Photomeson Production in Intermediate Coupling*†

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The wave functions for the ground state of a physical nucleon and for the scattering state of a physical nucleon and a pion are used to compute photopion production cross sections in an intermediate-coupling approximation. Since the mesons included in the physical nucleon suffice to account for an approximately correct value of the anomalous magnetic moment, no use is made in this theory of a correction term involving experimental values of the latter. The p-wave part of the photoproduction matrix element is arrived at naturally, largely from the interaction of the meson currents with the electromagnetic field. Differential cross sections are obtained for the production of π^+ and π^0 by photons of 200, 260, 335, and 440 Mev in the laboratory system. It is also pointed out that in the intermediate-coupling theory, the ambiguity in defining a gauge-invariant Hamiltonian is nontrivial. Different procedures for generalizing the meson-nucleon interaction term into a form that is gauge invariant in an extended-source theory lead to substantially different p-wave components of the photoproduction cross sections.

I. INTRODUCTION

BY making use of the Tomonaga method for dealing with intermediate-strength coupling between mesons and nucleons, approximate solutions have been obtained both for the ground state of a physical (clothed) nucleon and for the scattering state of a physical nucleon and a pion. This has been done for the case of a symmetric pseudoscalar meson field coupled to a fixed extended source through derivative coupling. The scattering state function obtained seems to give a correct description of low-energy scattering. In particular, it gives approximately the correct resonance in the scattering phase shift for the $I=J=\frac{3}{2}$ state.[1] (J is the spin, I the isotopic spin of the pion-physical nucleon combination.) The ground state solution of the physical nucleon has been used to evaluate the anomalous magnetic moment of the nucleon, on the assumption that the latter is correctly represented by the interaction of the meson current with a slowly varying magnetic field.[2] The result of that calculation approximately agrees with the experimental values. Since the anomalous moment is approximately right, and since the interaction current term must give the correct Kroll-Ruderman limit, it can reasonably be expected that the probability of free meson creation due to the interaction of the photon with the physical nucleon will be given at least to the right order of magnitude. Moreover, due to the approximately correct behavior of the scattering state function, it can also be expected that the relative amount of Born approximation terms and "enhancement" terms will be about right at relatively low energies.

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[1] Friedman, Lee, and Christian, Phys. Rev. **100**, 1494 (1955) (hereafter referred to as FLC).
[2] K. Haller and M. H. Friedman, Phys. Rev. **100**, 1501 (1955) (hereafter referred to as HF).

II. WAVE FUNCTIONS

The ground state wave function for a physical nucleon in a spin, isotopic spin state l, will be written

$$\mathfrak{N}_l=\Psi_0|l\rangle. \tag{1}$$

Here $|l\rangle$ represents a Dirac spinor for spin and isotopic spin l. $\mathfrak{N}_l$ represents a physical nucleon of the same spin and isotopic spin. Ψ_0 is a spinor operator function given in FLC. Matrix elements M_Θ of an operator Θ taken between two physical nucleon ground states,

$$M_\Theta=(\mathfrak{N}_l|\Theta|\mathfrak{N}_l),$$

can be written symbolically:

$$M_\Theta=\langle l|(\Psi_0^\dagger\Theta\Psi_0)|l\rangle.$$

Here the matrix element $(\Psi_0^\dagger\Theta\Psi_0)$ already includes integration over the nine-dimensional reduced meson space of FLC. It is still, however, a function of spinor operators. It will generally be written

$$(\Psi_0^\dagger\Theta\Psi_0)=[\![\Theta]\!].$$

The incoming scattering state wave function for a physical nucleon in spin and isotopic spin state m, and a meson of momentum $\mathbf{p}'$ in charge state μ, when written in the momentum representation, is given by

$$\Psi_m^{(-)}(\mu,\mathbf{p}')=\sum_{\mathbf{q},\alpha,l}\{\langle l|\chi(\mu,\alpha;\mathbf{p}',\mathbf{q})|m\rangle a_\alpha^*(\mathbf{q})\Psi_0|l\rangle -\sum_n\langle l|[\![a_\alpha^*(\mathbf{q})]\!]|n\rangle \times\langle n|\chi(\mu,\alpha;\mathbf{p}',\mathbf{q})|m\rangle\Psi_0|l\rangle\}. \tag{2}$$

The functions $\chi(\mu,\alpha;\mathbf{p}',\mathbf{q})$ are obtained from a variational principle and are given by

$$\chi(\mu,\alpha;\mathbf{p}',\mathbf{q})=\delta_{\mu,\alpha}\delta_{\mathbf{p}',\mathbf{q}}+\sum_{I,J}\frac{G_{I,J}(p',q)P_{I,J}(\mu,\alpha;\mathbf{p}',\mathbf{q})}{\omega_q-\omega_{p'}+i\epsilon}, \tag{3}$$

where the summation extends over I, $J=\frac{1}{2},\frac{3}{2}$. The $\delta_{i,j}$ here are the usual Kronecker delta functions. The

$P_{I,J}(\mu,\alpha;\mathbf{p}',\mathbf{q})$ appearing in Eq. (3) are the projection operators

$$P_{I,J}(\mu,\alpha;\mathbf{p}',\mathbf{q})=\mathcal{S}_J(\mathbf{p}',\mathbf{q})\mathcal{T}_I(\mu,\alpha), \tag{4}$$

and are

$$\mathcal{S}_{\frac{1}{2}}(\mathbf{p}',\mathbf{q})=\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{p}'/qp',$$
$$\mathcal{S}_{\frac{3}{2}}(\mathbf{p}',\mathbf{q})=(3\mathbf{p}'\cdot\mathbf{q}-\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{p}')/qp',$$
$$\mathcal{T}_{\frac{1}{2}}(\mu,\alpha)=\tfrac{1}{3}\tau_\alpha\tau_\mu,$$
$$\mathcal{T}_{\frac{3}{2}}(\mu,\alpha)=\delta_{\mu,\alpha}-\tfrac{1}{3}\tau_\alpha\tau_\mu.$$

The $G_{I,J}(p',q)$ are given by

$$G_{I,J}(p',q)=\alpha_{I,J}(p')g(q)+\beta_{I,J}(p')\omega_q g(q), \tag{5}$$

where the $g(q)$ are

$$g(q)=-Lqu(q\leqslant K)/[\omega_q]^{\frac{1}{2}}[\omega_q+\lambda],$$

and L is defined by[2]

$$L=\left(\frac{4\pi}{3}\right)^{\frac{1}{2}} f\frac{\langle l|\,[\![\tau_\alpha\sigma_i x_{i\alpha}]\!]\,|l\rangle}{\langle l|\,[\![p_{i\alpha}p_{i\alpha}-x_{i\alpha}x_{i\alpha}-1]\!]\,|l\rangle}.$$

The quantities $\alpha_{I,J}(p')$ and $\beta_{I,J}(p')$ are given by the equations

$$\alpha_{IJ}(p')=g(p')\left\{\frac{\frac{1}{2}A_{IJ}\omega_{p'}+B_{IJ}+\frac{1}{4}(A_{IJ})^2\Omega}{1-A_{IJ}-(A_{IJ}\omega_{p'}+B_{IJ})\langle g^2\rangle+\frac{1}{4}(A_{IJ})^2[1+(\omega_{p'}-\Omega)\langle g^2\rangle]}\right\}, \tag{6}$$

and

$$\beta_{IJ}(p')=g(p')\left\{\frac{\frac{1}{2}A_{IJ}-\frac{1}{4}(A_{IJ})^2}{1-A_{IJ}-(A_{IJ}\omega_{p'}+B_{IJ})\langle g^2\rangle+\frac{1}{4}(A_{IJ})^2[1+(\omega_{p'}-\Omega)\langle g^2\rangle]}\right\},$$

where

$$\Omega=\textstyle\sum_{\mathbf{k}}\omega_k[g(k)]^2,$$
$$\langle g^2\rangle=\textstyle\sum_{\mathbf{k}}[g(k)]^2[\omega_k-\omega_{p'}+i\epsilon]^{-1},$$

and the A_{IJ} and B_{IJ} are as defined in FLC.

