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Photon Decay of Hyperons*

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The photon mode of decay of the Σ^+ and Λ^0 is examined. The lifetime and angular distributions of the decay products are written in terms of the three physical parameters of the problem without restriction as to invariance under space reflection, charge conjugation, or time reversal. An estimate of these parameters is made by use of perturbation theory, and, although ambiguous, it indicates that this mode may soon be experimentally detectable.

INTRODUCTION

UNTIL recently, the limited number of Λ, Σ, and Ξ particles that have been observed has focused attention on the analysis of their main mode of decay, *i.e.*, by the emission of a π meson. In the near future, it will be of considerable interest to establish and analyze alternative, less frequent, modes of decay which have been theoretically predicted for hyperons. On such group of less frequent hyperon decays modes is that arising from a universal Fermi interaction.[1] It is the purpose of this paper to discuss, in some detail, another mode[2] which should, in some cases, compete favorably with the Fermi decay mode. This decay occurs through the emission of a photon,

$$Y_1 \rightarrow Y_2 + \gamma. \tag{1}$$

In the case of the Σ^0, this decay proceeds solely through strong interactions and is experimentally known to have a lifetime $<10^{-11}$ sec.[3] For other hyperons, this decay must proceed through a combination of weak and strong interactions (since strangeness is not conserved in the process) and hence its rate might be expected to be[4] some fraction (of the order of the fine structure constant) of the main decay mode. The decays which will be analyzed in this paper are the following:

$$\Sigma^+ \rightarrow p + \gamma, \tag{1'}$$

$$\Lambda^0 \rightarrow n + \gamma. \tag{1''}$$

The analysis will proceed in the following manner. In the first section, the general form of the matrix element, as a function of two complex parameters, will be established, without restriction as to invariance under space reflection, charge conjugation, or time reversal. The lifetime for the decay, as well as angular distributions in the case of polarized particles, will be written in terms of these two complex parameters and then it will be shown that the process can be completely characterized by three real physical parameters.

It then will be assumed that the Lagrangian responsible for the decay *does not* contain the photon mode of hyperon decay as a primary interaction. In general, when calculating the matrix element for the photon mode of decay from such a Lagrangian, the integrands of some of the integrals are expected to have poles. This will be demonstrated explicitly in the Appendix by a second-order perturbation calculation. Since the contribution of a pole to the matrix element is an imaginary quantity, it will give rise to terms in the angular distributions which give the appearance of violating time-reversal invariance. In the second section it will be shown that these poles correspond to the succession of two real processes.

Finally, estimates of the branching ratio of the photon mode to pion mode of hyperon decay will be made on the basis of the perturbation calculation.

I. PHENOMENOLOGICAL APPROACH

Subject to the requirement of invariance under the proper Lorentz group, the most general matrix element for the decay process (1) can be written in terms of the four-momenta of the initial fermion, p_1, and the final fermion, p_2, and the polarization vector for the electromagnetic field, e_μ [note that since only one photon is emitted in (1), the matrix element must be homogeneous of degree one in the four-vector e_μ]:

$$\begin{aligned} M = {} & (a_1+ia_2\gamma_5)\gamma_\mu e_\mu + (a_3+ia_4\gamma_5)p_{1\mu}e_\mu \\ & + (a_5+ia_6\gamma_5)p_{2\mu}e_\mu + (a_7+ia_8\gamma_5)\gamma_{\mu\nu}p_{1\mu}e_\nu \\ & + (a_9+ia_{10}\gamma_5)\gamma_{\mu\nu}p_{2\mu}e_\nu + (a_{11}+ia_{12}\gamma_5)\gamma_{\mu\nu\rho}p_{1\mu}p_{2\nu}e_\rho, \end{aligned} \tag{2}$$

where the γ_μ are the Dirac matrices,[5] the $\gamma_{\mu\nu}$ and $\gamma_{\mu\nu\rho}$ are the antisymmetric combinations of two and three γ_μ [*e.g.*, $\gamma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$], respectively, and the a_i are arbitrary functions (complex) of the invariants which may be formed from the three vectors p_1, p_2,

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[1] See, for example, R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] See, for example, M. Kawaguchi and K. Nishijima, Progr. Theoret. Phys. (Japan) **15**, 182 (1956), and C. Iso and M. Kawaguchi, Progr. Theoret. Phys. (Japan) **16**, 177 (1956).

[3] Plano, Samios, Schwartz, and Steinberger, Nuovo cimento **5**, 216 (1957).

[4] An event has been found by George, Herz, Noon, and Solntseff [Nuovo cimento **3**, 94 (1955)] in which a single charged particle emitted from a K^--produced star comes to rest and emits a proton of 26-Mev kinetic energy. They tentatively interpret this as a hyperdeuteron. M. Goldhaber (private communication) has pointed out that the energy of the proton coincides with the energy (26.5 Mev) expected in the decay mode $\Sigma^+ \rightarrow p + \gamma$.

