Empirical Model for Ultrarelativistic Nucleon-Nucleon Collisions

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The interactions of ultrarelativistic ($\geqslant 10^{12}$ ev) primary cosmic-ray protons with nucleons in photographic plates observed thus far show that in the majority of the cases the secondaries are emitted isotropically from two centers; this fact and the observation that the average transverse momentum of the secondaries is independent both of the energy of the primary nucleon and of the energy of the secondaries themselves suggest the following empirical model. When a nucleon-nucleon collision takes place at these energies, two "bodies" are formed, that move in opposite directions with respect to the center of momentum of the system, together with the two original nucleons, stripped of the energy necessary to create the "bodies." Each "body" emits, in its own system of reference, about half of the total number of secondaries, each secondary having an average energy of $\sim$1 Bev, independent of its nature. The inelasticity of the collision is thus determined by the number of secondaries and by the velocity of each "body" in the center-of-momentum system of the two original nucleons. Accurate determinations of the energy, in the laboratory system, of the secondaries of these interactions could provide a sensitive test for the model, but at present there are too few measurements available.

I. INTRODUCTION

THE problem of the collision of a nucleon of ultrarelativistic energy, say, above 10^{12} ev, with another nucleon at rest has been treated theoretically by several authors[1–5] who proposed a variety of models. However, the experimental evidence obtained thus far, mostly from collisions of high-energy nucleons of the cosmic radiation with nucleons and nuclei in photographic plates flown at high altitudes, shows that none of these models is capable of explaining all the observed features.[6]

Here, instead of trying to fit the experimental result to a theory, we shall follow the inverse path, and starting from the experimental observations, build a model that is in reasonable agreement with the data, putting off the worry about its theoretical justification.

II. KINEMATICAL TRANSFORMATIONS

Before going *in medias res*, let us first consider some properties of the relativistic transformations that correlate the events observed in the laboratory system (LS) to a system in uniform motion with respect to it.[7]

Let

$$\gamma_c = \frac{\text{total energy}}{\text{total rest mass}} = \frac{1}{(1-\beta_c^2)^{\frac{1}{2}}}, \quad (\beta_c = v_c/c)$$

be the Lorentz factor, in the LS, of a system, C, that is going to disintegrate into $2n$ secondary particles, and let $\bar{\gamma}_i$ $(1 \leqslant i \leqslant 2n)$ be the Lorentz factor of each particle in the system of reference where C is at rest. If ϑ_i is the polar angle of emission, in the C system, of the ith particle with respect to the direction of motion of C (the Z axis); then the corresponding angle Θ_i in the LS satisfies the relation

$$\tan\Theta_i = \frac{1}{\gamma_c} \frac{\sin\vartheta_i}{\cos\vartheta_i + \beta_c/\bar{\beta}_i} \xrightarrow[\beta_c = \bar{\beta}_i = 1]{} \frac{1}{\gamma_c} \tan(\vartheta_i/2). \quad (1)$$

We shall assume that the asymptotic equation holds, remembering that deviations from the exact treatment become large, when $\beta_c/\bar{\beta}_i \neq 1$, only for values of $\vartheta_i \approx \pi$.

(a) If, on the average, the $2n$ particles are emitted in the C system isotropically, then the fraction of them that falls within the angle ϑ is

$$F = \tfrac{1}{2}(1-\cos\vartheta) = \sin^2(\vartheta/2),$$

and in the LS the same fraction will be found within the angle Θ that satisfies the following relation:

$$\gamma_c^2 \tan^2\Theta = F/(1-F). \quad (2)$$

In a graph (Fig. 1) where $\log[F/(1-F)]$ is plotted against $\log \tan\Theta$, the $2n$ points are distributed along a straight line with *slope 2* that crosses the axis of the abscissas at a point characterized by an angle $\Theta(\frac{1}{2})$ that satisfies the relation

$$\log\gamma_c = -\log \tan\Theta(\tfrac{1}{2}). \quad (3)$$

(b) If the particle distribution in the C system, instead of being isotropic, is mostly forward and backward, but symmetric with respect to a plane normal to the Z axis and not zero at $\vartheta = \pi/2$, then the $2n$ points in the same graph will distribute themselves along a line not far from straight, with *slope smaller than 2*. The intercept always satisfies Eq. (3).

The distribution predicted by Heisenberg, e.g., gives[8]

[1] W. Heisenberg, *Vortrage über Kosmische Strahlung* (Springer-Verlag, Berlin, 1953), p. 148.
[2] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).
[3] E. Fermi, Phys. Rev. **81**, 683 (1951).
[4] S. Takagi, Progr. Theoret. Phys. (Kyoto) **7**, 123 (1952).
[5] S. S. Belenki and L. D. Landau, Suppl. Nuovo cimento **3**, 15 (1956).
[6] See, e.g., the discussion on the subject at the 1957 Varenna Conference [Suppl. Nuovo cimento (to be published)].
[7] The treatment followed here has been started by N. M. Dulles and W. D. Walker [Phys. Rev. **93**, 215 (1954)] and developed by Ciok, Coghen, Gierula, Holynski, Jurak, Miesowicz, Saniewska, Stanisz, and Pernegr [Nuovo cimento **8**, 166 (1958)].
[8] See L. V. Lindern, Nuovo cimento **5**, 491 (1957).