III. GAUGE INVARIANCE OF THE THEORY

It is well known that the simple procedure for generalizing theories with point interactions to include an electromagnetic field is not valid when the interaction of the nucleon and the meson field is extended over a finite region in space.[3] The part of the interaction Hamiltonian,

$$\mathsf{H}_\Sigma=(4\pi)^{\frac{1}{2}}f\int U(x)\tau_\alpha\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\phi_\alpha(\mathbf{x})d\mathbf{x}, \tag{7}$$

which deals with charged mesons can be written

$$\mathsf{H}=\sqrt{2}(4\pi)^{\frac{1}{2}}f\int U(x)\tau_{(+)}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\phi(\mathbf{x})d\mathbf{x}+\text{H.c.}, \tag{8}$$

where $\tau_{(+)}=\frac{1}{2}(\tau_1+i\tau_2)$, $\phi=(2)^{-\frac{1}{2}}(\phi_1-i\phi_2)$, f is the nonrenormalized coupling constant, and $U(x)$ is a spherically symmetric normalized nucleon source function. It is immediately obvious that the procedure of replacing $\boldsymbol{\nabla}\phi$ by $(\boldsymbol{\nabla}-ie\mathbf{A})\phi$ and $\boldsymbol{\nabla}\phi^*$ by $(\boldsymbol{\nabla}+ie\mathbf{A})\phi^*$ does not give rise to a gauge-invariant theory unless $U(x)$ is a delta function. When $U(x)$ is not a delta function, it is easily possible to generalize H to include an electromagnetic field in a gauge-invariant fashion, but it is not possible to do so uniquely. We may, for example, write a gauge-invariant form H_a, defined by

$$\mathsf{H}_a=\sqrt{2}(4\pi)^{\frac{1}{2}}f\int d\mathbf{x}\;U(x)\tau_{(+)}\left\{\exp\left[-ie\int_0^x\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}\right]\right\}\boldsymbol{\sigma}\cdot[\boldsymbol{\nabla}_{\mathbf{x}}-ie\mathbf{A}(\mathbf{x})]\phi(\mathbf{x})+\text{H.c.}, \tag{9}$$

which satisfies the requirements of (1) gauge invariance and (2) reducing to H (where $\boldsymbol{\nabla}$ has already been replaced by $(\boldsymbol{\nabla}-ie\mathbf{A})$ in H) for the case of a delta-function source. It is, however, possible to amend this form in an infinite number of ways, and still to preserve both of these properties: A term $\int\mathfrak{H}\cdot\mathbf{n}ds$ may be added to $\int_0^{\mathbf{x}}\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}$ in the exponential. Here $\mathfrak{H}$ is the magnetic field, $\int ds$ indicates a surface integral over any area entirely enclosed in the volume of the nucleon source, and $\mathbf{n}$ is the unit normal to the surface. The effect of this term is to make the path over which the line integral is to be performed entirely arbitrary. Also, a term H' where

$$\mathsf{H}'=c\int d\mathbf{x}\int_0^1 d\xi\{U(x)\tau_{(+)}\boldsymbol{\sigma}\cdot[\mathbf{x}\times\mathfrak{H}(\xi\mathbf{x})]\mathfrak{g}(\xi)\}, \tag{10}$$

may be added to H_a. Here c is an arbitrary constant and $\mathfrak{g}(\xi)$ is an arbitrary function of ξ. In all of these possible additions to H_a, $\mathfrak{H}$ is the only function of $\mathbf{A}$ that appears, and these additions are therefore manifestly gauge invariant. They are all scalars and they go to zero in the case of a delta-function source. However, each one of this infinity of possible alterations of H_a can in principle lead to different values of physically measurable quantities.

It can be hoped that the effects of this ambiguity in defining the most general form of a gauge-invariant meson-nucleon interaction Hamiltonian can be minimized either by showing that the physical consequences of the ambiguity are small; or at least, by finding one particularly simple specific form, which, for *a priori* reasons, is more plausible than all the others. That the latter is not the case can be easily shown by choosing c and $\mathfrak{g}(\xi)$ in H' to be $c=ie\sqrt{2}(4\pi)^{\frac{1}{2}}ef$ and $\mathfrak{g}(\xi)=\xi$, respectively. Then, since $\boldsymbol{\nabla}_{\mathbf{x}}\int_{0,l}^{\mathbf{x}}\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}$ (here the $\int_{0,l}^{\mathbf{x}}$ indicates that the path chosen is a straight line) can be

[3] R. H. Capps and W. G. Holladay, Phys. Rev. **99**, 931 (1955); for earlier discussion of a related problem, see R. G. Sachs, Phys. Rev. **74**, 433 (1948).

written

$$\nabla_{\mathbf{x}}\int_{0,\,l}^{\mathbf{x}} \mathbf{A}(\mathbf{y})\cdot d\mathbf{y}=\mathbf{A}(\mathbf{x})-\mathbf{x}\times\int_0^1 \xi d\xi\, \mathfrak{H}(\xi\mathbf{x}),$$

it can be shown that for this choice of path and of c and $\mathfrak{g}(\xi)$, the equation $\mathsf{H}_a+\mathsf{H}'=\mathsf{H}_b$ holds, where

$$\mathsf{H}_b=\sqrt{2}(4\pi)^{\frac{1}{2}}f\int d\mathbf{x}\, U(x)\tau_{(+)}\boldsymbol{\sigma} \cdot\nabla_{\mathbf{x}}\left\{\phi(\mathbf{x})\left[\exp\left(-ie\int_0^{\mathbf{x}} \mathbf{A}(\mathbf{y})\cdot d\mathbf{y}\right)\right]\right\}+\text{H.c.} \quad (11)$$

There is no compelling *a priori* reason for preferring either H_a or H_b and both have been used by different authors. H_a is used by Enoch, Sachs, and Wali[4] to calculate photopion production cross sections. $\mathsf{H}_a{}^0$, given by

$$\mathsf{H}_a{}^0=\sqrt{2}(4\pi)^{\frac{1}{2}}f\int d\mathbf{x}\, U(x)\tau_{(+)}\boldsymbol{\sigma}\cdot[\nabla-ie\mathbf{A}(\mathbf{x})]\phi(\mathbf{x})+\text{H.c.},$$

which accounts for by far the largest contributions to H_a, is used by Chew and Low in a computation of pion photoproduction cross sections.[5] H_b is used in various computations of the anomalous magnetic moment of the nucleon.[6] The question of whether the differences between the various forms are large enough to be important can now be raised. In the case of the choice of different paths for the line integral in the exponential of H_a or H_b, calculations carried out by the author indicate that these differences can be expected to be small.[7] For this reason, the simple choice of a straight-line path will henceforth be understood in this paper. The ambiguity in the meson-nucleon interaction Hamiltonian due to the H' term is, however, large. It can quickly be shown that the physical consequences of choosing H_b instead of H_a are pronounced. For example, in the case of a constant magnetic field, $\mathbf{A}(\mathbf{x})=\frac{1}{2}(\mathbf{x}\times\mathfrak{H})$ and $\int_{0}^{\mathbf{x}}{}_{,\,l}\,\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}=0$. H_b for this case reduces to (8) and there is no interaction current contribution to the magnetic moment. In HF, a calculation based on this interaction term gives an approximately correct value of the anomalous magnetic moment. H_a, however, gives rise to an interaction current term

$$H^{\text{Int}}=-ie\sqrt{2}(4\pi)^{\frac{1}{2}}f\int d\mathbf{x}\, U(x)\tau_{(+)}\boldsymbol{\sigma} \cdot\mathbf{A}(\mathbf{x})\phi(\mathbf{x})+\text{H.c.} \quad (12)$$

This term, if included in the calculation of the magnetic moment, would contribute an additional anomalous moment of $-2.5\tau_3$ nuclear magnetons, a result entirely inconsistent with the experimental facts. Since the program for the present calculation involves the use of a theory which predicts the correct anomalous magnetic moment, H_b is clearly the proper choice here. However, since the consequences of using H_a are also of interest (partially insofar as they provide a measure of the ambiguity involved in defining the interaction term), the meson photoproduction cross sections resulting from the use of H_a will also be calculated.

IV. THE TRANSITION MATRIX ELEMENT

The matrix element for the production of a single meson and the annihilation of a photon is

$$M=\langle\Psi_m{}^{(-)}(\mu,\mathbf{p}')|H|\mathfrak{N}_n\eta_{\mathrm{i}}(\mathbf{p})\rangle, \quad (13)$$

where H is the part of the complete Hamiltonian that is first order in the electric charge, and where $\eta_{\mathrm{i}}(\mathbf{p})$ is the normalized state vector for a single photon of momentum $\mathbf{p}$ and polarization i. Substitution of (2) into (13) gives rise to the following expression for M where for simplicity M is written $M=M(1)+M(2)$:

$$M(1)=\sum_{\mathbf{q},\alpha,l}\langle l|\,[\![a_\alpha(\mathbf{q})\bar{H}]\!]\,|n\rangle\langle m|\chi^\dagger(\mu,\alpha;\,\mathbf{p}',\mathbf{q})|l\rangle, \quad (14)$$

$$M(2)=-\sum_{\mathbf{q},\alpha,l}\langle l|\,[\![\bar{H}]\!]\,|n\rangle \times\langle m|\chi^\dagger(\mu,\alpha;\,\mathbf{p}',\mathbf{q})\,[\![a_\alpha(\mathbf{q})]\!]\,|l\rangle. \quad (15)$$

Here $\bar{H}$ denotes $H|\eta_{\mathrm{i}}(\mathbf{p})\rangle$.

The Hamiltonian H can be expressed as the sum of a meson current term and an interaction current term, H^M and H^I, respectively. Thus H is

$$H=H^M+H^I. \quad (16)$$

The meson current Hamiltonian is

$$H^M=e\int d\mathbf{x}\,\mathbf{A}\cdot(\phi_1\nabla\phi_2-\phi_2\nabla\phi_1); \quad (17)$$

in the momentum representation $\mathbf{A}$ is

$$\mathbf{A}(\mathbf{x})=\sum_{\mathrm{i}=1,2;\,\mathbf{p}}[2p]^{-\frac{1}{2}}[b^{\mathrm{i}}(\mathbf{p})+b^{\mathrm{i}*}(-\mathbf{p})]\boldsymbol{\varepsilon}_{\mathbf{p}}{}^{\mathrm{i}}e^{i\mathbf{p}\cdot\mathbf{x}}, \quad (18)$$

where $b^{\mathrm{i}}(\mathbf{p})$ and $b^{\mathrm{i}*}(\mathbf{p})$ are annihilation and creation operators, respectively, for photons of momentum $\mathbf{p}$ and polarization i. (The $\mathbf{p}$ and i index on the $\boldsymbol{\varepsilon}$ will generally be suppressed.) The interaction current Hamiltonian which originates from H_b can conveniently be written as the sum

$$H^I=H^{Ia}+H^{Ib}. \quad (19)$$

Here the H^{Ia} parts involve the meson field ϕ but not its derivatives; the H^{Ib} parts include only the derivatives of ϕ. H^{Ia} can again be conveniently subdivided as

$$H^{Ia}=H_A{}^{Ia}+H_B{}^{Ia}. \quad (20)$$

[4] Enoch, Sachs, and Wali, Phys. Rev. **108**, 433 (1957).

