{[\sum_{L_1}(j_2\|L_1\|j_1)^2][\sum_{L_2}(j_3\|L_2\|j_2)^2]$$
$$+\tau_1B[(j_1+1)/3j_1]^{\frac{1}{2}}[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_1(L_1L_1'j_2j_1)][\sum_{L_2}(j_3\|L_2\|j_2)^2](\mathbf{p}_r\cdot\mathbf{k}_1)$$
$$+\tau_2B(-)^{j_1+j_2+1}[(j_1+1)(2j_1+1)(2j_2+1)/3j_1]^{\frac{1}{2}}[\sum_{L_1}(-)^{L_1}W(j_1j_1j_2j_2;1L_1)(j_2\|L_1\|j_1)^2]$$
$$\times[\sum_{L_2,L_2'}(j_3\|L_2\|j_2)(j_3\|L_2'\|j_2)F_1(L_2L_2'j_3j_2)(\mathbf{p}_r\cdot\mathbf{k}_2)]\}d\Omega_{\text{recoil}}d\Omega_{\gamma1}d\Omega_{\gamma2}. \quad (12)$$

(e) Circularly polarized gamma-resonant gamma angular correlation:

$$P(\theta,\tau_1,P_r,K_1,K_2)dK_1dK_2d\Omega_{\gamma1}d\Omega_{\gamma2} \quad P=0 \quad \text{for} \quad P_r<|K_1\cos\theta+K_2|,$$
$$=\{[\sum_{L_1}(j_2\|L_1\|j_1)^2]+\tau_1B[(j_1+1)/3j_1]^{\frac{1}{2}}[\sum_{L_1,L_1'}(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)F_1(L_1L_1'j_2j_1)$$
$$\times(\cos\theta/P_r)(K_1\cos\theta+K_2)]\}[\sum_{L_2}(j_3\|L_2\|j_2)^2]dK_1dK_2d\Omega_{\gamma1}d\Omega_{\gamma2} \quad \text{for} \quad P_r\geqslant|K_1\cos\theta+K_2|. \quad (13)$$

(f) The relative intensity of the resonant gamma rays:

$$P=0 \quad \text{for} \quad K_2>P_r+K_1,$$
$$P(\tau_2,P_r,K_1,K_2)dK_2d\Omega_{\gamma2}=\{[\sum_{L_1}(j_2\|L_1\|j_1)^2][\sum_{L_2}(j_3\|L_2\|j_2)^2]$$
$$+\tau_2B(-)^{j_1+j_2+1}[(j_1+1)(2j_1+1)(2j_2+1)/3j_1]^{\frac{1}{2}}[\sum_{L_1}(-)^{L_1}W(j_1j_1j_2j_2;1L_1) \quad (14)$$
$$\times(j_2\|L_1\|j_1)^2][\sum_{L_2,L_2'}(j_3\|L_2\|j_2)(j_3\|L_2'\|j_2)F_1(L_2L_2'j_3j_2)]h_3\}dK_2d\Omega_{\gamma2},$$

with

$$\begin{aligned} h_3&=0 && \text{for} \quad K_1>P_r+K_2\\ &=K_2/P_r && \text{for} \quad P_r\geqslant K_1+K_2\\ &=(P_r-K_1+K_2)/2P_r && \text{for} \quad P_r+K_1\geqslant K_2\geqslant P_r\sim K_1.\end{aligned}$$

6. CONCLUDING REMARKS

Although a formula for the absolute intensity of resonant gamma rays for the beta-gamma decay has not been deduced, it is easily obtained from $\frac{1}{2}[\sum_\tau\int I(K)P(\tau,E_0,K)dK]d\Omega_\gamma$. Here we use the expression in Eq. (3) for $P(\tau,E_0,K)$. The result of this integration is to be used in the analysis of experiment 4. In this case, the a and c terms are relevant for determination of the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$. We have no such terms for K capture.

In Eq. (3) the integrals over the energy of the beta ray for g_2 and g_3 are equal and negative, if we assume either the electron mass$=0$, or nuclear charge$=0$ [namely $F(Z,E)=1$].[14] Thus, the circular polarization of the resonant gamma ray in experiment 8 could be large for T and zero for A.[16] Here $C_i=-C_i'$ for STP and $C_i=C_i'$ for VA are adopted. Taking into account the effect of the nuclear charge, it would be large for T and small, but not equal to zero, for A. Furthermore, circular polarizations for T and A have different (same) signs in the electron (positron) decay. The same statement holds for the interference terms of the Fermi and Gamow-Teller parts, with the replacements $T\rightarrow VA$ and $A\rightarrow ST$. The resonant gamma ray from K capture-gamma-gamma decay has its maximum circular polarization when $K_2=P_r$ and $K_1=0$.