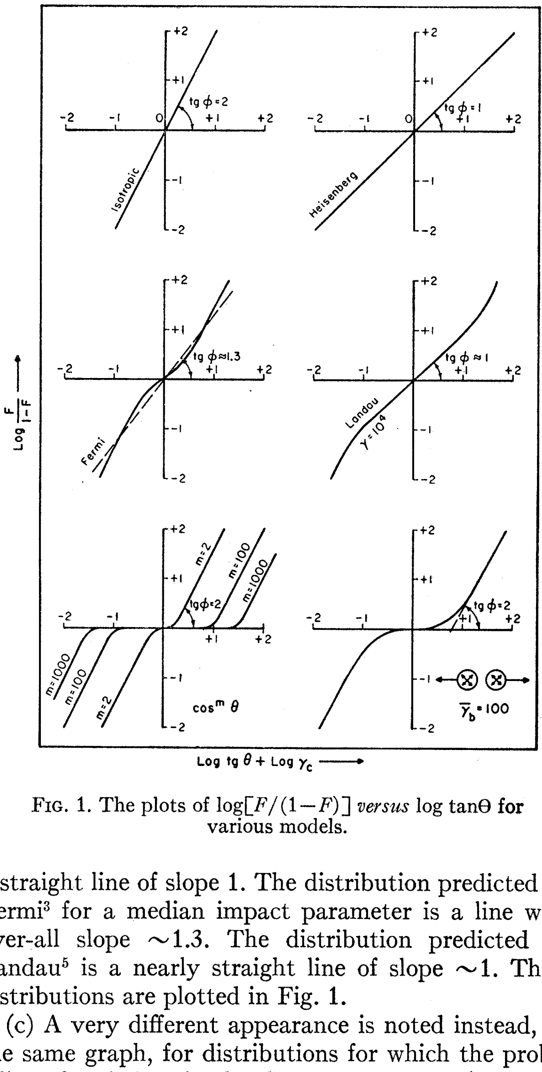

FIG. 1. The plots of $\log[F/(1-F)]$ *versus* $\log \tan\Theta$ for various models.

a straight line of slope 1. The distribution predicted by Fermi[3] for a median impact parameter is a line with over-all slope ~ 1.3. The distribution predicted by Landau[5] is a nearly straight line of slope ~ 1. These distributions are plotted in Fig. 1.

(c) A very different appearance is noted instead, on the same graph, for distributions for which the probability of emission, in the C system, at $\vartheta = \pi/2$ is zero. Consider the distribution

$$\mathcal{P}(\vartheta)d\Omega \propto \cos^m\vartheta \, d(\cos\vartheta),$$

with m even. Curves for several m's are plotted in Fig. 1. Far from the zero ordinate they degenerate into two straight branches, one representing the n forward particles, the other the n backward ones. Each branch has a slope $= 2$ and each intercept of the straight line with the axis of the abscissas is away from the point defined by Eq. (3) by the amount $\frac{1}{2}\log(m+1)$. The separation between the two branches is thus $\log(m+1)$.

(d) The $\cos^m\vartheta$ distribution, however, is rather difficult to justify on any physical basis. There is another distribution that eventually leads, in the LS, to a very similar result, i.e., to two separate branches, and has an immediate physical meaning; it is the following.

Assume that in the C system two equal "bodies" of particles, b_1 and b_2, are emitted in opposite directions along the Z axis, each with Lorentz factor $\bar{\gamma}_b$; assume furthermore that each "body" eventually disintegrates into n particles, and that these particles are emitted, in the b system, isotropically and with Lorentz factors $\bar{\gamma}_i \gg 1$. The distribution of the $2n$ particles in the plot of Fig. 1 will then again give two branches, each with asymptotic slope 2 (the isotropy in each center), symmetric with respect to the point defined by Eq. (3) (see Fig. 1 where the curve refers to the case of $\bar{\gamma}_b = 100$).[9]

A result of this kind is predicted by the model of Takagi,[4] and, as we shall see, is also strongly suggested by the experimental evidence. We shall further develop the kinematic relations for this case in Chap. IV.

III. EXPERIMENTAL INFORMATION

(a) Transverse Momentum

Nishimura[10] first pointed out that the analysis of jets in nuclear emulsions and the study of nuclear cascades in extensive air showers indicate that, on the average, the transverse momentum of the secondary particles produced in ultrarelativistic interactions is around 0.5 Bev, independent of the primary energy. This result has by now been confirmed by various authors,[11] so that it can be considered as a well-grounded experimental fact.

Note that, if the near constancy of the average transverse momentum is true for nucleon-nucleon (N-N) collisions, it must be true also in the case of collisions between a nucleon and a nucleus, independent of whether the nucleon makes one or more encounters in crossing the nucleus. This is in fact confirmed by the measurements of the Japanese group[12] in "emulsion chambers," where the primary interactions occurred in carbon.

(b) *N-N* Collisions; Angular Distribution of the Secondary Particles

In this section we want to consider only examples of nucleon-nucleon collisions, and the problem arises of selecting them among the variety of interactions that can be present in photographic plates. In fact, the jets found in plates can also be due (a) to particles other than nucleons (mesons, hyperons, heavy nuclei); (b) to collisions with several nucleons in a nucleus (composite collisions).

In order to discriminate against possibility (a), we shall consider only jets observed in plates flown at balloon altitudes and produced by singly charged particles entering the emulsion stack isolated. As far as we know, the singly ionizing primary cosmic radiation

[9] When $\bar{\gamma}_b = 1$, the two branches merge together and a single line of slope 2 results.