[5] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1579 (1956) (hereafter referred to as CL II).

[6] See reference 2; see also M. H. Friedman, Phys. Rev. **97**, 1123 (1954); also H. Miyazawa, Phys. Rev. **101**, 1564 (1956).

[7] The smallness of the difference between $\int_0^{\mathbf{x}}{}_{c_1}\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}$ and $\int_0^{\mathbf{x}}{}_{c_2}\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}$ can be understood in the following way: Since $\oint\mathbf{A}(\mathbf{y})\cdot d\mathbf{y}=\int\mathfrak{H}(\mathbf{y})\cdot\mathbf{n}ds$ and since the area integrated over is entirely within the volume of the nucleon source, the linear dimensions are limited to R, the radius of the latter. The area is therefore of order $\sim R^2$. From the differentiation of $\mathbf{A}$, a factor of $\sim p$ is obtained and the area integral is smaller than a term of order $|\mathbf{A}|R$ by a factor of approximately $R/\lambda_{(\text{photon})}$.

The individual terms are

$$H_A{}^{I_a}=-(4\pi)^{\frac{1}{2}}ef\int d\mathbf{x}\; U(x)\boldsymbol{\sigma}\cdot\mathbf{A}(\mathbf{x})\epsilon_{\lambda\beta3}\tau_\lambda\phi_\beta(\mathbf{x}),$$

$$H_B{}^{I_a}=-(4\pi)^{\frac{1}{2}}ef\int d\mathbf{x}\int_0^1 \xi d\xi\; U(x)\boldsymbol{\sigma}\cdot[\mathbf{x}\times\mathfrak{H}(\xi\mathbf{x})]\epsilon_{\lambda\beta3}\tau_\lambda\phi_\beta(\mathbf{x}),$$

$$H^{I_b}=-(4\pi)^{\frac{1}{2}}ef\int d\mathbf{x}\int_0^1 d\xi\; U(x)\mathbf{x}\cdot\mathbf{A}(\xi\mathbf{x})\epsilon_{\lambda\beta3}\tau_\lambda\boldsymbol{\sigma}\cdot\nabla_x\phi_\beta(\mathbf{x}).$$

Here $\epsilon_{\lambda\beta3}$ is the antisymmetric isotropic tensor, (i.e., $+1$ for every even, -1 for every odd permutation of λ, β, 3).

The alternate form of the interaction current, originating from H_a, is given by

$$H_{\mathrm{Alt}}{}^{I}=H_A{}^{I_a}+H^{I_b}. \qquad (21)$$

Hence it is clear that the difference between the two forms of the interaction Hamiltonian is the presence of $H_B{}^{I_a}$ in H^I.

In the expression for $M(1)$, the matrix element $[\![a_\alpha(\mathbf{q})\bar{H}]\!]$ appears. Hereafter it will be denoted by the symbol Π. Π will be written

$$\Pi=\Pi^M+\Pi^{I_a}+\Pi^{I_b},$$

where each of these parts of Π involves only the $\bar{H}^M$, $\bar{H}^{I_a}$, and $\bar{H}^{I_b}$ parts of $\bar{H}$, respectively. In a similar manner, $M(1)$ and $M(2)$ will be written

$$M(\mathrm{i})=M^M(\mathrm{i})+M^{I_a}(\mathrm{i})+M^{I_b}(\mathrm{i}) \quad (\mathrm{i}=1,2).$$

TABLE I. Table of integrals. The energy is that of the photon in the laboratory system. The meson energy is computed under the assumption that the origin of the momentum space is the mass center of the pion-nucleon system. R indicates the real, I the imaginary part of the integral. If only one number is cited, the integral is real.

Integral		200 Mev	260 Mev	335 Mev	440 Mev
$\mathfrak{K}^\alpha$	R	1.4944	1.8172	2.0756	2.2984
	I	0.0665	0.1880	0.4651	0.8829
$\mathfrak{K}^\beta$	R	6.1029	6.9984	8.1829	9.5887
	I	0.0839	0.2834	0.8792	2.0473
$\mathfrak{G}_a$		1.1524	1.0853	0.9616	0.7981
$\mathfrak{G}_b{}^\alpha$	R	0.5536	0.5789	0.5730	0.5096
	I	0.0426	0.0990	0.1849	0.2670
$\mathfrak{G}_b{}^\beta$	R	1.8513	1.9581	2.0448	1.9797
	I	0.0538	0.1480	0.3496	0.6192
$\mathfrak{G}_c{}^\alpha$	R	0.2762	0.3602	0.4958	0.6347
	I	0.0029	0.0160	0.0428	0.900
$\mathfrak{G}_c{}^\beta$	R	1.1748	1.5154	2.0929	2.7304
	I	0.0037	0.0190	0.0809	0.2087
$\mathfrak{L}_a$		0.8183	1.0109	1.3231	1.6782
$\mathfrak{L}_b{}^\alpha$		0.1935	0.2600	0.3976	0.6360
$\mathfrak{L}_b{}^\beta$		1.0626	1.4029	2.0747	3.1531
$\mathfrak{P}_a$		−1.9867	−1.8458	−1.6109	−1.3304
$\mathfrak{P}_b{}^\alpha$		−0.4717	−0.4773	−0.4873	−0.5074
$\mathfrak{P}_b{}^\beta$		−2.5822	−2.5653	−2.5320	−2.5068
$\mathfrak{L}_{A,a}$		1.6024	1.9616	2.5171	3.0856
$\mathfrak{L}_{A,b}{}^\alpha$		0.3960	0.5368	0.8394	1.4422
$\mathfrak{L}_{A,b}{}^\beta$		2.1024	2.7709	4.1039	6.4306

V. THE MESON CURRENT CONTRIBUTION

The contribution from the meson current part of H to the pion photoproduction matrix element will be considered here. The quantities $M^M(1)$ and $M^M(2)$ which together comprise this contribution will be computed in turn.

The quantity Π^M which appears in $M^M(1)$ is obtained by first permuting meson variables until all annihilation operators are to the right of all creation operators.[8] Then the matrix element is written in the momentum representation, the Tomonaga approximation is made,[2,8] and the matrix elements in the reduced space of FLC are evaluated.[9] This results in the following expression for Π^M:

$$\Pi^M=-\frac{2}{9}\frac{(6)^{\frac{1}{2}}e\mathfrak{A}_1{}^a\boldsymbol{\varepsilon}\cdot\mathbf{q}(\mathbf{q}-\mathbf{p})\cdot\boldsymbol{\sigma}g(|\mathbf{q}-\mathbf{p}|)}{[2p\omega_q\omega_{(|\mathbf{q}-\mathbf{p}|)}]^{\frac{1}{2}}|\mathbf{q}-\mathbf{p}|}-\frac{(54)^{\frac{1}{2}}\pi eL^2\mathfrak{G}_a(p)g(q)}{(2\pi)^3(2p)^{\frac{1}{2}}|\mathbf{q}|}\times\{[\mathfrak{A}_1{}^c\delta_{\alpha,3}-i(\bar{f}/48)\Delta(\alpha)](\boldsymbol{\varepsilon}\times\mathbf{p})\cdot\boldsymbol{\sigma}+\tfrac{1}{2}[(\mathfrak{A}_3{}^c-\mathfrak{A}_2{}^c-\bar{f}/12)\Delta(\alpha)+i(\bar{f}/24)\delta_{\alpha,3}]\times[(\boldsymbol{\varepsilon}\times\mathbf{p})\times\mathbf{q}]\cdot\boldsymbol{\sigma}\}, \qquad (22)$$

where $\Delta(\alpha)=\delta_{2,\alpha}\tau_1-\delta_{1,\alpha}\tau_2$. The function $\mathfrak{G}_a(p)$ results from the integration over momentum space; its value is given in Table I. The constants $\mathfrak{A}_1{}^a$, $\mathfrak{A}_1{}^b$, $\mathfrak{A}_1{}^c$, $\mathfrak{A}_2{}^c$, $\mathfrak{A}_3{}^c$ are integrals over the reduced space of FLC. Their values are given in the Appendix.

In computing $M^M(1)$ it is convenient to express it as the sum

$$M^M(1)=\bar{M}^M(1)+\textstyle\sum_{I,J} M_{IJ}{}^M(1).$$

$\bar{M}^M(1)$ refers to the matrix elements taken between the ground state and the plane wave part of the portion of $\Psi_m{}^{(-)}(\mu,\mathbf{p}')$ which appears in $M(1)$. $M_{IJ}{}^M(1)$ refers to the part with spin and isotopic spin J and I, respectively. It can be seen that $\bar{M}^M(1)$ represents the part of $M^M(1)$ in which the final state is taken as the free-particle state, and thus is the Born approximation to $M^M(1)$. The $M_{IJ}{}^M(1)$ represent the "enhancement" of $M^M(1)$ beyond its Born approximation value, due to the entry of the photoproduced meson into a pion-nucleon scattering state.