[5] The representation of the γ's is $\gamma_i = \beta\alpha_i$, $\gamma_0 = \beta$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_0 = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The scalar product of two four-vectors is $a\cdot b = a_0 b_0 - \mathbf{a}\cdot\mathbf{b}$.

and k (the four-momentum of the photon). The matrix element (2) allows for noninvariance under space reflection, charge conjugation, and time reversal. It should be noted that in order to introduce polarized fermions into the problem, it is necessary only to flank the matrix element on the left and/or right by the appropriate spin projection operator.

The matrix element (2) must, of course, be gauge invariant. By performing a gauge transformation on (2), and by using the relations

$$p_1 = p_2 + k, \quad k^2 = 0, \tag{3}$$

it is possible to show that M must have the form

$$M = \tfrac{1}{2}(A + iB\gamma_5)\gamma_{\mu\nu} f_{\mu\nu} = (A + iB\gamma_5)\boldsymbol{ke}, \tag{4}$$

where $\boldsymbol{a} \equiv \gamma_\mu a_\mu$, $f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu$, and A and B are unknown, invariant functions of the pertinent mass and coupling constants of the problem.

If it were assumed that the decay Lagrangian contained a term of the form of (4), then by the usual rules it would be easy to see that the requirement of invariance under space reflection, P (or CT by the Lüders-Pauli theorem), demands that $AB=0$ (i.e., either $A=0$ or $B=0$); invariance under charge conjugation, C, demands $\mathrm{Re}(AB^*)=0$; and invariance under time reversal, T, demands $\mathrm{Im}(AB^*)=0$.

If, however, the interaction (4) is not considered to be primary (i.e., if no such term appears in the original Lagrangian) but is assumed to be compounded from other terms in the Lagrangian, *e.g.*, the pion mode of hyperon decay *plus* the electromagnetic interaction Lagrangian, then no such simple statements can be made concerning invariance under time reversal. In the following, this latter assumption will be made, and it will be seen in the Appendix that, starting from a Lagrangian which is invariant under time reversal, A and B will *not* fulfill the above conditions for time-reversal invariance.

From the matrix element (4), it is simple to calculate the transition probability for fermions of spin $\frac{1}{2}$. The result is

$$1/\tau = (|A|^2 + |B|^2)(m_2^3/\pi)\sinh^3\omega, \tag{5}$$

where, for later convenience, ω has been defined as

$$\omega = \ln(m_1/m_2). \tag{6}$$

In order to test time-reversal, charge-conjugation, and space-reflection invariance properties, it is necessary to consider the angular distribution arising when some or all of the particles are polarized. Only the cases when all three particles are polarized will be discussed, the other results following immediately from these general cases.

In the case of a circularly polarized photon, the differential transition probability is

$$P \propto |A - B\hat{k}\cdot\epsilon|^2[1 - \mathrm{sech}\,\omega(\hat{k}\cdot s_1)(\hat{k}\cdot s_2) - \hat{k}\cdot\epsilon(\mathrm{sech}\,\omega\hat{k}\cdot s_2 - \hat{k}\cdot s_1)], \tag{7}$$

where $\hat{k}$ is a unit 3-vector (space components) in the direction of the photon propagation, s_1 and s_2 are unit three-vectors in the directions of the spins of the initial and final baryon, respectively, and ϵ is a unit three-vector in the direction of the "spin" of the photon. ϵ must necessarily be parallel to $\hat{k}$ and hence $\hat{k}\cdot\epsilon(=\pm 1)$ describes the helicity (± 1) of the photon.

In the case of a linearly polarized photon, the differential transition probability is

$$\begin{aligned} P \propto \{ &(AA^* + BB^*)[1 - \mathrm{sech}\,\omega(\hat{k}\cdot s_1)(\hat{k}\cdot s_2)] \\ &+ (A^*B + AB^*)[\mathrm{sech}\,\omega\hat{k}\cdot s_2 - k\cdot s_1] \\ &+ (AA^* - BB^*)[s_1\cdot s_2 - 2s_1\cdot es_2\cdot e - \hat{k}\cdot s_1\hat{k}\cdot s_2] \\ &+ i(A^*B - AB^*)[s_2\times s_1\cdot\hat{k} - 2s_1\cdot es_2\times e\cdot\hat{k}], \end{aligned} \tag{8}$$

where e is the photon polarization.