[10] J. Nishimura, Soryushiron Kenkyu 12, 24 (1956).

[11] See, e.g., the discussion on the subject at the Varenna Conference [Suppl. Nuovo cimento (to be published)]; and the article by Edwards, Losty, Perkins, Pinkau, and Reynolds, Phil. Mag. 3, 237 (1958).

[12] Report No. 7 from the Institute for Nuclear Studies, University of Tokyo, March 1, 1958 (unpublished).

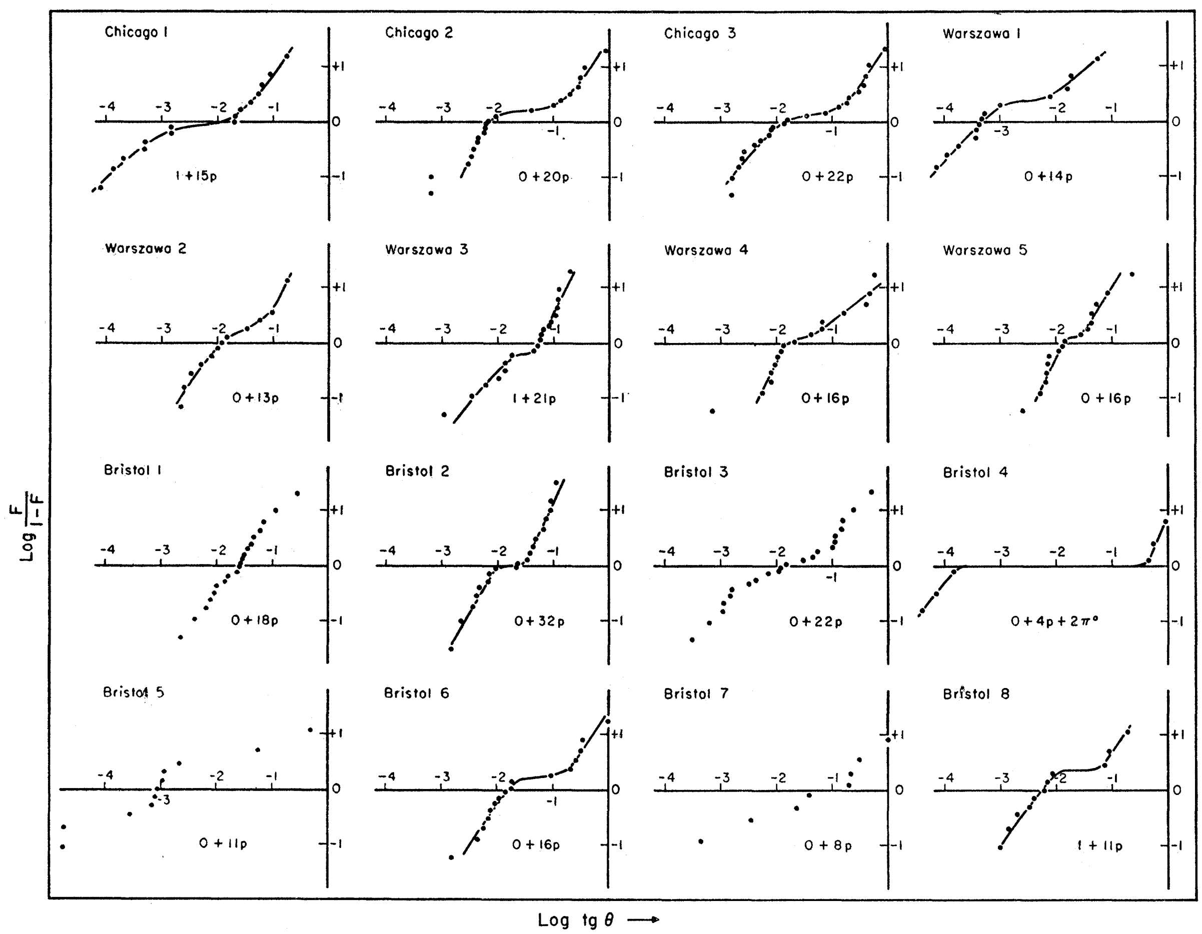

FIG. 2. The $\log[F/(1-F)]$ plots of high-energy jets that could be examples of N-N collisions. Not all the secondaries are plotted whenever their number is large. Chicago 1: Glasser, Haskins, Schein, and Lord, Phys. Rev. **99**, 1955 (1955). Chicago 2, 3: M. Schein (private communication). Warszawa 1: Ciok, Danysz, Gierula, Jurak, Miesowicz, Pernegr, Vrana, and Wolter, Nuovo cimento **6**, 1409 (1957). Warszawa 2, 3, 4, 5: Ciok, Coghen, Gierula, Holynoki, Jurak, Miesowicz, Samienska, Stanisz, and Pernegr, reference 7; and M. Miesowicz (private communication). Bristol 1, 2, 3, 4, 5, 6, 7, 8: Edwards, Losty, Perkins, Pinkau, and Reynolds (reference 11), and D. Perkins (private communication).

consists overwhelmingly of protons. The possibility still remains that the "primary" is a secondary of an interaction which occurred higher in the air, hence possibly a meson; however, the probability for this to happen is rather small for plates flown with balloons. Neutral particles are excluded since they are certainly of secondary origin.