The matrix element $\bar{M}^M(1)$ is

$$\bar{M}^M(1)=\sum_{\mathbf{q},\alpha}\langle m|\delta_{\mathbf{p}',\mathbf{q}}\delta_{\mu,\alpha}\Pi^M|n\rangle. \qquad (23)$$

[8] T. D. Lee and D. Pines, Phys. Rev. **92**, 883 (1953).

[9] For further details refer to the doctoral dissertation by K. Haller at Columbia University, 1958 (University Microfilms, Ann Arbor, Michigan).

It can be directly obtained by replacing the $\mathbf{q}$ and α by $\mathbf{p}'$ and μ, respectively, in (22). The matrix element $M_{\frac{1}{2},\frac{1}{2}}{}^M(1)$ is

$$M_{\frac{1}{2},\frac{1}{2}}{}^M(1)=\sum_{\mathbf{q},\alpha}\left\langle m\left|\frac{G_{\frac{1}{2},\frac{1}{2}}{}^*(p',q)}{\omega_q-\omega_{p'}-i\epsilon}P_{\frac{1}{2},\frac{1}{2}}(\mu,\alpha;\mathbf{p}',\mathbf{q})\Pi^M\right|n\right\rangle. \quad (24)$$

This becomes

$$M_{\frac{1}{2},\frac{1}{2}}{}^M(1)=\{[\boldsymbol{\varepsilon}\times\mathbf{p}]\cdot\mathbf{p}'+i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]\}\{2c_1R_{\frac{1}{2},\frac{1}{2}}{}^{Gb}+c_2[\mathcal{A}_1{}^c-2\mathcal{A}_2{}^c+2\mathcal{A}_3{}^c-(f/4)]R_{\frac{1}{2},\frac{1}{2}}{}^K\}\times(\delta_{\mu,3}+i\Delta(\mu)), \quad (25)$$

where

$$c_1=\frac{2(6)^{\frac{1}{2}}\pi eL^2\mathcal{A}_1{}^a}{27(2\pi)^3p'(2p)^{\frac{1}{2}}},$$

$$c_2=\frac{4(54)^{\frac{1}{2}}\pi^2eL^4\mathfrak{G}_a(p)}{(2\pi)^69p'(2p)^{\frac{1}{2}}},$$

$$R_{IJ}{}^{Gb}=\alpha_{IJ}{}^*\mathfrak{G}_b{}^\alpha+\beta_{IJ}{}^*\mathfrak{G}_b{}^\beta,$$

$$R_{IJ}{}^K=\alpha_{IJ}{}^*\mathfrak{K}^\alpha+\beta_{IJ}{}^*\mathfrak{K}^\beta.$$

$\mathfrak{G}_b{}^\alpha$, $\mathfrak{G}_b{}^\beta$, $\mathfrak{K}^\alpha$, $\mathfrak{K}^\beta$ (and later, $\mathfrak{G}_c{}^\alpha$, $\mathfrak{G}_c{}^\beta$) result from the integration over momentum space. Their values are given in Table I.

In a similar manner, the other I, J, states contribute as follows:

$$M_{\frac{3}{2},\frac{1}{2}}{}^M(1)=\{\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})+i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]\}\{c_1R_{\frac{3}{2},\frac{1}{2}}{}^{Gb}+c_2[\mathcal{A}_1{}^c+\mathcal{A}_2{}^c-\mathcal{A}_3{}^c+(f/16)]R_{\frac{3}{2},\frac{1}{2}}{}^K\}[2\delta_{\mu,3}-i\Delta(\mu)], \quad (26)$$

$$M_{\frac{1}{2},\frac{3}{2}}{}^M(1)=[2c_1\{[\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})+2i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}]R_{\frac{1}{2},\frac{3}{2}}{}^{Gb}-3[i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}]R_{\frac{1}{2},\frac{3}{2}}{}^{Gc}\}+c_2\{2\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})-i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]\}[\mathcal{A}_1{}^c+\mathcal{A}_2{}^c-\mathcal{A}_3{}^c+(\bar{f}/16)]R_{\frac{1}{2},\frac{3}{2}}{}^K][\delta_{\mu,3}+i\Delta(\mu)],$$

where

$$R_{I,J}{}^{Gc}=\alpha_{IJ}{}^*\mathfrak{G}_c{}^\alpha+\beta_{IJ}{}^*\mathfrak{G}_c{}^\beta,$$

$$M_{\frac{3}{2},\frac{3}{2}}{}^M(1)=[c_1\{[\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})+2i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}]R_{\frac{3}{2},\frac{3}{2}}{}^{Gb}-3[i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}]R_{\frac{3}{2},\frac{3}{2}}{}^{Gc}\}+c_2\{2\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})-i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]\}[\mathcal{A}_1{}^c-\tfrac{1}{2}\mathcal{A}_2{}^c+\tfrac{1}{2}\mathcal{A}_3{}^c-(\bar{f}/8)]R_{\frac{3}{2},\frac{3}{2}}{}^K][2\delta_{\mu,3}-i\Delta(\mu)].$$

Contributions from $M^M(2)$ to M^M will now be considered. It can be shown that

$$\langle m|\chi^\dagger(\mu,\alpha;\mathbf{p}',\mathbf{q})[\![a_\alpha(\mathbf{q})]\!]|l\rangle=i(\tfrac{3}{2})^{\frac{1}{2}}(2/9)\mathcal{A}_1{}^a\langle m|(\boldsymbol{\sigma}\cdot\mathbf{p}'/p')\tau_\mu|l\rangle[g(p')-(L^2/2\pi^2)R_{\frac{1}{2},\frac{1}{2}}{}^K]. \quad (27)$$

It is to be noted that the only contributions that are made to this term come from the plane wave and the $I=J=\frac{1}{2}$ state. The matrix element $[\![\bar{H}^M]\!]$ is

$$[\![\bar{H}^M]\!]=(ie/3)(\boldsymbol{\varepsilon}\times\mathbf{p})\cdot\boldsymbol{\sigma}\tau_3(2p)^{-\frac{1}{2}}\mathcal{A}_1{}^b(L^2/4\pi^2)\mathfrak{G}_a(p). \quad (28)$$

The expression for $M^M(2)$ thus becomes

$$M^M(2)=-e(\tfrac{3}{2})^{\frac{1}{2}}\frac{2L^2\mathcal{A}_1{}^a\mathcal{A}_1{}^b\mathfrak{G}_a(p)}{(27)(4\pi^2)}\left\{\frac{\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})+i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]}{(2p)^{\frac{1}{2}}p'}\right\}\times[g(p')-(L^2/2\pi^2)R_{\frac{1}{2},\frac{1}{2}}{}^K][\delta_{\mu,3}+i\Delta(\mu)]. \quad (29)$$

VI. THE INTERACTION CURRENT CONTRIBUTION

The same procedure that was followed in computing the meson current contribution to the photoproduction matrix element will now be invoked in this section for the case of the M^I part of the latter. Namely, $M^I(1)$ and $M^I(2)$ will be individually computed and $M^I(1)$ will be represented as the sum:

$$M^I(1)=\bar{M}^I(1)+\textstyle\sum_{I',J'}M_{I',J'}{}^I(1).$$

The matrix element $\Pi_A{}^{Ia}$ is:

$$\Pi_A{}^{Ia}=(2\pi)^3p^{-\frac{1}{2}}c_0\rho u(|\mathbf{q}-\mathbf{p}|)\omega_q{}^{-\frac{1}{2}}\epsilon_{\lambda\alpha3}\tau_\lambda\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}+6\pi Lg(q)\mathfrak{L}_{A,a}(p)p^{-\frac{3}{2}}q^{-\frac{1}{2}}\times\{[(\mathcal{A}_1{}^e+\mathcal{A}_6{}^e-\mathcal{A}_2{}^e)\epsilon_{\lambda\alpha3}\tau_\lambda+2i\mathcal{A}_7{}^e\delta_{\alpha3}]\mathbf{q}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}+[(\mathcal{A}_2{}^e-\mathcal{A}_5{}^e)\epsilon_{\lambda\alpha3}\tau_\lambda+2i\mathcal{A}_8{}^e\delta_{\alpha3}]\boldsymbol{\sigma}\cdot\mathbf{p}\mathbf{q}\cdot\boldsymbol{\varepsilon}+[2\mathcal{A}_9{}^e\delta_{\alpha3}+i(\mathcal{A}_7{}^e-\mathcal{A}_8{}^e)\epsilon_{\lambda\alpha3}\tau_\lambda](\boldsymbol{\varepsilon}\times\mathbf{p})\cdot\mathbf{q}\}. \quad (30)$$

$\mathfrak{L}_{A,a}(p)$ [and, later, $\mathfrak{L}_{A,b}{}^\alpha(p,p')$ and $\mathfrak{L}_{A,b}{}^\beta(p,p')$] result from the integration over momentum space. Their values are given in Table I. Here $c_0=ef(4\pi)^{\frac{1}{2}}/2(2\pi)^3$. The $\mathcal{A}_n{}^e$ are integrals over the reduced space; their values are given in the Appendix. $u(k)$ is the cutoff function

$$u(k)=1 \quad \text{for} \quad k\leq K,$$
$$u(k)=0 \quad \text{for} \quad k>K.$$

Again $\bar{M}_A{}^{Ia}(1)$ is just $\Pi_A{}^{Ia}$ with $\mathbf{p}'$ and μ substituted for $\mathbf{q}$ and α, respectively. Since $|\mathbf{p}'-\mathbf{p}|<K$ for all values of the energy for which this calculation is at all valid, $[\bar{M}_A{}^{Ia}]_s$, the s-wave part of $\bar{M}_A{}^{Ia}$, reduces to

$$[\bar{M}_A{}^{Ia}]_s=\{(2\pi)^3c_0\rho\epsilon_{\lambda\alpha3}\tau_\lambda/[p\omega_{p'}]^{\frac{1}{2}}\}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}, \quad (31)$$

where ρ is the ratio of the renormalized to the unrenormalized meson nucleon coupling constant.[10] $[\bar{M}_A{}^{Ia}]_s$ is the only part of the entire photomeson production

[10] Reference 1, Eq. (41); also G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).

matrix element that does not disappear at the threshold for pion production. It can be seen that it corresponds exactly to the result obtained from a weak-coupling treatment of this theory, when the renormalized coupling constant is substituted for the unrenormalized one.[11] The pion photoproduction matrix element obtained here is thus consistent with the requirements of the Kroll-Ruderman theorem.

