Theory of Reactions $H^3(d,n)He^4$ and $He^3(d,p)He^4$*†

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The reactions $H^3(d,n)He^4$ and $He^3(d,p)He^4$ are fitted by using one-level and one-level-plus-background dispersion formulas and taking into account the implications of the principle of charge symmetry of nuclear forces. Fits are found by using a variety of penetration parameters and channel radii. The schematic treatment of Breit and the $\mathfrak{R}$-matrix formalism of Wigner and Eisenbud are used to obtain the one-level-plus-background formulas and fits made by using them are compared with those obtained by previous workers using one-level formulas. The connection of these reaction fits with recent $n+He^4$ scattering data is discussed.

1. INTRODUCTION

THE reactions $H^3(d,n)He^4$ and $He^3(d,p)He^4$ show resonances at bombarding energies of 0.110 Mev and 0.425 Mev, respectively, and provide information on the comparison of reactions involving mirror nuclei. While there are a number of tests of the charge symmetry of nuclear forces in the Coulomb energy of mirror nuclei, the information available on the similarity of reaction parameters for mirror nuclei is much less abundant. It appeared of interest therefore to examine such fits as could be made to the two reactions by dispersion-type formulas in order to see whether allowance for barrier penetration will result in evidence for similarity of the processes involved in the nuclear interior. The form of the barrier penetration factor used matters in the comparisons and it appeared of interest, therefore, to make it employing these factors in the form in which they arose in the work of Ostrofsky, Breit, and Johnson,[1] Konopinski and Bethe,[2] Kapur and Peierls,[3] Breit,[4,5] and Wigner and Eisenbud.[6] These workers have found that different combinations of the regular and irregular radial wave functions could be used to describe the barrier penetration effects. These functions are usually combined with other quantities referring to the external or channel region to form a quantity termed the penetration parameter. The disintegration probabilities and partial level widths which occur in a one-level dispersion form such as that obtained by Breit and Wigner[7] are usually assumed to be a product of the penetration parameter and a second factor which may be called the inherent channel width. The latter quantity is supposed to depend upon the properties of the compound nucleus and to be insensitive to properties of the channels such as the bombarding energy. Breit[5] has shown that with some interpretations of a dispersion formula representation of data, the factorization of the partial widths into the two factors is not generally valid unless some simplifying assumptions concerning the nuclear interactions are made. For this reason some caution would seem to be desirable when the results of fitting a nuclear resonance to a one-level formula with penetration parameters are interpreted. Furthermore, the single-level formula is not expected to be accurate and the many-level formulas involve the employment of an infinite number of levels with parameters which are not readily accessible to experimental determination. For these reasons one may expect that from dispersion-type formulas, fits having no physical reality can be secured, and that the possibility of a test of the fits employing the assumption of approximate charge symmetry[8] can be helpful in ruling out the spurious ones. The study reported provides also some information regarding the practical applicability of different types of resonance formulas and of barrier penetration factors. It is not exhaustive because, in some of the forms used,[5] the Coulomb function must be evaluated at a distance at which the interaction takes place. For simplicity this distance was taken to be at the nuclear surface and the information regarding the effect of using smaller distances is not complete.

Although some of the formulas used employ a schematic representation of the reaction, the lack of complete realism does not matter for the charge symmetry tests and the exact meaning of the replacement of the complicated many-body wave function by the simpler two-body function does not matter for the present work.

The total cross sections of the reactions $H^3(d,n)He^4$

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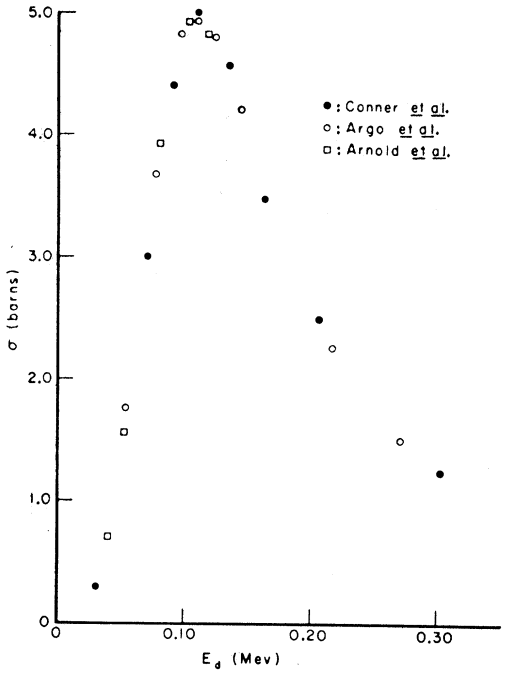

FIG. 1. The experimental reaction cross section for $H^3(d,n)He^4$ as a function of the laboratory energy of the incident deuteron. Points are shown for three different groups of workers.