To eliminate possibility (b) we shall consider only jets accompanied by no more than one heavily ionizing particle. Though the possibility still remains that some of the cases thus selected are due to composite collisions, it seems unlikely that the contamination is large.

Finally we shall consider only cases where the energy of the primary protons, in the LS, is $\geqslant 1000$ Bev.

From the published literature and from private communications, 16 events satisfying the above conditions have been collected out of about 50 jets with apparent primary energy $\geqslant 1000$ Bev. They are plotted in Fig. 2 in the way discussed in Sec. II.

It is evident from an examination of Fig. 2 that in most cases the relativistic secondaries are separated into two groups as if they were emitted, in the center-of-momentum (c.m.) system of the collision, not by a single center but by two bodies, as described in Sec. II(d). The evidence is so striking that we are going to analyze these events in a slightly different manner, more adjusted to the model.

Instead of considering all the relativistic particles produced in the collision together, let us divide them into two groups: the forward group, b_1, and the backward group, b_2 (the narrow and the wide cones).

Let n_1 and n_2 be the number of particles falling in each group[13] and let us analyze them in terms of log tanΘ

[13] The symmetry between the two nucleons in the c.m. system suggests that, in first approximation, $n_1 \approx n_2$: however, especially when the total number of secondaries is small, large fluctuations can be expected. The subdivision of the tracks into the two categories must be decided upon inspection of the $F/(1-F)$ plot.

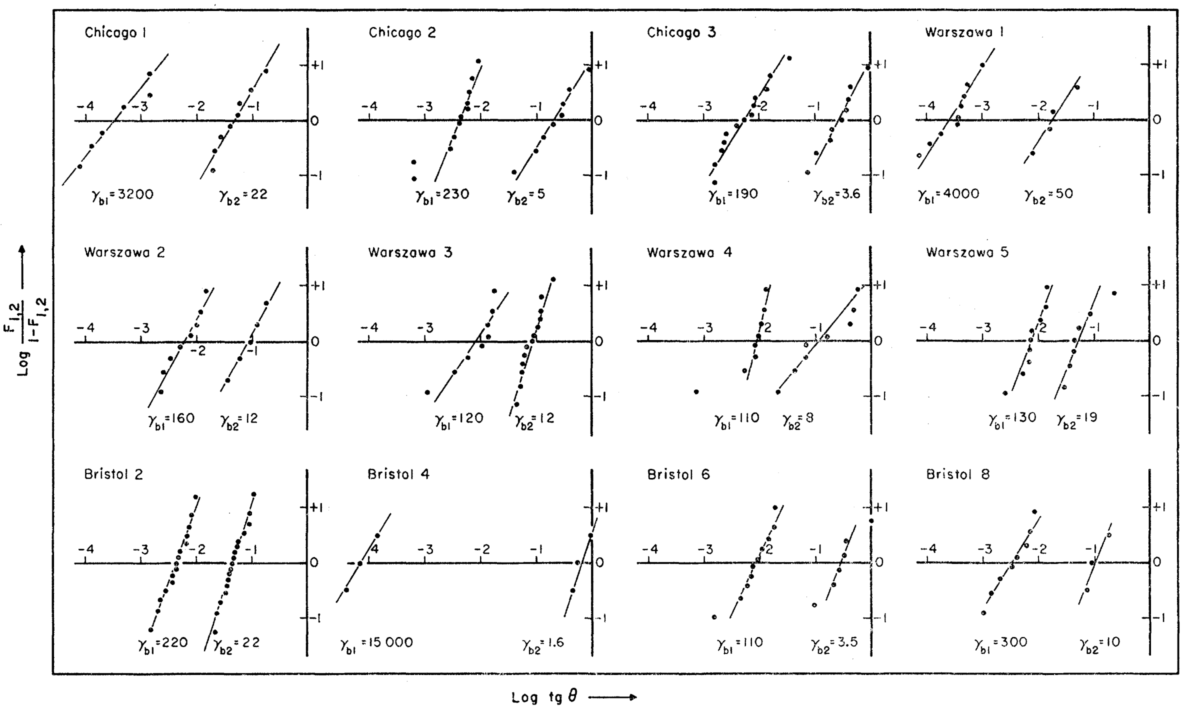

FIG. 3. The $\log[F_1/(1-F_1)]$ and $\log[F_2/(1-F_2)]$ plots of the events of Fig. 1 that show two separate branches.

versus $\log[F_1/(1-F_1)]$ and *versus* $\log[F_2/(1-F_2)]$, where

$$F_1=(\text{No. of particles in } b_1 \text{ within } \Theta)/n_1,$$

$$F_2=(\text{No. of particles in } b_2 \text{ within } \Theta)/n_2.$$

The results are plotted in Fig. 3.

The fact that the experimental points in Fig. 3 lie quite consistently along straight lines shows that the distributions of the particles in each group is not zero at $\vartheta=\pi/2$. The isotropy in each group would correspond to a slope of 2. The average value of the slopes of the forward jets plotted in Fig. 3 is 2.0 and that of the backward jets is 2.1.[14]

Among the cases examined in Fig. 2, four (Bristol 1, 3, 5, and 7) do not show the two branches. These could be cases of composite collisions; this is particularly suggested by the first two.[15] The other two are more likely the result of anomalous fluctuations in the distribution of the ionizing secondaries.