Matrix elements $M_{A;I,J}{}^{Ia}(1)$ are now computed in the same manner as were the corresponding $M_{IJ}{}^{M}(1)$. The results of this are

$$M_{A;\frac{1}{2},\frac{1}{2}}{}^{Ia}(1)=2c_3[i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}-i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}-\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]$$
$$\times[R_{A;\frac{1}{2},\frac{1}{2}}{}^{L}\rho-c_4\mathfrak{L}_{A,a}(p)(-\mathcal{A}_1{}^e+2\mathcal{A}_2{}^e-\mathcal{A}_5{}^e-\mathcal{A}_6{}^e+2\mathcal{A}_7{}^e-2\mathcal{A}_8{}^e+\mathcal{A}_9{}^e)R_{\frac{1}{2},\frac{1}{2}}{}^{K}][\delta_{\mu,3}+i\Delta(\mu)],\quad(32)$$

where $c_3=\frac{2}{3}\pi Lc_0/p'p^{\frac{3}{2}}$, and where $c_4=4\pi L^2/(2\pi)^3$. Here $R_{A;I,J}{}^{L}=\alpha_{I,J}{}^{*}\mathfrak{L}_{A,b}{}^{\alpha}+\beta_{I,J}{}^{*}\mathfrak{L}_{A,b}{}^{\beta}$.

$$M_{A;\frac{3}{2},\frac{1}{2}}{}^{Ia}(1)=c_3[i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}-i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]$$
$$\times[R_{A;\frac{3}{2},\frac{1}{2}}{}^{L}\rho+c_4\mathfrak{L}_{A,a}(p)(\mathcal{A}_1{}^e-2\mathcal{A}_2{}^e+\mathcal{A}_5{}^e+\mathcal{A}_6{}^e+\mathcal{A}_7{}^e-\mathcal{A}_8{}^e+2\mathcal{A}_9{}^e)R_{\frac{3}{2},\frac{1}{2}}{}^{K}][2\delta_{\mu,3}-i\Delta(\mu)],$$

$$M_{A;\frac{1}{2},\frac{3}{2}}{}^{Ia}(1)=-2c_3\{[2i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}-\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]R_{A;\frac{1}{2},\frac{3}{2}}{}^{L}\rho$$
$$+c_4\mathfrak{L}_{A,a}(p)[(2\mathcal{A}_1{}^e-\mathcal{A}_2{}^e-\mathcal{A}_5{}^e+2\mathcal{A}_6{}^e-\mathcal{A}_7{}^e-2\mathcal{A}_8{}^e+\mathcal{A}_9{}^e)i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}$$
$$+(\mathcal{A}_1{}^e+\mathcal{A}_2{}^e-2\mathcal{A}_5{}^e+\mathcal{A}_6{}^e-2\mathcal{A}_7{}^e-\mathcal{A}_8{}^e-\mathcal{A}_9{}^e)i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}$$
$$+(-\mathcal{A}_1{}^e+2\mathcal{A}_2{}^e-\mathcal{A}_5{}^e-\mathcal{A}_6{}^e-\mathcal{A}_7{}^e+\mathcal{A}_8{}^e-2\mathcal{A}_9{}^e)\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]R_{\frac{1}{2},\frac{3}{2}}{}^{K}\}[\delta_{\mu,3}+i\Delta(\mu)],$$

$$M_{A;\frac{3}{2},\frac{3}{2}}{}^{Ia}(1)=c_3\{[2i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}-\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]R_{A;\frac{3}{2},\frac{3}{2}}{}^{L}\rho$$
$$+c_4\mathfrak{L}_{A,a}(p)[(2\mathcal{A}_1{}^e-\mathcal{A}_2{}^e-\mathcal{A}_5{}^e+2\mathcal{A}_6{}^e+5\mathcal{A}_7{}^e+\mathcal{A}_8{}^e-2\mathcal{A}_9{}^e)i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}$$
$$+(\mathcal{A}_1{}^e+\mathcal{A}_2{}^e-2\mathcal{A}_5{}^e+\mathcal{A}_6{}^e+\mathcal{A}_7{}^e+5\mathcal{A}_8{}^e+2\mathcal{A}_9{}^e)i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}$$
$$-(\mathcal{A}_1{}^e-2\mathcal{A}_2{}^e+\mathcal{A}_5{}^e+\mathcal{A}_6{}^e+4\mathcal{A}_7{}^e-4\mathcal{A}_8{}^e-4\mathcal{A}_9{}^e)\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')]R_{\frac{3}{2},\frac{3}{2}}{}^{K}\}[2\delta_{\mu,3}-i\Delta(\mu)].$$

The consequences of including the $H_B{}^{Ia}$ part of the interaction current Hamiltonian H^I will now be considered. When the matrix elements $\bar{M}_B{}^{Ia}$, $M_{B;I,J}{}^{Ia}$ are evaluated and added to $\bar{M}_A{}^{Ia}$, $M_{A;I,J}{}^{Ia}$, respectively, the results are

$$\bar{M}^{Ia}(1)=c_0\left\{\frac{(2\pi)^3\rho u(|\mathbf{p}'-\mathbf{p}|)\epsilon_{\lambda\mu3}\tau_\lambda\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}}{(p\omega_{p'})^{\frac{1}{2}}}+\frac{6\pi Lg(p')\mathfrak{L}_a(p)}{(p^3p')^{\frac{1}{2}}}\right.$$
$$\left.\times i[\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}][(\mathcal{A}_1{}^e+\mathcal{A}_6{}^e-\mathcal{A}_5{}^e)\Delta(\mu)+2i(\mathcal{A}_7{}^e+\mathcal{A}_8{}^e)\delta_{\mu,3}]\right\},$$

$$M_{\frac{1}{2},\frac{1}{2}}{}^{Ia}(1)=M_{\frac{3}{2},\frac{1}{2}}{}^{Ia}(1)=0,\qquad(33)$$

$$M_{\frac{1}{2},\frac{3}{2}}{}^{Ia}(1)=-6c_3[\delta_{\mu,3}+i\Delta(\mu)][i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}][R_{\frac{1}{2},\frac{3}{2}}{}^{L}\rho+c_4\mathfrak{L}_a(p)R_{\frac{1}{2},\frac{3}{2}}{}^{K}(\mathcal{A}_1{}^e-\mathcal{A}_5{}^e+\mathcal{A}_6{}^e-\mathcal{A}_7{}^e-\mathcal{A}_8{}^e)],$$

$$M_{\frac{3}{2},\frac{3}{2}}{}^{Ia}(1)=3c_3[2\delta_{\mu,3}-i\Delta(\mu)][i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}][R_{\frac{3}{2},\frac{3}{2}}{}^{L}\rho+c_4\mathfrak{L}_a(p)R_{\frac{3}{2},\frac{3}{2}}{}^{K}(\mathcal{A}_1{}^e-\mathcal{A}_5{}^e+\mathcal{A}_6{}^e+2\mathcal{A}_7{}^e+2\mathcal{A}_8{}^e)].$$

The quantities $\mathfrak{L}_a(p)$, $\mathfrak{L}_b{}^{\alpha}(p,p')$, and $\mathfrak{L}_b{}^{\beta}(p,p')$ $(R_{IJ}{}^{L}=\alpha_{IJ}{}^{*}\mathfrak{L}_b{}^{\alpha}+\beta_{IJ}{}^{*}\mathfrak{L}_b{}^{\beta})$, result from the integration over momentum space. Their values are given in Table I.

It is to be noted that the s-wave parts of $M_B{}^{Ia}$ (and later, of M^{Ib}) disappear. This is the case because all s-wave terms of $M_B{}^{Ia}$ and M^{Ib} contain the expression $\nabla_{\mathbf{p}'}u(|\mathbf{p}'-\xi\mathbf{p}|)$ and $|\mathbf{p}'-\xi\mathbf{p}|<K$ for all cases here considered.

A comparison of the numerical values indicates that the p-wave parts of the $M_A{}^{Ia}$ and $M_B{}^{Ia}$ terms strongly interfere with each other. Although each of them, singly, is somewhat greater in magnitude than the meson current contribution, the M^{Ia} terms are only a small correction to the latter. This interference accounts for the fact that when H_b is used, the dominant contributions to the p-wave part of the pion photoproduction matrix element are from the meson current. $M^I(2)$ gives no contribution here since $[\![\bar{H}^I]\!]$ is identically zero. This identity is closely related to the disappearance of the magnetic moment of the interaction current.