In analogy to the analysis of the pion mode of decay by Lee and Yang,[6] it is possible to characterize the photon mode of decay by just three real parameters. One of these parmeters may be taken as $|A|^2 + |B|^2$ which is, according to (5), proportional to the lifetime. Another parameter can be defined by noting that the angular distribution of the decay photon from a completely polarized hyperon at rest is [from (7)]

$$P \propto (1 - \alpha_\gamma \cos\chi)d\Omega, \tag{9}$$

where $d\Omega$ is the solid angle of the photon momentum and χ is the angle between this momentum, k, and the spin s_1 of the baryon. [Note that for the pion mode $P \propto (1 + \alpha\cos\chi)d\Omega$.] The second parameter is thus α which is defined (in terms of A and B) as

$$\alpha_\gamma = \frac{2\,\mathrm{Re}(AB^*)}{|A|^2 + |B|^2}. \tag{10a}$$

Another way of measuring the asymmetry parameter is to measure the ratio of the numbers of positive- to negative-helicity photons, for arbitrary polarizations of the fermions. Thus

$$\frac{N_R}{N_L} = \frac{1 - \alpha_\gamma}{1 + \alpha_\gamma}, \quad \text{or} \quad \alpha_\gamma = \frac{N_L - N_R}{N_L + N_R}. \tag{10b}$$

The third parameter could be chosen as either

$$\beta_\gamma = \frac{-2\,\mathrm{Im}(A^*B)}{|A|^2 + |B|^2}, \tag{11a}$$

or

$$\gamma_\gamma = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2}, \tag{11b}$$

since there exists the relation $\alpha^2 + \beta^2 + \gamma^2 = 1$. However, following the procedure of Lee and Yang, the third parameter will be taken as φ_γ defined by

$$\beta_\gamma = (1 - \alpha_\gamma^2)^{\frac{1}{2}}\cos\varphi_\gamma, \quad \gamma_\gamma = (1 - \alpha_\gamma^2)^{\frac{1}{2}}\sin\varphi_\gamma. \tag{12}$$

[6] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957).

The geometrical significance of this parameter can be seen in the following way. Let $\hat{k}$ define the z axis and let the linear polarization which is accepted be perpendicular to s_1 (*i.e.*, $s_1 \cdot e = 0$) and lie along the x axis. Then φ_γ is the azimuthal angle of the transverse component of the nucleon spin in the rest system of the hyperon.

It is interesting that the only way of testing invariance under time reversal involves detecting the linear polarization of the photon as well as the initial and final baryon spins. The cases in which the photon polarization is not detected [derived from (7) or (8)] or in which the photon polarization is circular [(7)] do not contain terms which afford the opportunity of detecting such an invariance. It should again be emphasized that the experimental detection of the last terms in (8) would mean noninvariance under time reversal *only* if the interaction (4) were considered primary.

In the following, it will be assumed that (4) is compounded from a Lagrangian in which the pion decay modes of the hyperons are primary. This Lagrangian may be written[7]

$$\mathcal{L} = \bar{\psi}_1(p - m_1)\psi_1 + \bar{\psi}_2(p - m_2)\psi_2 + f\bar{\psi}_2(ia + b\gamma_5)\psi_1\varphi - e\sum_{k=1}^{2}\psi_k A\psi_k + \sum_{k=1}^{2} g_k\bar{\psi}_k\gamma_5\psi_k\varphi + ie\left[\frac{\partial\varphi^*}{\partial x^\mu}\varphi - \varphi^*\frac{\partial\varphi}{\partial x^\mu}\right]A^\mu + \mathcal{L}_0 + \text{H.c.}, \quad (13)$$

where the isotopic spin indices have been suppressed, $\mathcal{L}_0$ is the free boson Lagrangian, φ is the pion wave function, and e, f, and g are, respectively, the electromagnetic, weak decay, and mesonic coupling constants. (Charge independence is assumed for the strong pion-baryon coupling.)

In order to gain some insight into the relationship between the Lagrangian (13) and the matrix element (4), it might be well to examine some of the symmetries which the Lagrangian (13) possesses. In addition, these symmetries will help to simplify some of the calculations of the Appendix. For example, the connection between the parameters A and B and the parameters of the Lagrangian (of which the A and B are functions) may be found by noting that a symmetry that is obeyed by the total Lagrangian and which leaves the commutation rules unchanged will also be valid for the matrix element of the decay. Thus, consider the transformation

$$\psi_1 \to e^{i\alpha}\psi_1, \quad a \to ae^{-i\alpha}, \quad b \to be^{-i\alpha}, \quad (14a)$$

under which the Lagrangian is invariant. In order that the matrix element be invariant, A and B must transform as

$$A \to Ae^{-i\alpha}, \quad B \to Be^{-i\alpha}. \quad (14b)$$

Now, to a good approximation, it can be assumed that the weak interaction occurs only once, and thus A and B depend linearly on a and b. Further, consider another transformation under which the Lagrangian is invariant:

$$\psi_1 \to \gamma_4\psi_1; \quad \psi_2 \to \gamma_4\psi_2; \quad a \to -a; \quad p_i \to -p_i; \quad A_i \to -A_i; \quad \varphi \to -\varphi, \quad (15a)$$

for $i = 1, 2, 3$. Then, from the matrix element, one has

$$A \to A; \quad B \to -B. \quad (15b)$$

Combining the results of (14b) and (15b), it can be seen that

$$A = fbK_2(m_1, m_2, g_1, g_2, \mu^2), \quad B = faK_1(m_1, m_2, g_1, g_2, \mu^2), \quad (16)$$

where K_1 and K_2 are unknown functions of the masses and strong-coupling constants. Explicit use of the invariance of the Lagrangian under change in sign of the meson mass has been made. (The dependence on μ^2 will be understood but not explicitly noted in the following.)