(c) Composite Collisions

In Fig. 4 we have analyzed some jets likely produced in composite collisions because the number of heavy prongs is high and/or because the primary is not singly ionizing. It is gratifying to find that in most of them no structure is visible, and the secondary particles are distributed with continuity over all angles. In the examples of Bern and of Bristol, however, it would be possible to separate the forward and the backward branches. Presumably, notwithstanding the interaction with several nucleons, especially the forward jet manages to maintain some of its structure. This is also often observed in collisions due to alpha particles and to singly ionizing particles with several heavy secondary prongs (not reproduced in Fig. 4).

The analysis of all these cases is very complex and, at least for the time being, it seems impossible to utilize them for studying the properties of N-N collisions.

(d) Secondary Collisions

In Fig. 5 we have collected 8 cases of high-energy (≥ 1000 Bev) secondary collisions, i.e., jets produced by particles generated in another jet in the same stack, irrespective of the number of heavy prongs and of the charge of the secondary.

Some show the two-branched structure, some do not. This inhomogeneity makes it very difficult to interpret these events. E.g., are the cases where a single branch is observed examples of emission from a unique center in the c.m. system, or of emission of secondaries by one "body" only? Questions of this kind are here pertinent, because many of the secondary interactions must be due to mesons, and could be fundamentally different from N-N collisions. Besides, it always remains to be decided whether the collisions are single or composite.

[14] Note that a $\cos^m\vartheta$ distribution, in the plot of Fig. 3, would not give two straight lines, but instead for b_1 a line bent to the left in the upper part of the figure, and for b_2 a line bent to the right in the lower part. Looking at the experimental points, this does not seem to be the case.

[15] The possibility also exists that for some cases $\bar{\gamma}_b\approx 1$.

This situation is very unfortunate, because it makes it impossible to utilize these interactions for studying in detail how the energy is distributed among the most energetic secondaries.

IV. EMPIRICAL MODEL

The results of the previous chapter suggest the model for N-N collisions described in Fig. 6, where the notations are equivalent to those used in Sec. II.

In the c.m. system, after the collision, four units are present: the two original nucleons, identified with the subscript a, traveling in opposite directions with Lorentz factors $\bar{\gamma}_a$, and two clouds of particles, b_1 and b_2, whose c.m. move in opposite directions with Lorentz factors $\bar{\gamma}_b$. From these two clouds, n_1 and n_2 ($n_1 \approx n_2$) particles emerge, mesons and nucleons, with an angular distribution roughly isotropic in the b system, each of them having, on the average, momentum $\langle p \rangle \approx 1$ Bev/c.

The average transverse momentum for all the n_1+n_2 particles is thus, also in the LS[16]

$$\langle p_\perp \rangle = (\pi/4)\langle p \rangle \approx 0.78 \text{ Bev}/c.$$

The Lorentz factors of the particles in the b systems vary from ~ 7 for π mesons to ~ 2.5 for K mesons and ~ 1.5 for nucleons. The asymptotic formula (1) we are going to use can thus be expected to be at fault, especially for the nucleons; however, the general features of the collision depend mostly on the mesons, that seem the preponderant product, and for them the approximation is not very bad.

In the LS the two branches of secondaries give rise to the narrow jet (b_1) and to the wide jet (b_2). Their Lorentz factors, in the LS, can be evaluated with the help of Eq. (3). Since the distributions of the n_1 and n_2 particles is isotropic in each of the b systems, the analysis of the two jets in terms of $\log \tan\Theta$ *versus* $\log[F_1/(1-F_1)]$ and $\log[F_2/(1-F_2)]$ is expected to give two straight lines with slope ~ 2.

One cannot insist too strongly on the isotropy in the b systems (the slope of 2) when $n_{1,2}$ is not large, since in that case the emission of nucleon pairs and of heavy mesons could presumably introduce great disturbances.[17] However, the criterion of requiring at least two separate bunches of particles is essential.

The information about the interaction is thus reduced to the three numbers: γ_{b1}, γ_{b2}, and N, the total number of ionizing particles. Following the current belief that most of the secondaries are mesons and that $\sim\frac{1}{3}$ of them are neutral, we shall put

$$N \approx \tfrac{2}{3}(n_1+n_2). \tag{4}$$

From these numbers it is possible to deduce γ_c, $\bar{\gamma}_b$, $\bar{\gamma}_a$, and ρ, the inelasticity of the collision.

From the relativistic transformations, one finds

$$\gamma_{b1} = \gamma_c \bar{\gamma}_b (1+\beta_c \bar{\beta}_b) \xrightarrow[\beta_c = \bar{\beta}_b = 1]{} 2\gamma_c \bar{\gamma}_b,$$

$$\gamma_{b2} = \gamma_c \bar{\gamma}_b (1-\beta_c \bar{\beta}_b) \rightarrow (\gamma_c^2 + \bar{\gamma}_b^2)/\gamma_{b1},$$

where, in the second equation, we have put $\beta = 1-(1/2\gamma^2)$. Solving for γ_c and $\bar{\gamma}_b$, one obtains

$$\begin{aligned} \gamma_c &\approx (\gamma_{b1}\gamma_{b2})^{\frac{1}{2}}, \\ \bar{\gamma}_b &\approx \tfrac{1}{2}(\gamma_{b1}/\gamma_{b2})^{\frac{1}{2}}. \end{aligned} \tag{5}$$

The inelasticity parameter is

$$\rho = \frac{\text{total energy in the two ``bodies''}}{\text{total energy in the c.m. system}}.$$

In first approximation the total energy of each particle in the b systems is ~ 1 Bev $\approx 1Mc^2$. Then, remembering