The final contribution to M^I, namely M^{Ib}, will now

[11] R. E. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952), Chap. I.

TABLE II. Coefficients in the expression for the transition matrix elements. The coefficients are those of Eqs. (38). Those without subscripts are computed with H^I as the interaction current Hamiltonian. Those with the Alt subscripts are computed with the $H_{\text{Alt}}{}^I$ interaction current Hamiltonians. The energy is that of the photon in the laboratory system. R indicates the real, I the imaginary part of the coefficient. Where either R or I is omitted, the coefficient is pure imaginary or pure real, respectively.

Coefficient		200 Mev	260 Mev	335 Mev	440 Mev
$\mathfrak{A}_1{}^{(+)}$	I	0.4620	0.3782	0.2918	0.2298
$\mathfrak{A}_2{}^{(+)}$	I	−1.259	−1.031	−0.7954	−0.6265
$\mathfrak{A}_3{}^{(+)}$	R	−0.04740	−0.05671	−0.00676	−0.00171
	I	−0.00654	−0.01392	−0.01287	−0.00030
$\mathfrak{A}_4{}^{(+)}$	R	0.04744	0.05576	−0.00493	0.01753
	I	0.00706	0.03101	0.02750	0.01393
$\mathfrak{A}_5{}^{(+)}$	R	0.06003	0.07413	−0.00637	0.00830
	I	0.00565	0.04702	0.03255	−0.00583
$\mathfrak{A}_{3;\text{Alt}}{}^{(+)}$	R	−0.15383	−0.14950	−0.03140	−0.00839
	I	−0.00790	−0.05169	−0.05959	−0.02318
$\mathfrak{A}_{4;\text{Alt}}{}^{(+)}$	R	0.12042	0.09622	0.01409	0.02303
	I	0.00971	0.03430	0.04080	0.00100
$\mathfrak{A}_{5;\text{Alt}}{}^{(+)}$	R	0.01620	0.14488	−0.01770	−0.02566
	I	0.01206	0.08284	0.09914	0.02273
$\mathfrak{A}_3{}^{(0)}$	R	0.06548	0.07087	0.00585	−0.00798
	I	0.00187	0.04045	0.01978	−0.00298
$\mathfrak{A}_4{}^{(0)}$	R	−0.05471	−0.04745	0.01849	0.01751
	I	−0.00943	−0.02363	0.05118	−0.02517
$\mathfrak{A}_5{}^{(0)}$	R	−0.12684	−0.14911	0.01402	−0.02138
	I	−0.01540	−0.07131	−0.07659	0.02573
$\mathfrak{A}_{3;\text{Alt}}{}^{(0)}$	R	0.11872	0.16728	−0.01349	−0.03098
	I	0.01058	0.10040	0.10470	0.04061
$\mathfrak{A}_{4;\text{Alt}}{}^{(0)}$	R	−0.12969	−0.10586	0.03242	0.03228
	I	−0.01441	−0.07271	−0.08734	−0.00123
$\mathfrak{A}_{5;\text{Alt}}{}^{(0)}$	R	−0.16494	−0.34014	0.00634	−0.03583
	I	−0.01901	−0.16883	−0.19290	0.06331

be considered. The expression for Π^{Ib} is

$$\Pi^{Ib}=c_0\left\{(2\pi)^3\rho\int_0^1\frac{d\xi\,\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\varepsilon}\cdot[\boldsymbol{\nabla}_q u(|\mathbf{q}-\xi\mathbf{p}|)]}{(p\omega_q)^{\frac{1}{2}}}\epsilon_{\lambda\alpha3}\tau_\lambda\right.$$
$$+\frac{6\pi Lg(q)\mathfrak{P}_a(p)}{(p^3q)^{\frac{1}{2}}}\times[\mathbf{q}\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}+\mathbf{q}\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}]$$
$$\left.\times[2(\mathcal{A}_7{}^e+\mathcal{A}_8{}^e)\delta_{\alpha3}-(\mathcal{A}_1{}^e-\mathcal{A}_5{}^e+\mathcal{A}_6{}^e)i\Delta(\alpha)]\right\}.\quad(34)$$

$\mathfrak{P}_a(p)$ (and later $\mathfrak{P}_b{}^\alpha(p,p')$ and $\mathfrak{P}^\beta(p,p')$) result from the integration over momentum space. Their values are given in Table I. The expressions for the $M_{IJ}{}^{Ib}$ terms are

$$M_{\frac{1}{2},\frac{1}{2}}{}^{Ib}=M_{\frac{3}{2},\frac{1}{2}}{}^{Ib}=0,$$
$$M_{\frac{1}{2},\frac{3}{2}}{}^{Ib}=-6c_3[\delta_{\mu,3}+i\Delta(\mu)]$$
$$\times[i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}][R_{\frac{1}{2},\frac{3}{2}}{}^P\rho+c_4\mathfrak{P}_a(p)R_{\frac{1}{2},\frac{3}{2}}{}^K$$
$$\times(\mathcal{A}_1{}^e-\mathcal{A}_5{}^e+\mathcal{A}_6{}^e-\mathcal{A}_7{}^e-\mathcal{A}_8{}^e)],\quad(35)$$
$$M_{\frac{3}{2},\frac{3}{2}}{}^{Ib}=3c_3[2\delta_{\mu,3}-i\Delta(\mu)]$$
$$\times[i\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}+i\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma}][R_{\frac{3}{2},\frac{3}{2}}{}^P\rho+c_4\mathfrak{P}_a(p)R_{\frac{3}{2},\frac{3}{2}}{}^K$$
$$\times(\mathcal{A}_1{}^e-\mathcal{A}_5{}^e+\mathcal{A}_6{}^e-\mathcal{A}_7{}^e-\mathcal{A}_8{}^e)].$$

Here $R_{IJ}{}^P=\alpha_{IJ}{}^*\mathfrak{P}_b{}^\alpha+\beta_{IJ}{}^*\mathfrak{P}_b{}^\beta$. The numerical values of these quantities are again comparable to the M^{Ia} contribution, and are significantly smaller than the contributions from the meson current Hamiltonian.

VII. CONTRIBUTIONS FROM THE ALTERNATE FORM OF THE INTERACTION CURRENT

Few new computations are required here since the matrix elements $M_{\text{Alt}}{}^I(1)$ are just the sum of $M_A{}^{Ia}(1)$ and $M^{Ib}(1)$. It will be noted here that the $M_A{}^{Ia}(1)$ are in general somewhat larger than the meson current contributions. Since the $M_B{}^{Ib}(1)$, which in the last section were shown to interfere strongly with $M_A{}^{Ia}(1)$, are not included in this alternate form of the interaction Hamiltonian, the contributions of this interaction current term to the photoproduction matrix element are very large.

It should be noted that $M_{\text{Alt}}{}^I(2)$ must also be included now since $[\![\bar{H}_{\text{Alt}}{}^{Ia}]\!]$ is not zero. This is related to the

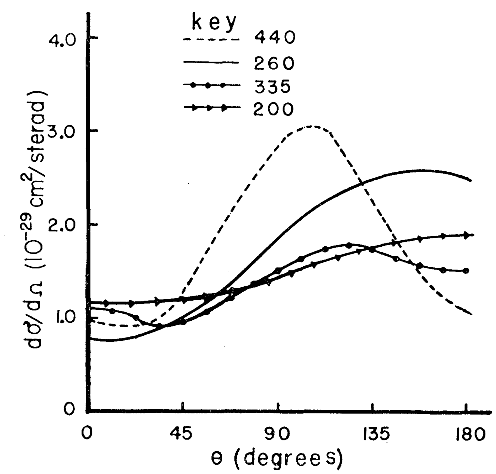

FIG. 1. Differential cross sections for $\pi^{(+)}$ photoproduction computed with interaction current Hamiltonian H^I. $\theta=\cos^{-1}[\mathbf{p}'\cdot\mathbf{p}/p'p]$. Numbers indicated in key denote energy of photon, in laboratory system, in Mev.

aforementioned fact that a large magnetic moment contribution is given by this form of the interaction current. $[\![\bar{H}_{\text{Alt}}{}^{Ia}]\!]$ is given by

$$[\![H_{\text{Alt}}{}^{Ia}]\!]=i(12)^{\frac{1}{2}}(2\pi)^{\frac{3}{2}}efLp^{-\frac{3}{2}}(\boldsymbol{\varepsilon}\times\mathbf{p})\cdot\boldsymbol{\sigma}\tau_3\mathfrak{L}_{A,a}(p)\mathcal{A}_2{}^d,\quad(36)$$

where $\mathcal{A}_2{}^d$ is a reduced space integral; its value is given in the Appendix. The value of $M_{\text{Alt}}{}^I(2)$ is therefore:

$$M_{\text{Alt}}{}^I(2)=-(18)^{\frac{1}{2}}(2/9)(2\pi)^{-\frac{3}{2}}ef\mathfrak{L}_{A,a}(p)\mathcal{A}_1{}^a\mathcal{A}_2{}^d$$
$$\times\{\mathbf{p}'\cdot(\boldsymbol{\varepsilon}\times\mathbf{p})+i\boldsymbol{\sigma}\cdot[\mathbf{p}'\times(\boldsymbol{\varepsilon}\times\mathbf{p})]\}$$
$$\times[g(p')-(L^2/2\pi^2)R_{\frac{1}{2},\frac{1}{2}}{}^K].\quad(37)$$

$M_{\text{Alt}}{}^I(2)$ is numerically small relative to $M_{\text{Alt}}{}^I(1)$.