In experiments 1–3, 5–7, 9, and 10, we can choose a suitable geometry and/or energy of the beta ray to give large effects for both STP and VA. As an example, we discuss experiment 7 in detail. The resonant gamma ray in coincidence with the beta ray has its maximum polarization at $\theta=\pi/2$ and $K=q$ [see Eq. (1)]. Furthermore, if we assume a Gamow-Teller transition for the beta decay and a pure multipolarity for the gamma ray, the degree of the circular polarization becomes $(L_1+1)^{-1}$, $(j+2)[(j+1)(L_1+1)]^{-1}$, and $(j-1)[j(L_1+1)]^{-1}$ for $j\rightarrow j\pm1\rightarrow j\pm1\pm L_1$, $j\rightarrow j+1\rightarrow j+1-L_1$, and $j\rightarrow j-1\rightarrow j-1+L_1$, respectively. Thus the best decay scheme is $j\rightarrow j+1\rightarrow j$ with a dipole transition. (The

[16] A. M. Bincer and J. Weneser have also mentioned this. We wish to thank Dr. Bincer and Dr. Weneser for a valuable discussion.

same decay scheme is also best for experiment 8.) In this case, numerical values of the degree of circular polarization are 100%, 83%, 75%, 70%, 67%, and 64% for $j=0, \frac{1}{2}, 1, \frac{3}{2}, 2$, and $\frac{5}{2}$, respectively. The helicities of the emitted neutrino and antineutrino are equal (opposite) to that of the gamma ray coming out at $\theta=\pi/2$ in the decay scheme $j\to j\pm1\to j\pm1\mp L_1$ ($j\to j\pm1\to j\pm1\pm L_1$).

Explicit formulas for possible experiments in decays involving beta-gamma-gamma cascades discussed in Appendix II and a description of suitable nuclei will be published in the near future.

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APPENDIX I. ANGULAR INTEGRALS EVALUATED UNDER RESONANCE CONDITIONS

First, we present the results of angular integrals evaluated under resonance conditions in the case of K capture-gamma-gamma decay. For the calculation of the gamma-resonant gamma angular correlation, the following two integrals are necessary:

$$\begin{aligned}J_1&=\int\delta(-P_r(\mathbf{p}_r\cdot\mathbf{k}_2)+K_1(\mathbf{k}_1\cdot\mathbf{k}_2)+K_2)d\Omega_{\text{recoil}}\\&=2\pi/P_r \qquad \text{for } P_r\geqslant|K_1\cos\theta+K_2|,\\&=0 \qquad \text{for } P_r<|K_1\cos\theta+K_2|.\end{aligned}\tag{A1}$$

$$\begin{aligned}J_2&=\int(\mathbf{p}_r\cdot\mathbf{k}_2)\delta(-P_r(\mathbf{p}_r\cdot\mathbf{k}_2)+K_1(\mathbf{k}_1\cdot\mathbf{k}_2)+K_2)d\Omega_{\text{recoil}}\\&=2\pi(K_1\cos\theta+K_2)/P_r^2 \quad \text{for } P_r\geqslant|K_1\cos\theta+K_2|,\\&=0 \qquad \text{for } P_r<|K_1\cos\theta+K_2|.\end{aligned}\tag{A2}$$

For calculation of the intensity of resonant gamma rays, we define I_i by

$$I_i=\int f_i\delta(-P_r(\mathbf{p}_r\cdot\mathbf{k}_2)+K_1(\mathbf{k}_1\cdot\mathbf{k}_2)+K_2)d\Omega_{\text{recoil}}d\Omega_{\gamma1}.$$

Here, f_i is a function of $(\mathbf{p}_r\cdot\mathbf{k}_1)$, $(\mathbf{p}_r\cdot\mathbf{k}_2)$, and $(\mathbf{k}_1\cdot\mathbf{k}_2)$. In general, the integrals with $f_i=(\mathbf{p}_r\cdot\mathbf{k}_2)(\mathbf{k}_1\cdot\mathbf{k}_2)^n$ and $(\mathbf{p}_r\cdot\mathbf{k}_1)(\mathbf{k}_1\cdot\mathbf{k}_2)^{n-1}$ give the same result. The six integrals involving the lowest powers of $\mathbf{k}_1$ and $\mathbf{k}_2$ are as follows:

$$\begin{aligned}f_1&=1,\\I_{1a}&=8\pi^2/K_1,\\I_{1b}&=4\pi^2(P_r+K_1-K_2)/P_rK_1,\\I_{1c}&=8\pi^2/P_r.\end{aligned}\tag{A3}$$

$$\begin{aligned}f_2&=(\mathbf{k}_1\cdot\mathbf{k}_2),\\I_{2a}&=-8\pi^2K_2/K_1^2,\\I_{2b}&=-2\pi^2[K_1^2-(K_2-P_r)^2]/P_rK_1^2,\\I_{2c}&=0.\end{aligned}\tag{A4}$$

$$\begin{aligned}f_3&=(\mathbf{p}_r\cdot\mathbf{k}_2),\\I_{3a}&=0,\\I_{3b}&=2\pi^2[P_r^2-(K_2-K_1)^2]/P_r^2K_1,\\I_{3c}&=8\pi^2K_2/P_r^2.\end{aligned}\tag{A5}$$