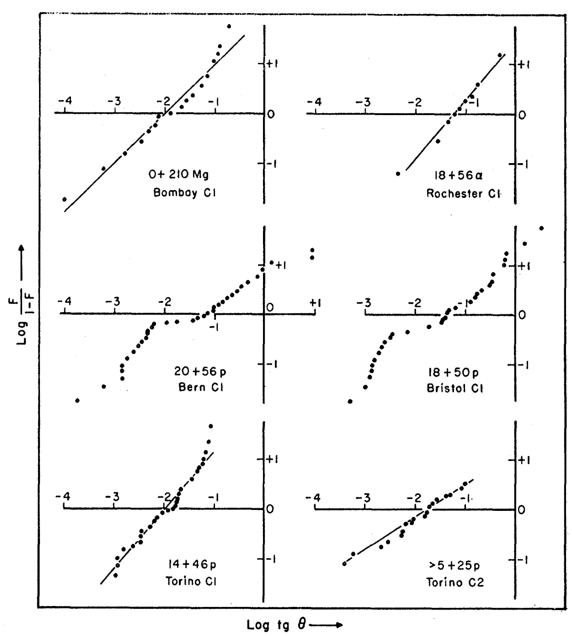

FIG. 4. The $\log[F/(1-F)]$ plots of the events that could be examples of composite collisions. Not all the secondaries are plotted whenever their number is large. Bombay $C1$: Lal, Pal, Peters, and Swarmi, Proc. Indian Acad. Sci. **36**, 75 (1952). Rochester $C1$: Bradt, Kaplon, and Peters, Helv. Phys. Acta, **23**, 24 (1950). Bern $C1$: Hanni, Lang, Teucher, and Winzeler, Nuovo cimento **4**, 1473 (1956). Bristol $C1$: Edwards, Losty, Perkins, Pinkau, and Reynolds, reference 11; and D. Perkins (private communication). Torino $C1$ and $C2$: G. Wataghin (private communication).

[16] Actually the experimental evidence discussed thus far only requires a transverse momentum of $\sim\frac{1}{2}$ Bev/c and a quasi-isotropic distribution in the b systems. The energy distribution in these systems could be different from that assumed, and besides could depend on the nature of the particles emitted. Only energy measurements of the secondaries can determine whether our choice, besides being the simplest, is also reasonable. Unfortunately, as will be discussed in Chap. V, the energy measurements available thus far are not sufficient for deciding this question.

[17] The narrow jet is expected to include also the two original nucleons (a_1 and a_2); since these are moving at small angles to the Z axis, their contribution tends to decrease the slope of the narrow jet.

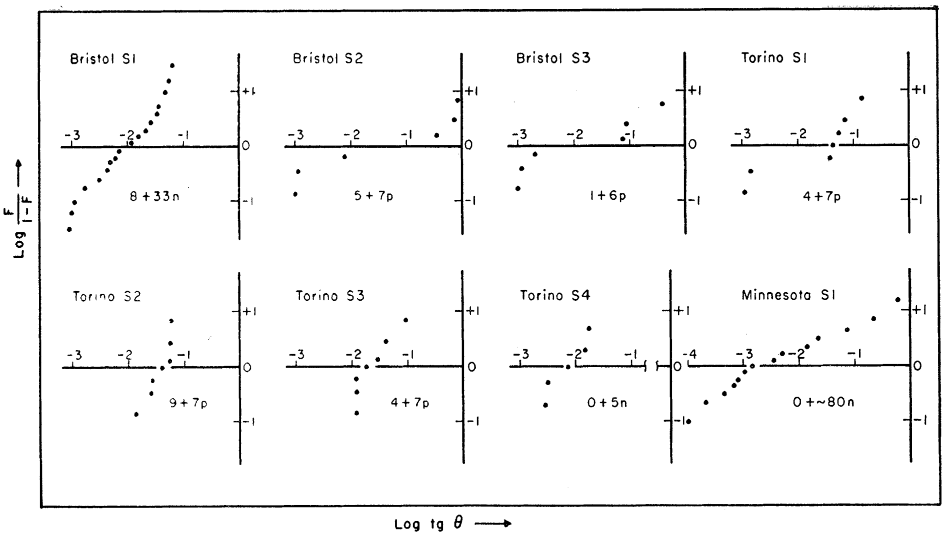

FIG. 5. The $\log[F/(1-F)]$ plots of jets produced by energetic secondaries of other jets. Bristol $S1$, $S2$, $S3$: Edwards, Losty, Perkins, Pinkau, and Reynolds, reference 11, and D. Perkins (private communication). Torino $S1$, $S2$, $S3$, $S4$: G. Wataghin (private communication). Minnesota $S1$: S. Freier (private communication).

(4), one deduces

$$\rho\approx\frac{(n_1+n_2)\bar{\gamma}_b}{2\gamma_c}=\frac{n_1+n_2}{4\gamma_{b2}}\approx\frac{3}{8}\frac{N}{\gamma_{b2}}. \tag{6}$$

Large values of ρ are thus associated with large apertures, in the LS, of the backward cone; the reason is that, if ρ is large, the b bodies move fast in the c.m. system, and b_2 then moves slowly in the LS.