Next, consider the transformation

$$\psi_1 \to \gamma_5\psi_1, \quad \psi_2 \to \gamma_5\psi_2, \quad \varphi \to -\varphi, \quad m_1 \to -m_1, \quad m_2 \to -m_2, \quad (17)$$

which leaves the Lagrangian invariant.[8] The condition on the matrix element is

$$K_i(-m_1, -m_2, g_1, g_2) = -K_i(m_1, m_2, g_1, g_2). \quad (18)$$

Under

$$\psi_1 \to \gamma_5\psi_1, \quad m_1 \to -m_1, \quad g_1 \to -g_i, \quad a \to -ib, \quad b \to -ia, \quad (19)$$

it follows that

$$K_1(-m_1, m_2, -g_1, g_2) = -K_2(m_1, m_2, g_1, g_2). \quad (20)$$

The Lagrangian is also invariant under

$$\varphi \to -\varphi; \quad g_1 \to -g_1; \quad g_2 \to -g_2; \quad f \to -f, \quad (21)$$

and therefore from the matrix element one finds

$$K_i(m_1, m_2, -g_1, -g_2) = -K_i(m_1, m_2, g_1, g_2). \quad (22)$$

Thus, if $g_1 = 0$, $g_2 = 0$, then $K_i = 0$.

[7] An alternative Lagrangian would treat the four-fermion interaction as primary. The lowest-order matrix element giving rise to the photon mode of decay is, by the Furry theorem, zero unless the four-fermion interaction is tensor or vector. For the vector case, the matrix element involves an integral which appears in the evaluation of vacuum polarization in quantum electrodynamics and which has been shown, by the regulator method, to be identically zero. The tensor case involves an integral which can be evaluated by the regulator method. The result is logarithmically divergent.

[8] These symmetry properties of the theory have been used before. See, for example, A. Salam, Nuclear Phys. 4, 687 (1957).

From dimensional arguments, A and B must be inversely proportional to a mass. It can be seen from the perturbation calculation in the Appendix that the natural mass dependence is

$$A \propto 1/(m_1+m_2), \quad B \propto 1/(m_1-m_2). \tag{23}$$

Because of (16) and (23), it is convenient to introduce the new parameters A' and B' defined by

$$A = fbA'/(m_1+m_2), \quad B = faB'/(m_1-m_2). \tag{24}$$

If it is assumed that the asymmetry parameter in the pion mode of decay is a maximum, then there exists a relation between a and b, namely

$$|b|^2 = \left(\frac{\cosh\omega - \frac{1}{2}\eta e^{-\omega}+1}{\cosh\omega - \frac{1}{2}\eta e^{-\omega}-1}\right)|a|^2. \tag{25}$$

If the pion mode of decay has a vector-pseudovector weak interaction, then the third term in the Lagrangian (9) is to be replaced by[9]

$$h\bar{\psi}_2(ic+d\gamma_5)\gamma_\mu\psi_1\partial_\mu\varphi. \tag{26}$$

In this case the condition between c and d for a maximum asymmetry parameter for the pion mode is

$$|d|^2 = \left(\frac{\cosh\omega - \frac{1}{2}\eta e^{-\omega}+1}{\cosh\omega - \frac{1}{2}\eta e^{-\omega}-1}\right)\left(\frac{\cosh\omega-1}{\cosh\omega+1}\right)|c|^2. \tag{27}$$

If now the branching ratio for the photon mode to the neutral pion mode of hyperon decay is calculated, it can be shown that for both scalar-pseudoscalar and vector-pseudovector weak pion coupling, the branching ratio assumes the form

$$B = \frac{\{(|A'|^2+|B'|^2)(\sinh^2\omega - \frac{1}{2}\eta e^{-\omega}) + (|A'|^2-|B'|^2)\frac{1}{2}\eta e^{-\omega}\}C\sinh\omega}{[(\cosh\omega - \frac{1}{2}\eta e^{-\omega})^2-1]^{\frac{3}{2}}}, \tag{28}$$

where C is the charged-to-neutral branching ratio for the pion mode of decay.