$$\begin{aligned}f_4&=(\mathbf{p}_r\cdot\mathbf{k}_1),\\I_{4a}&=8\pi^2P_r/3K_1^2,\\I_{4b}&=2\pi^2[2P_r^3+2K_1^3+K_2^3\\&\qquad-3K_2(P_r^2+K_1^2)]/3P_r^2K_1^2,\\I_{4c}&=8\pi^2K_1/3P_r^2.\end{aligned}\tag{A6}$$

$$\begin{aligned}f_5&=(\mathbf{k}_1\cdot\mathbf{k}_2)^2,\\I_{5a}&=8\pi^2(3K_2^2+P_r^2)/3K_1^3,\\I_{5b}&=4\pi^2(P_r^3+K_1^3-K_2^3+3P_rK_2^2\\&\qquad-3P_r^2K_2)/3P_rK_1^3,\\I_{5c}&=8\pi^2/3P_r.\end{aligned}\tag{A7}$$

$$\begin{aligned}f_6&=(\mathbf{p}_r\cdot\mathbf{k}_1)(\mathbf{k}_1\cdot\mathbf{k}_2),\\I_{6a}&=-16\pi^2P_rK_2/3K_1^3,\\I_{6b}&=\pi^2(3P_r^4-3K_1^4-K_2^4+6P_r^2K_2^2\\&\qquad-8P_r^3K_2+4K_1^3K_2)/3P_r^2K_1^3,\\I_{6c}&=8\pi^2K_2/3P_r^2.\end{aligned}\tag{A8}$$

Subscripts a, b, and c of the I_i's denote inequalities involving P_r, K_1, and K_2 as follows:

$$\begin{aligned}a&:\ K_1\geqslant P_r+K_2,\\b&:\ P_r+K_1\geqslant K_2\geqslant P_r\sim K_1,\\c&:\ P_r\geqslant K_1+K_2.\end{aligned}$$

The I_i's ($i=1$ through 6) are equal to zero if $K_2>P_r+K_1$. In decay schemes where nuclear spins and multipolarities of gamma rays have high values, integrals containing higher powers of $\mathbf{k}_1$ and $\mathbf{k}_2$ are necessary.

In the case of beta-gamma decay, corresponding angular integrals obtain with the following substitutions: $-P_r\mathbf{p}_r\to\mathbf{q}$, $K_1\mathbf{k}_1\to\mathbf{p}$, $\mathbf{k}_2\to\mathbf{k}$, $P_r\to q$, $K_1\to p$, and

$K_2 \rightarrow K$, both in the results for the integrals and in the energy inequalities.

APPENDIX II. LIST OF POSSIBLE EXPERIMENTS IN BETA-GAMMA-GAMMA DECAY

In the decay scheme $j\text{-}^{\beta}\!\!\rightarrow j_1\text{-}^{\gamma_1}\!\!\rightarrow j_2\text{-}^{\gamma_2}\!\!\rightarrow j_3$, the following measurements determine the values of the ratios $|C_S/C_V|$ and $|C_T/C_A|$ in beta interactions:

1. the absolute intensity of the resonant gamma rays and the nuclear level width;
2. the circular polarization of the resonant gamma ray;
3. the beta-recoil directional correlation;
4. the beta-ray spectrum in coincidence with the recoil;
5. the beta-resonant gamma directional correlation;
6. the beta-ray spectrum in coincidence with the resonant gamma ray;
7. the circular polarization of the resonant gamma ray in coincidence with the beta ray;
8. the directional correlation of the first (second) gamma ray and the recoil;
9. the circular polarization of the first (second) gamma ray in coincidence with the recoil;
10. the circular polarization of the first gamma ray in coincidence with the resonant second gamma ray;
11. the circular polarization of the resonant second gamma ray in coincidence with the first gamma ray;
12. the (first gamma)-(resonant second gamma) directional correlation;
13. the beta-(first gamma)-(resonant second gamma) directional correlation;
14. the beta-ray spectrum in coincidence with the first and resonant second gamma rays;
15. the circular polarization of the first (resonant second) gamma ray in coincidence with the beta and resonant second (first) gamma rays;
16. the beta-(recoil nucleus)-first (second) gamma directional correlation;
17. the circular polarization of the first (second) gamma ray in coincidence with the beta ray and recoil nucleus;
18. the beta-ray spectrum in coincidence with the recoil nucleus and the first (second) gamma ray;
19. the (recoil nucleus)-gamma-gamma directional correlation;
20. the circular polarization of the first (second) gamma ray in coincidence with the recoil nucleus and the second (first) gamma ray;
21. the beta-(recoil nucleus)-gamma-gamma directional correlation;
22. the circular polarization of the first (second) gamma ray in coincidence with the recoil nucleus, beta ray, and the second (first) gamma ray;
23. the beta-ray spectrum in coincidence with the recoil nucleus and the first and second gamma rays.