In the same way, there follows

$$\bar{\gamma}_a\approx\gamma_c-\tfrac{1}{2}(n_1+n_2)\bar{\gamma}_b=\gamma_c(1-\rho),$$

and

$$\begin{aligned}\gamma_{a1}&\approx\gamma(1-\rho),\\ \gamma_{a2}&\approx 1+\tfrac{1}{2}\rho/(1-\rho)\approx 1.\end{aligned} \tag{7}$$

The second equation in (7) is obviously wrong, as a consequence of the use of asymptotic equations; however, γ_{a2} is expected to be always rather small.

In the LS, the energy of the particles in the two jets is related to their angle of emission in the b systems, ϑ, by the relation $(\bar{\beta}\approx 1)$

$$E\approx\gamma_{bi}(1+\cos\vartheta_i)\ \text{Bev}, \tag{8}$$

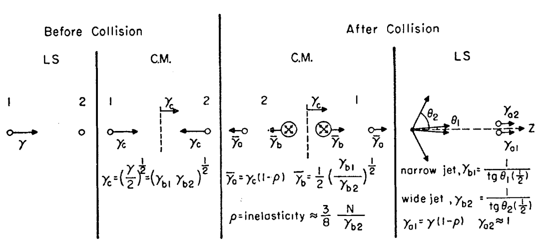

FIG. 6. A schematic diagram of the empirical model. The equations are justified in the text.

where $i=1$, 2. The relation between ϑ_i and the corresponding angle Θ in the LS is given by Eq. (1), here rewritten as

$$\tan\Theta=(1/\gamma_{bi})\tan(\vartheta_i/2). \tag{9}$$

If average energies in the LS are considered, the particles emitted in the forward quadrants of each of the two bodies have $\langle E_{\text{forward}}\rangle=\frac{3}{2}\gamma_{bi}$ Bev, while for the backward quadrants $\langle E_{\text{backward}}\rangle=\frac{1}{2}\gamma_{bi}$ Bev. The ratio,

$$\langle E_{\text{forward}}\rangle/\langle E_{\text{backward}}\rangle=3, \tag{10}$$

is independent of the value assumed for the average energy of emission of the secondaries in the frame of reference of each b body (1 Bev in this article). These quantities can be directly compared with the experimental results.

The transverse momentum is instead

$$p_\perp\approx\gamma_{bi}(1+\cos\vartheta_i)\sin\Theta=\gamma_{bi}\left(\frac{2}{1+\tan^2(\vartheta_i/2)}\right)\sin\Theta\ \frac{\text{Bev}}{c},$$

and for angles for which $\sin\Theta\approx\tan\Theta$, using (9),

$$p_\perp\approx\frac{2}{1/(\gamma_{bi}\tan\Theta)+\gamma_{bi}\tan\Theta}\ \text{Bev}/c. \tag{11}$$

If the emission in the b-centers is isotropic, relation (2) holds for each center of emission and

$$p_\perp=2\bigg/\left[\left(\frac{1-F_i}{F_i}\right)^{\frac{1}{2}}+\left(\frac{F_i}{1-F_i}\right)^{\frac{1}{2}}\right]\ \text{Bev}/c.$$

Ninety percent of the secondaries $(F_i=0.95)$ are thus expected to have $1\leqslant p_\perp\leqslant 0.43$ Bev/c.

TABLE I. Characteristics of the events of Fig. 3.

Event		γ_{b1}	γ_{b2}	$\tan\phi$ 1	$\tan\phi$ 2	γ_c	$\bar{\gamma}_b$	ρ	Primary energy (ev)
Chicago 1	$1+15p$	3200	22	1.2	1.7	260	6.0	0.25	1.3 10^{14}
Chicago 2	$0+20p$	230	5	2.6	1.6	34	3.4	1.5	2.2 10^{12}
Chicago 3	$0+22p$	190	3.6	1.6	1.8	26	3.6	2.3	1.3 10^{12}
Warszawa 1	$0+14p$	4000	50	1.6	1.6	450	4.5	0.10	3.8 10^{14}
Warszawa 2	$0+13p$	160	12	1.8	1.8	44	1.8	0.40	3.6 10^{12}
Warszawa 3	$1+21p$	120	12	1.5	3.1	38	1.6	0.65	2.7 10^{12}
Warszawa 4	$0+16p$	110	8	4.0	1.2	95	3.7	0.75	1.7 10^{12}
Warszawa 5	$0+16p$	130	19	2.6	2.6	50	1.3	0.32	4.7 10^{12}
Bristol 2	$0+32p$	220	22	2.7	3.0	71	1.6	0.55	9.5 10^{12}
Bristol 4	$0+4p$ $(+2\pi^0)$	15 000	1.6	1.7	2.7	150	48	1.0	4.2 10^{13}
Bristol 6	$0+16p$	110	3.5	2.1	2.6	20	2.8	1.7	0.75 10^{12}
Bristol 8	$1+11p$	300	10	1.6	2.4	55	2.7	0.41	5.7 10^{12}

The model described here does not differ from that proposed by Takagi[4] as far as the assumption of two centers of emission of the secondary particles is concerned; it differs substantially from it, however, in insisting that all the secondaries have approximately the same momentum and that the two originary nucleons persist after the interaction. It is thus possible to define unequivocally the inelasticity of the collision.