VIII. RESULTS

The matrix elements resulting from this calculation can be expressed in the following form:

$$M(\pi^+) = -\Delta(\mu)\{\mathfrak{A}_1^{(+)}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon} + \mathfrak{A}_2^{(+)}[\omega_{(|\mathbf{p}'-\mathbf{p}|)}(\omega_{(|\mathbf{p}'-\mathbf{p}|)}+\lambda)]^{-1}\boldsymbol{\varepsilon}\cdot\mathbf{p}'(\mathbf{p}'-\mathbf{p})\cdot\boldsymbol{\sigma} + \mathfrak{A}_3^{(+)}\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma} + \mathfrak{A}_4^{(+)}\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma} - i\mathfrak{A}_5^{(+)}\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')\}, \quad (38a)$$

$$M(\pi^0) = i\delta_{\mu,3}\{\mathfrak{A}_3^{(0)}\mathbf{p}'\cdot\mathbf{p}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma} + \mathfrak{A}_4^{(0)}\mathbf{p}'\cdot\boldsymbol{\varepsilon}\mathbf{p}\cdot\boldsymbol{\sigma} - i\mathfrak{A}_5^{(0)}\boldsymbol{\varepsilon}\cdot(\mathbf{p}\times\mathbf{p}')\}. \quad (38b)$$

The numerical values of these coefficients are tabulated in Table II. The matrix elements for which the alternate form of the interaction current has been used are also tabulated for purposes of comparison. $\mathfrak{A}_1^{(+)}$ (the Kroll-Ruderman term) and $\mathfrak{A}_2^{(+)}$ (part of the Born approximation term of the meson current Hamiltonian) are unchanged by the substitution of $\bar{H}_{\mathrm{Alt}}^I$ for $\bar{H}^I$. The coefficients computed with $\bar{H}_{\mathrm{Alt}}^I$ are reported as $\mathfrak{A}_{3;\mathrm{Alt}}$,

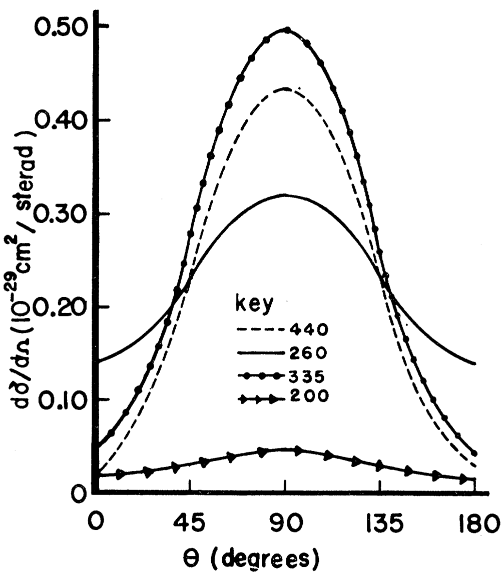

FIG. 2. Differential cross sections for $\pi^{(0)}$ photoproduction computed with interaction current Hamiltonian $\bar{H}^I$. $\theta=\cos^{-1}[\mathbf{p}'\cdot\mathbf{p}/p'p]$. Numbers indicated in key denote energy of photon, in laboratory system, in Mev.

$\mathfrak{A}_{4;\mathrm{Alt}}$, and $\mathfrak{A}_{5;\mathrm{Alt}}$ and replace $\mathfrak{A}_3$, $\mathfrak{A}_4$, and $\mathfrak{A}_5$, respectively, in $M_{\mathrm{Alt}}(\pi^+)$ and $M_{\mathrm{Alt}}(\pi^0)$.

Differential cross sections for π^+ and π^0 production are plotted in Figs. 1 and 2. The cross sections resulting from the use of $\bar{H}_{\mathrm{Alt}}^I$ appear in Figs. 3 and 4. For comparison, experimental values for pion photoproduction cross sections are given in Table III.

A number of features of these results are of interest. In the case of π^+ production, the angular dependence of the cross sections for 200-, 260-, and 335-Mev photons has a definite resemblance to that of the experimental curves.[12] Their magnitudes are ~ one to two times those experimentally obtained. It must be remembered, in this connection, that the quantity $(f\rho)^2$, obtained

[12] For a bibliography of experimental results, see M. Ross, Phys. Rev. **103**, 760 (1956), footnote 1; also reference 4, footnote 1.

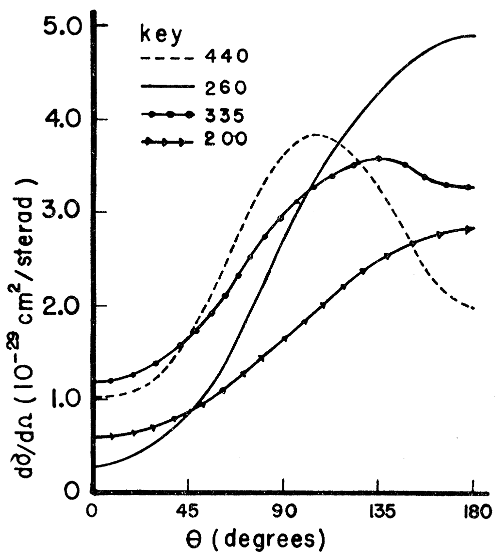

FIG. 3. Differential cross sections for $\pi^{(+)}$ photoproduction computed with interaction current Hamiltonian H_{Alt}^I. $\theta=\cos^{-1}[\mathbf{p}'\cdot\mathbf{p}/p'p]$. Numbers indicated in key denote energy of photon, in laboratory system, in Mev.

from the scattering data, is only an approximate one and originates from a resonance fitting of a single $(I=J=\frac{3}{2})$ scattering state, with only p-state mesons included. Experimental determinations of the coupling constant, made by fitting the Kroll-Ruderman expression for the threshold production of π^+ and π^- to experimental data for meson photoproduction from deuterium, gives a value of $(f\rho)^2$ about half as large as that used in this paper.[13] The absence of the peak at 90° could be ascribed to the appearance of $[\omega_{(|\mathbf{p}'-\mathbf{p}|)}(\omega_{(|\mathbf{p}'-\mathbf{p}|)}+\lambda)]$

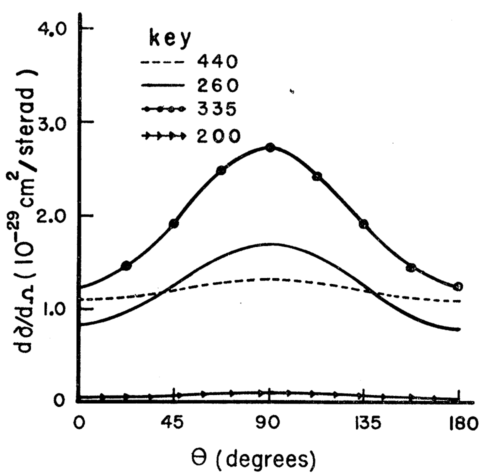

FIG. 4. Differential cross sections for $\pi^{(0)}$ photoproduction computed with interaction current Hamiltonian H_{Alt}^I. $\theta=\cos^{-1}[\mathbf{p}'\cdot\mathbf{p}/p'p]$. Numbers indicated in key denote energy of photon, in laboratory system, in Mev.

[13] H. A. Bethe and F. De Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. II, p. 279; see also G. Bernardini and E. L. Goldwasser, Phys. Rev. **95**, 875 (1954).

TABLE III. Table of experimental data[a] ($d\sigma/d\Omega$ in 10^{-29} cm^2/sterad).