II. UNITARITY OF THE S MATRIX

As has been shown by Kawaguchi and Nishijima[2] and Iso and Kawaguchi,[2] it is possible to analyze the photon mode of hyperon decay by use of the unitarity of the S matrix. The procedure is to substitute $S=1+R$ into the unitarity condition, $S^\dagger S=1$, and then form the matrix elements. The result is

$$-\langle p\gamma|R^\dagger+R|Y\rangle = \sum_{N\pi}\langle p\gamma|R^\dagger|N\pi\rangle\langle N\pi|R|Y\rangle.$$

The sum is restricted to a sum over a nucleon and a single pion state because this is the main process on the energy shell (the intermediate state $N\gamma$ is smaller by a factor $\sim 1/137$). By introducing the reduced R-matrix elements $\langle f|R|i\rangle = (2\pi)^4\delta^4(p_f-p_i)R_{fi}$, the above relation may be written

$$\begin{aligned}-(2\pi)^4\delta(p_f-p_i)[(R^\dagger)_{p\gamma,Y}+R_{p\gamma,Y}]\\ = (2\pi)^4\delta^4(p_f-p_i)\Sigma D_{N\pi}R_{N\pi,Y}R_{N\pi,p\gamma}^*,\end{aligned} \tag{29}$$

where $D_{N\pi}$ is the two-particle (nucleon-pion) density of states and the sum still contains an angular integration as well as a sum over energy and spin. Since the initial and final states are eigenstates of the total angular momentum, the R matrix is symmetric and the left-hand side of (29) is proportional to the real part of the R matrix.

From a perturbation theory point of view, the left-hand side of (29) is equal to the contribution of the poles calculated in the Appendix. The right-hand side, which corresponds to the succession of the two real processes of hyperon decay by emission of a pion and of the inverse of pion photoproduction, is the product of the R matrices for each process, calculated by perturbation theory to lowest order, and summed over the available states with the appropriate density function, $D_{N\pi}$.

Now, according to the Kroll-Ruderman theorem,[10] meson photoproduction near threshold is correctly predicted by perturbation theory. In addition, the weak-coupling pion mode of hyperon decay is also given correctly by perturbation theory, provided, in analogy to meson photoproduction, the renormalized decay coupling constant is used. But (29) is an S-matrix element between real (i.e., clothed) baryons. Therefore, it is to be expected that near the threshold for the photon mode of hyperon decay, the real part of the S-matrix element is given correctly, to all orders in the meson coupling constant, by the contribution of the poles in the weak-coupling theory, provided the renormalized coupling constants are used. Apparently, no such statement can be made concerning the imaginary part of the S-matrix element.

III. PERTURBATION ESTIMATES

Although a perturbation calculation involving the strong mesonic coupling constant usually cannot be expected to give quantitatively correct results, it is

[9] If the technique of Feynman is used [R. P. Feynman, Phys. Rev. **76**, 769 (1949), Appendix D], an "equivalence" can be demonstrated between the vector-pseudovector matrix elements, arising from (26), and the scalar-pseudoscalar matrix elements, arising from (13), which holds in lowest order perturbation theory. This "equivalence" is $(fa)^2=(hc)^2(m_1-m_2)^2$; $(fb)^2=(hd)^2 \times(m_1+m_2)^2$. In determining the branching ratio (28), this "equivalence" was assumed to hold to all orders in the strong-coupling constant.

[10] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

TABLE I. Branching ratios (photon mode to neutral pion mode) for scalar-pseudoscalar weak coupling and pseudoscalar strong coupling for various values of the real parts of the parameters and for the perturbation value of the imaginary part.

Hyperon	Re $A'=0$ Re $B'=0$	Re $A'=$Im A' Re $B'=$Im B'	Re A' and Re B' given by perturbation theory
Λ^0	$\sim 1/850$	$\sim 1/425$	$\sim 1/21$
Σ^+ [case (1)]	$\sim 1/2500$	$\sim 1/1250$	$\sim 1/670$
Σ^+ [case (2)]	$\sim 1/1100$	$\sim 1/550$	$\sim 1/66$

sometimes profitable to examine the properties predicted by such a calculation. This is done in the Appendix.

Table I lists the values of the branching ratio as predicted by perturbation theory (see the Appendix). Since the unitarity condition shows that the real part of the S-matrix element [ImA' and ImB'] is given by perturbation theory, the value calculated for this quantity in the Appendix is used throughout the table. On the other hand, the imaginary part of the S-matrix element is most probably incorrectly predicted by perturbation theory. Therefore the branching ratios are listed for several different values of this quantity.

In Table II, the branching ratios are listed for the different combinations of weak and strong coupling, *i.e.*, scalar-pseudoscalar weak coupling with pseudoscalar strong coupling, etc., by using both the real and imaginary parts of the S-matrix element as calculated from perturbation theory. The branching ratio for the Λ is unchanged by the different couplings so that only the Σ branching ratios are given.