However, our model can be considered also from another point of view. Instead of giving the two *fire balls* a separate existence, they could be thought of as two bunches of secondaries emerging from a common center of interaction (the point of maximum approach between the two nucleons), in opposite directions and with relatively small spread in angle and energy—the equivalent of two collimated beams of particles with small energy straggling. The b bodies would, in this case, be the centers of momentum of the two beams, and represent a convenient way of visualizing a very anisotropic distribution of particles coming out from a single center. This way of thinking could perhaps be reconciled with the model suggested by Heisenberg[1] or with that suggested by Lewis, Oppenheimer, and Wouthuysen.[2]

V. DISCUSSION

Table I has been constructed with the 12 events of Fig. 3 interpreted as examples of N-N interactions according to the model presented in Chap. IV [Eqs. (5) and (6)].

With such a limited sample it is impossible to see any correlation among the various parameters. It would also be dangerous to consider a sample of this kind as unbiased, since both the way the plates are scanned searching for jets and the criteria that prompt their publication are still somewhat personal.

Several comments are pertinent.

In interpreting these interactions with our model it must be realized that some of the consequences are purely "geometrical," i.e., follow from the fact that the secondaries are emitted isotropically from each of the two b bodies; others instead are "dynamical," i.e., depend on how the energy is shared among the secondaries. While the geometrical description is on rather sound ground, since it follows directly from the existence of the two branches in the $\log[F_i/(1-F_i)]$ plot (Fig. 3) and from the value ~ 2 of the slopes, the dynamics could be substantially different from that here postulated, provided it satisfies the constancy of the average transverse momentum.

Considering first the "geometrical" aspects of the examples of Table I, the values of $\bar{\gamma}_b$ seem to cluster around 4 though large deviations are present. The number of secondaries seems to be characteristically 15–20, for energies around $10^{12}-10^{14}$ ev.[18]

The evaluation of the inelasticity parameter, ρ, involves dynamical assumptions. The fact that in some cases ρ turns out to be larger than 1 could be an indication that our choices are wrong. However, large fluctuations both in the number of neutral particles produced and in the average energy per particle are expected in individual cases, and could be the cause of the anomalous results.[19] The other values of ρ span all the range from 0.1 to 1. No obvious correlation seems to exist between ρ and the number of secondaries.

The real test of the dynamical assumptions can come only from accurate measurements of the energy of the secondaries; these measurements could check Eqs. (8) and (11). Unfortunately, the situation, for the time being, is not favorable, as discussed below.

The energy of a secondary can be estimated by the following methods:

(a) for particles of sufficiently long lifetime, from the analysis of the interactions that the secondary occasionally produces while crossing the rest of the stack;

(b) for π^0's, from the analysis of the electromagnetic cascade produced by the two photons into which they decay;

(c) for all ionizing particles, from the measurement of

[18] The values of the primary energies given by our model do not practically differ from those obtained with the classical method, since both are based on the symmetry of the collision in the c.m. system.

[19] Also the value of 1 Bev chosen for the average energy per particle in the b systems could be too high. A value around $\frac{2}{3}$ Bev could give a better agreement with the available data.

their scattering, either absolute or relative to another particle belonging to the same jet.

Method (a), as discussed in Sec. III(d), can give only orders of magnitude. Method (b) is perhaps the most reliable, when the energy is large ($>10^{11}$ ev) but thus far it has been utilized only for few π^0's emitted by the jets of Table I. Method (c) can have the widest application and is particularly useful when applied to the particles emitted backward, in the c.m. system, which in general have relatively small energies in the LS. Unfortunately, for these particles absolute scattering measurements must be used, as they rarely are found in close pairs, and the emulsion distortions can introduce systematic errors.

Coming to the actual measurements, in the Chicago 1 event, the momenta of all particles in the wide cone were deduced from their absolute scattering and found to range between 0.3 and 6 Bev/c (average ~ 2 Bev/c) while from Table I the expected momenta are around 20 Bev/c. This could be a serious discrepancy if one could be sure that emulsion distortions have not impaired the measurements. As a matter of fact, this doubt exists, since the average transverse momentum of these particles, as deduced from the scattering measurements, turns out to be 0.09 Bev/c, at least 5 times smaller than the expected average.

The secondaries of Bristol 4 are all measured (two are π^0's) and the energies in the narrow cones are around 3000 Bev while the expected average is 10 000; those in the wide cone are $\sim\frac{2}{3}$ Bev, in agreement with the expected value. There is also indication that, in each b body, the energy of the particles going forward is larger than that of the particles going backward.

Three secondaries in the forward cone of Bristol 2 give energies around 450 Bev [method (a)] against the expected 220.

Keeping in mind the uncertainties of the energy measurements and the possibility of fluctuations in the energy distribution among the secondaries, the situation does not seem hopeless, though there is as yet no conclusive check for the model.

As a final comment, the most interesting feature of the model is the possible presence of the two *fire balls* detached from the originary nucleons and from which the secondaries, all with about the same momentum, emerge. It suggests that the final products of N-N interactions are rather insensitive to the primary energy, once the ultrarelativistic region is reached, as the high-frequency components of the interactions never succeed in emerging from the region of interaction.

A conclusion of this kind could have the practical consequence of removing some urgency to the building of accelerators of always higher energies. It would in fact make it plausible that the present panorama of the elementary particles is already rather complete, if by "particle" we mean a property of matter that lasts long enough to be singled out.

VI. ACKNOWLEDGMENTS

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The first motivation of the model presented here came from the discussion contained in the article by the Polish group (reference 7), sent in preprint form by Professor M. Danysz. Professor M. Miesowicz, later, made available further information about the Polish jets. We want to point out that several of the conclusions reached by us were also reached independently in Warsaw.

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