Photon energy in lab. system (Mev)	Center-of-mass pion angle (degrees) 0	40	59	75	93	107	123	135	148	159	165	180
					π^+ photoproduction data							
200 BB[b]	...	0.70 ±0.11	0.85 ±0.05	0.95 ±0.13	0.98 ±0.06	1.01 ±0.08	1.05 ±0.06	1.19 ±0.19	...	0.95 ±0.07	...	...
200 JLP[c]	0.66 ±0.35	0.78 ±0.45	0.88 ±0.50	0.97 ±0.52	1.04 ±0.52	1.07 ±0.52	1.08 ±0.49	1.07 ±0.46	1.06 ±0.41	1.05 ±0.38	1.05 ±0.37	1.04 ±0.35
260 TKW[d]	0.58 ±0.23	1.02 ±0.17	1.38 ±0.11	1.64 ±0.07	1.81 ±0.04	1.82 ±0.07	1.72 ±0.12	1.61 ±0.15	1.47 ±0.19	1.36 ±0.21	1.32 ±0.22	1.28 ±0.23
265 BB	...	0.80 ±0.10	1.16 ±0.11	1.82 ±0.17	...	1.94 ±0.10	...	1.99 ±0.18	1.84 ±0.13	1.70 ±0.24	1.69 ±0.11	1.48 ±0.15
265 JLP	0.18 ±0.20	0.77 ±0.26	1.28 ±0.30	1.66 ±0.32	1.94 ±0.31	2.02 ±0.32	1.98 ±0.30	1.88 ±0.27	1.75 ±0.24	1.65 ±0.22	1.61 ±0.21	1.56 ±0.20
320 TKW	1.17 ±0.19	1.50 ±0.14	1.76 ±0.09	1.93 ±0.06	1.99 ±0.04	1.93 ±0.06	1.77 ±0.10	1.62 ±0.13	1.46 ±0.16	1.34 ±0.17	1.29 ±0.18	1.24 ±0.19
350 TKW	1.18 ±0.18	1.36 ±0.13	1.48 ±0.09	1.55 ±0.06	1.54 ±0.04	1.47 ±0.06	1.34 ±0.09	1.22 ±0.12	1.09 ±0.15	1.01 ±0.17	0.98 ±0.17	0.94 ±0.18
440 TKW	0.88 ±0.14	0.82 ±0.10	0.76 ±0.07	0.70 ±0.05	0.61 ±0.03	0.54 ±0.05	0.45 ±0.07	0.40 ±0.09	0.35 ±0.11	0.32 ±0.12	0.30 ±0.13	0.29 ±0.14
					π^0 photoproduction data							
270 GOS[e]	...	...	1.41 ±0.29	1.24 ±0.18	1.31 ±0.13	1.29 ±0.19	1.17 ±0.31	1.04 ±0.40	0.89 ±0.50	0.79 ±0.56	0.75 ±0.59	0.70 ±0.62
320 OW[f]	...	...	...	2.30 ±0.25	2.62 ±0.14	2.55 ±0.27	2.19 ±0.47	1.79 ±0.63	1.34 ±0.78	1.02 ±0.89	...	...
450 OW	...	...	0.74 ±0.12	0.83 ±0.07	0.82 ±0.04	0.73 ±0.08	0.56 ±0.13	0.40 ±0.17	0.24 ±0.21	0.13 ±0.24	...	...
450 WOT[g]	0.99 ±0.33	0.97 ±0.23	0.92 ±0.15	0.85 ±0.08	0.73 ±0.05	0.61 ±0.09	0.47 ±0.16	0.36 ±0.21	0.26 ±0.26	0.20 ±0.30	0.17 ±0.31	0.15 ±0.33

[a] Of the papers cited below, the following—TKW, GOS, OW, and WOT—obtain values of A_0, A_1, and A_2 in the equation $d\sigma/d\Omega = A_0 + A_1 \cos\theta + A_2 \cos^2\theta$. (JLP obtain B_0, B_1, and B_2 in the equation $d\sigma/d\Omega = B_0 + B_1 \cos\theta + B_2 \sin^2\theta$) by a least-squares fit of these constants to the data. The table entries corresponding to data given in these papers have been computed from the equations cited above and the constants reported in the respective papers.
[b] Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau, Nuovo cimento **4**, 323 (1956).
[c] Jenkins, Luckey, Palfrey, and Wilson, Phys. Rev. **95**, 179 (1954).
[d] Tollestrop, Keck, and Worlock, Phys. Rev. **99**, 1283 (1955).
[e] Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **97**, 188 (1955).
[f] D. C. Oakley and R. L. Walker, Phys. Rev. **97**, 1283 (1955).
[g] Walker, Oakley, and Tollestrop, Phys. Rev. **97**, 1279 (1955).

instead of $[\omega_{(|\mathbf{p}'-\mathbf{p}|)}]^2$ as the divisor of the $\mathfrak{A}_2^{(+)}$ coefficient. That the latter is correct is strongly indicated by Klein's low-energy theorem for the meson current term.[14] Moreover, the erroneous appearance of $(\omega+\lambda)$ instead of ω in the denominator of scattering amplitudes is a well-known feature of intermediate-coupling theory. It arises primarily from the fact that the identity[15]

$$(H-E_q)a_q^*\Psi_0 = V_q^{(0)}\Psi_0, \tag{39}$$

is violated in the intermediate-coupling theory by a factor of approximately $\omega/\omega+\lambda$.[16] The cross section for 440-Mev photons differs substantially from experimental data. However, the use of a nonrelativistic nonrecoil approximation at this energy is of questionable validity, and this curve cannot be taken too seriously.

[14] A. Klein, Phys. Rev. **99**, 998 (1955).
[15] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[16] R. Drachman (private communication). See also R. Stroffolini, Phys. Rev. **104**, 1146 (1956).

The π^+ cross sections obtained with H_{Alt}^I suffer from a great excess of p-wave contributions and are completely inconsistent with the data. They seem to indicate that the additional p-wave terms from H_{Alt}^I are incorrect.

In the case of π^0 production, the calculated cross sections again have somewhat similar angular dependence to that of the experimental curves and show the expected resonance at 335 Mev. It can be seen that in this case the cross sections obtained with H^I are too small by a factor of about 3–5, while those obtained with H_{Alt}^I are of about the right magnitude. This fact can, however, be understood in terms of the following relation proven by Chew and Low[17]:

$$\sigma_{\gamma\to\pi^0} = \frac{p}{p'}\left(\frac{e}{f\rho}\right)^2\left(\frac{g_p-g_n}{4M}\right)^2 \sigma_{\pi^0\to\pi^0}. \tag{40}$$

[17] Reference 5, Eq. (51).

where g_p and g_n refer to the magnetic moments of the proton and neutron, respectively. The factor $[(g_p-g_n)/f\rho]^2$ is too small by about a factor of 3 in the intermediate-coupling theory. If this correction is applied to the calculated π^0 production cross sections, then the result that $H_{\mathrm{Alt}}{}^I$ leads to an excess of p-wave contributions is again obtained.

It can be seen that the substitution of the H_b for the H_a interaction Hamiltonian results in marked changes in the values of both the anomalous magnetic moment and the meson photoproduction cross section. Since one or another specific form of the gauge-invariant interaction Hamiltonian is generally explicitly involved in the calculation of electromagnetic properties in extended-source theories,[4–6] it is of interest to note that the ambiguity involved in defining the most general form is large. Capps and Holladay[18] estimate that the use of H_a instead of H_b in the computation of the anomalous magnetic moment changes the latter quantity by $-0.13\tau_3$ nuclear magnetons. This figure is arrived at by assuming a 0.1 probability of finding a charged pion in the physical nucleon. Since the average number of mesons in the physical nucleon according to the intermediate-coupling theory is about 1.8,[19] the additional anomalous magnetic moment of $-2.5\tau_3$ nuclear magnetons due to H_a arrived at in this theory is not inconsistent with the former result. It is also of interest to note that the use of the interaction Hamiltonian H_a (as by Enoch, Sachs, and Wali) gives rise to line current terms that are a relatively small correction to the pion photoproduction matrix element. However, the use of the term H_b gives rise to such large line current contributions to the p-wave part of the photoproduc-matrix element, that they suffice to largely cancel the p-wave contributions of $H_A{}^{I_a}$.

Contrary to the result of Chew and Low,[5] the anomalous magnetic moment and the pion photoproduction cross section are not simply related by an approximately interaction-independent multiplicative constant in this theory. To verify this fact, the following calculation was made: A phenomenological term proportional to $\boldsymbol{\sigma}\cdot\mathfrak{H}$ was added to H_a, sufficient in magnitude to restore the anomalous magnetic moment to the value obtained with H_b. The photoproduction matrix element due to this term was then computed and added to $M_{\mathrm{Alt}}(\pi^+)$ and $M_{\mathrm{Alt}}(\pi^0)$. When this sum was compared with $M(\pi^+)$ and $M(\pi^0)$, it was found that substantial differences still existed. Part of the reason for the failure of the photoproduction matrix element and the anomalous moments to be proportional to each other may be inherent in features of the model used here for the physical nucleon and for the pion-nucleon scattering state: (1) the previously-mentioned failure of the identity [Eq. (39)] to hold; (2) errors due to the approximate treatment of the reduced space state function S^λ in FLC.[20] However, this above-mentioned failure of the proportionality relation to apply can also probably in part be explained by the fact that in this theory the intermediate states are not limited to those in which, at most, a single meson is allowed.

IX. ACKNOWLEDGMENTS

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APPENDIX

Numerical values of the reduced space integrals that enter into the expressions for $M(\pi^+)$ and $M(\pi^0)$ are listed below:

$\mathcal{A}_1{}^a=-0.4546$, $\mathcal{A}_1{}^b=-2.18$, $\mathcal{A}_1{}^c=0.1358$,
$\mathcal{A}_2{}^c=-0.4764$, $\mathcal{A}_3{}^c=0.0461$, $\mathcal{A}_1{}^d=-0.2493$,
$\mathcal{A}_1{}^e=0.4462$, $\mathcal{A}_2{}^e=-0.0805$, $\mathcal{A}_4{}^e=0.0007$,
$\mathcal{A}_5{}^e=0.2203$, $\mathcal{A}_6{}^e=-0.1918$, $\mathcal{A}_7{}^e=0.0516$,
$\mathcal{A}_8{}^e=-0.0945$, $\mathcal{A}_9{}^e=-0.2424$,

Numerical values of some other constants are[21] $f^2=0.847$, $K=5.772$, $\lambda=3.39$, $\rho=0.381$.

[18] R. H. Capps and W. G. Holladay, reference 3.

[19] R. Drachman and G. Feinberg (private communication).

[20] $S^\lambda(s_1,s_2,s_3)$ is approximated by $S^\lambda(s_1)$. All $\partial S^\lambda/\partial x_{i\alpha}$ are, however, treated exactly.

[21] Small differences between the values of some of these quantities as given here and in FLC represent corrections of some minor computational errors in the latter.