In both tables, two cases for the Σ are listed. This arises from an ambiguity in the relative phases of the couplings for $\Sigma^+ \rightarrow n+\pi^+$ and $\Sigma^+ \rightarrow p+\pi^0$. Case (1) corresponds to a relative phase difference of zero and case (2) to a difference of π. (See the Appendix for further discussion.)

TABLE II. Branching ratios from perturbation theory for various combinations of weak and strong couplings.

Hyperon \ Weak-strong coupling	$SP-P$	$SP-A$	$VA-P$	$VA-A$
Σ^+ [case (1)]	$\sim 1/670$	$\sim 1/460$	$\sim 1/530$	$\sim 1/370$
Σ^+ [case (2)]	$\sim 1/66$	$\sim 1/57$	$\sim 1/320$	$\sim 1/430$

Although the branching ratios for these decays cannot be unambiguously predicted, their existence is to be expected. It is hoped that with the increasing numbers of hyperons available, this experimental branching ratio can be determined.

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APPENDIX. PERTURBATION CALCULATION

It will be assumed in this section that the weak interaction which is indirectly responsible for the gamma-decay process is the pion mode of decay of the hyperon being considered and that its interaction is of the form $ia+b\gamma_5$. The strong coupling of the pion to the baryon will be assumed to be of the pseudoscalar symmetric type. This Lagrangian has been given in (13).

It will be convenient to discuss initially the decay of a neutral hyperon. The pertinent Feynman diagrams are shown in Figs. 1 and 2, and the corresponding matrix elements in momentum representation are

$$M^{1a}=\frac{g_2 fe}{16\pi^4 i}\bar{u}_2\left\{\int\frac{d^4q\,\gamma_5(\boldsymbol{p}_2-\boldsymbol{q}+m_2)\boldsymbol{e}(\boldsymbol{p}_1-\boldsymbol{q}+m_2)(ia+b\gamma_5)}{(q^2-\mu^2)[(p_2-q)^2-m_2^2][(p_1-q)^2-m_2^2]}\right\}u_1, \tag{A.1}$$

$$M^{2a}=\frac{g_1 fe}{16\pi^4 i}\bar{u}_2\left\{\int\frac{d^4q\,(ia+b\gamma_5)(\boldsymbol{p}_2-\boldsymbol{q}+m_1)\boldsymbol{e}(\boldsymbol{p}_1-\boldsymbol{q}+m_1)\gamma_5}{(q^2-\mu^2)[(p_2-q)^2-m_1^2][(p_1-q)^2-m_1^2]}\right\}u_1, \tag{A.2}$$

$$M^{1b}=-\frac{g_2 fe}{16\pi^4 i}\bar{u}_2\left\{\int\frac{d^4p\,\gamma_5(\boldsymbol{p}+m_2)(ia+b\gamma_5)(p_1+p_2-2p)\cdot e}{(p^2-m_2^2)[(p_1-p)^2-\mu^2][(p_2-p)^2-\mu^2]}\right\}u_1, \tag{A.3}$$

$$M^{2b}=-\frac{g_1 fe}{16\pi^4 i}\bar{u}_2\left\{\int\frac{d^4p\,(ia+b\gamma_5)(\boldsymbol{p}+m_1)\gamma_5(p_1+p_2-2p)\cdot e}{(p^2-m_1^2)[(p_1-p)^2-\mu^2][(p_2-p)^2-\mu^2]}\right\}u_1, \tag{A.4}$$

where q and μ are the four-momentum and rest mass, respectively, of the virtual π meson.

It is now profitable to notice that under the simultaneous operation of C and interchange of the indices 1 and 2, $M^{1a}\leftrightarrow M^{2a}$ and $M^{1b}\leftrightarrow M^{2b}$. Therefore, it is necessary to calculate explicitly only two of the four matrix elements, the others being obtained by the substitution $1\leftrightarrow 2$. In the following, the matrix elements (A.1) and (A.3) will be calculated explicitly.

Since the problem being considered involves a free

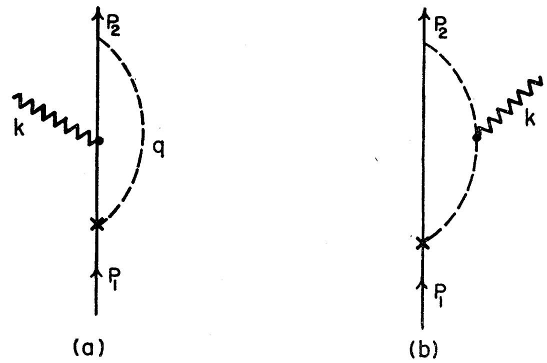

FIG. 1. Lowest order Feynnman diagram leading to the photon mode of decay in which the weak interaction occurs first.

photon, $k^2=0$, the matrix elements, (A.1) and (A.3), may be put into the following form:

$$M^{1a,1b}=\bar{u}_2[(A_0{}^{a,b}+iB_0{}^{a,b}\gamma_5)\mathbf{e}+(A^{a,b}+iB^{a,b}\gamma_5)\mathbf{ke}]u_1. \quad \text{(A.5)}$$

As seen in Sec. I, gauge invariance requires that $A_0{}^a+A_0{}^b$ and $B_0{}^a+B_0{}^b$ must each be identically zero. This may also be checked explicitly in this case by calculating the coefficients of $\mathbf{e}$ from (A.1)–(A.4).

The coefficients $A^{a,b}$ and $B^{a,b}$ are

$$\begin{aligned}
A^a&=\frac{g_2fe}{16\pi^2m_2}\int_0^1dx\int_0^1dy\,\frac{(e^\omega-1)x^2(1-x)y-x^3}{\Delta_a},\\
B^a&=\frac{g_2fe}{16\pi^2m_2}\int_0^1dx\int_0^1dy\,\frac{(e^\omega+1)x^2(1-x)y+x^3}{\Delta_a},\\
A^b&=\frac{g_2fe}{16\pi^2m_2}\int_0^1dx\int_0^1dy\,\frac{(e^\omega-1)x^2(1-x)y-x(1-x)^2}{\Delta_b},\\
B^b&=\frac{g_2fe}{16\pi^2m_2}\int_0^1dx\int_0^1dy\,\frac{(e^\omega+1)x^2(1-x)y+x(1-x)^2}{\Delta_b},
\end{aligned} \quad \text{(A.6)}$$

and

$$\Delta_a=x^2-(e^{2\omega}-1)x(1-x)y+\eta(1-x),$$
$$\Delta_b=(1-x)^2-(e^{2\omega}-1)x(1-x)y+\eta x,$$

where η is defined as

$$\eta=\mu^2/m_2{}^2. \quad \text{(A.7)}$$

Under the transformation $x\to1-x$, there results $\Delta_a\leftrightarrow\Delta_b$; therefore, only one denominator need be considered. Thus

$$A^{a,b}=\frac{bg_2fe}{16\pi^2m_2}\int_0^1dx\int_0^1dy\,\frac{[(e^\omega-1)x(1-x)y-x^2][x;\,(1-x)]}{x^2-(e^{2\omega}-1)x(1-x)y+\eta(1-x)}, \quad \text{(A.8)}$$

where the set $B^{a,b}$ may be obtained from the set $A^{a,b}$ by the transformation (19), *i.e.*, (20) becomes $A^{a,b}\to-B^{a,b}$. (The quantity to the left of the semicolon is to be associated with the A^a while that to the right with A^b.)

If (A.8) is integrated over y, $A^{a,b}$ becomes

$$A^{a,b}=\frac{bg_2fe}{16\pi^2m_2}\int_0^1\frac{dx[x;\,(1-x)]}{e^\omega+1}\left\{-1+\left[\frac{e^\omega x^2-\eta(1-x)}{(e^{2\omega}-1)x(1-x)}\right]\ln\left[\frac{e^{2\omega}x^2-(e^{2\omega}-1)x+\eta(1-x)}{x^2+\eta(1-x)}\right]\right\}. \quad \text{(A.9)}$$

If this expression is integrated by parts, then

$$A^a=\frac{bg_2fe}{16\pi^2m_2}\int_0^1\frac{dx}{e^\omega+1}\left\{-x+\frac{[\frac{1}{2}e^\omega x^2+(e^\omega+\eta)x+e^\omega\ln(1-x)](e^{2\omega}-1)[x^2-\eta(1-x)^2]}{[e^{2\omega}x^2-(e^{2\omega}-1)+\eta(1-x)][x^2+\eta(1-x)]}\right\}. \quad \text{(A.10)}$$

It should be noted that if

$$(e^\omega-1)^2\geqslant\eta, \quad [\text{i.e., } (m_1-m_2)^2\geqslant\mu^2], \quad \text{(A.11)}$$

the second terms in (A.10) have simple poles. These values of ω are such that the mass difference between the initial and final baryons is greater than a real pion mass. In this case, the poles then correspond to the emission and reabsorption of a real meson. As was seen in Sec. II, this corresponds to the successive real processes of baryon decay into a pion followed by the inverse of pion photoproduction. It is the contribution from these poles that gives rise to terms [the last terms in Eq. (8)] which have the *appearance* of violating time-reversal invariance. The same statements apply to the functions $B^{a,b}$. However, as was mentioned just before Eq. (A.5), only the terms corresponding to the diagrams of Fig. 1 have been considered so far (they correspond to the emission of the pion by the weak decay interaction). In order to find the contribution from the diagrams of Fig. 2, it is only necessary to substitute $e^\omega\to e^{-\omega}$, $g_1\leftrightarrow g_2$, $\eta\to\eta e^{-2\omega}$. As may be readily seen in this case, the second terms in the equations corresponding to (A.10) do not have poles. Thus, only the diagrams of Fig. 1 can contribute terms which give the appearance of violating time-reversal invariance. It should be noticed that in the limit as $\omega\to0$, A^a and A^b go into the functions B_1 and B_2 which were introduced by Bethe and de Hoffmann[11]

[11] H. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, 1955), Vol. II, p. 291.

(based on the work of Fried[12]) in order to represent the anomalous magnetic moment of the nucleon.

Upon evaluation of the integrals (A.10) it is possible to calculate the complete matrix element for each of the pertinent decays. For simplicity, all the weak pion-baryon coupling constants will be taken to have equal magnitudes. However, one ambiguous point still remains. This concerns the relative phase of the weak coupling of charged and neutral pions to the Σ. If it is assumed that time-reversal invariance is valid for the pion modes of Σ^+-decay, then this relative phase must be either zero or π (neglecting final-state interactions). There also exists the question as to the relative phases of the weak pion coupling to different baryons. For simplicity, this phase will be assumed to be zero. As a result, two limiting cases will be considered for the Σ: the relative phase is (1) zero, and (2) π. It will be assumed that global symmetry[13] is valid. As an example, decay (1′) proceeds in the following manner. (Note that if the $\Lambda-\Sigma$ mass difference is neglected, the intermediate Λ^0 and Σ^0 states cancel because of global symmetry.)

$$\begin{aligned}&\Sigma^+\to p+\pi^0\to p+\pi^0+\gamma\to p+\gamma,\\&\Sigma^+\to n+\pi^+\to n+\pi^++\gamma\to p+\gamma,\\&\Sigma^+\to\Sigma^++\pi^0\to\Sigma^++\pi^0+\gamma\to p+\gamma.\end{aligned}\tag{A.12}$$

The parameters in the matrix element (4) are then [assuming the branching ratio $(\Sigma^+\to p)/(\Sigma^+\to n)=1$]

$$\begin{aligned}A&=A^a-\sqrt{2}A^b+A^{2a},\\B&=B^a-\sqrt{2}B^b+B^{2a}.\end{aligned}\tag{A.13}$$

In case (2), they are

$$\begin{aligned}A&=A^a+\sqrt{2}A^b+A^{2a},\\B&=B^a+\sqrt{2}B^b+B^{2a}.\end{aligned}\tag{A.13$'$}$$

In these equations A^{2a} and B^{2a} are obtained from A^a and B^a, respectively, by interchanging the indices 1 and 2. The parameters for the decay (1″) are

$$\begin{aligned}A&=\sqrt{2}(A^a+A^b),\\B&=\sqrt{2}(B^a+B^b).\end{aligned}\tag{A.14}$$

[12] B. D. Fried, Phys. Rev. **88**, 1142 (1952).
[13] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

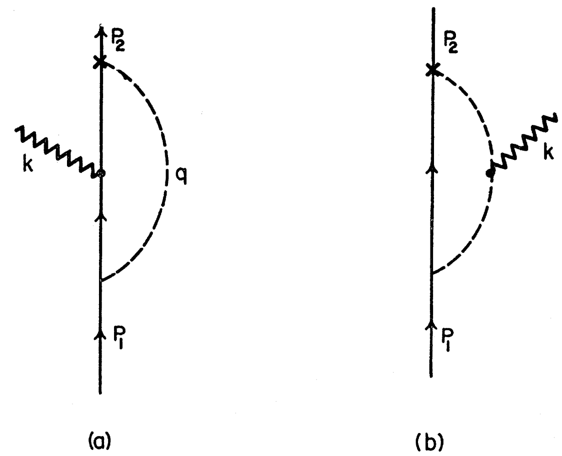

FIG. 2. Lowest order Feynman diagram leading to the photon mode of decay in which the strong interaction occurs first.

The numerical evaluation, assuming $m_{\Sigma^+}=2327m_e$, $m_{\Lambda^0}=2180m_e$, is for decay (1′), case (1)

$$\begin{aligned}A&=\frac{bgfe}{16\pi^2(m_1+m_2)}(-1.00-0.17i),\\B&=\frac{agfe}{16\pi^2(m_1-m_2)}(+0.27+0.71i);\end{aligned}\tag{A.15}$$

for decay (1′), case (2), it is

$$\begin{aligned}A&=\frac{bgfe}{16\pi^2(m_1+m_2)}(-3.62-0.36i),\\B&=\frac{agfe}{16\pi^2(m_1-m_2)}(0.86+1.04i),\end{aligned}\tag{A.16}$$

and for decay (1″), it is

$$\begin{aligned}A&=\frac{bg_2fe}{16\pi^2(m_1+m_2)}(-2.87+0.10i),\\B&=\frac{ag_2fe}{16\pi^2(m_1-m_2)}(+0.89-0.73i).\end{aligned}\tag{A.17}$$