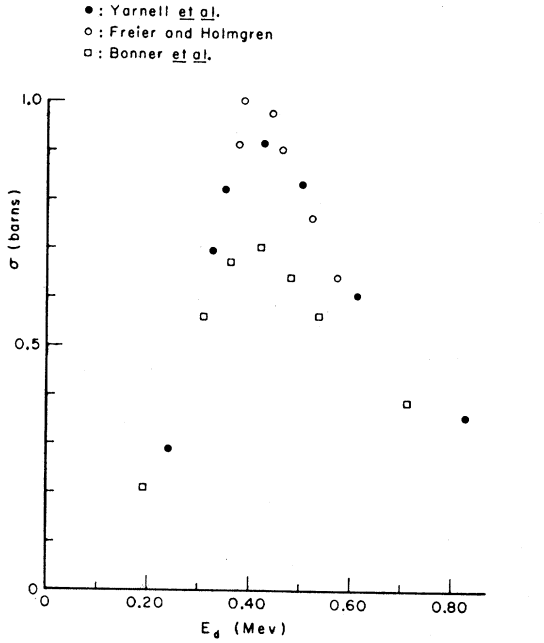

FIG. 2. The experimental reaction cross section for $He^3(d,p)He^4$ as a function of the laboratory energy of the incident deuteron. Points are shown for three different groups of workers. Near the maximum cross section the upper and lower sets of data differ by more than the estimated experimental uncertainties. The arithmetic mean of the two sets of points will be fitted.

and $He^3(d,p)He^4$ have been measured by several groups of workers[9–19] and are known to exhibit pronounced resonances near a bombarding energy of 0.110 Mev and 0.425 Mev, for the two reactions. The compound nuclei involved, He^5 and Li^5, are mirror nuclei so that apart from relatively small Coulomb effects one may expect the internal nuclear processes to be very similar. These nuclei may be supposed to possess well-separated levels because of their low mass number. Angular distribution measurements performed on the reaction neutrons and protons[12,14] have shown that they are distributed isotropically. It appears probable therefore that the part of the incident deuteron beam which is effective in causing a reaction has an orbital angular momentum of zero relative to the target. Some of the workers who have measured the cross sections have also fitted the reactions[12–14,18,19] and they found it necessary to assume that the compound state has an angular momentum of $\frac{3}{2}\hbar$. Conservation of parity and angular momentum then require that the relative orbital angular momentum in the exit channels of both reactions have the value $2\hbar$. In the present paper the values $L=0$ and 2 are therefore used for the initial and final channels, respectively.

Some typical experimental points are shown in Figs. 1 and 2, where it may be seen that the results on $H^3(d,n)He^4$ are in fair agreement with each other. The different sets of data on $He^3(d,p)He^4$ are in disagreement, significantly so near the peak. Since there was no reason for questioning the accuracy of any one set of data, the arithmetic mean of the cross sections given by the higher and lower sets was generally used for fitting purposes. All of the publications reported an experimental uncertainty of at least 8% in the cross sections of both reactions, and such an uncertainty in the mean value of the $He^3(d,p)He^4$ data around the peak would allow for either group of data being correct.

It seemed of interest to see if it was possible to fit these reactions with several different formalisms taking into account the restrictions on inherent widths in corresponding channels imposed by the principle of charge symmetry. The second part of the present paper presents fits obtained using the one-level formula of Breit and Wigner[7] with several forms of penetration parameter in the partial widths. In this part are also presented fits using a one-level formula obtained from the $\mathfrak{R}$-matrix formalism of Wigner and collabora-

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TABLE I. Fitting parameters and inherent widths for the form $k/(F_L^2+G_L^2)$.

(a) The fitting parameters obtained using a penetration parameter $k_i/(F_L^2+G_L^2)$ in both channels of each reaction.

	$b_1=5.0\times10^{-13}$ cm		$b_1=4.0\times10^{-13}$ cm	
	H^3	He^3	H^3	He^3
$\mathcal{G}_1$ (10^{-12} Mev cm)	0.20	0.20	0.23	0.23
Γ_2 (Mev)	0.030	0.030	0.030	0.030
E_0 (Mev)	0.080	0.290	0.080	0.290

(b) The inherent width $\mathcal{G}_2=[(F_L^2+G_L^2)/k]_{i=2}\Gamma_2$ in the exit channels at several channel radii.

b_2 (10^{-13} cm)	2.82		3.20		3.54	
	H^3	He^3	H^3	He^3	H^3	He^3
$\mathcal{G}_2$ (10^{-12} Mev cm)	0.0068	0.0068	0.0059	0.0059	0.0054	0.0053

tors.[6,20,21] The third part gives fits employing one-level-plus-background formulas obtained from the schematic treatment of Breit[5] and by means of the $\mathcal{R}$ matrix. In the latter section the effect of allowing the background to vary with the bombarding energy is examined. Finally, a discussion is given of the present fits and a comparison is made with fits obtained by previous workers.

2. FITS WITH ONE LEVEL FORMULAS

A. Fits with the Breit-Wigner Formula

The one-level formula obtained by Breit and Wigner[7] for the cross section of a nuclear reaction near an isolated resonance is

$$\sigma(i\rightarrow f)=(S\Lambda_i^2/\pi)\frac{\Gamma_i\Gamma_f}{(E_i-E_0)^2+(\Gamma_i+\Gamma_f)^2}. \quad (2.1)$$

The letters i and f denote the initial and final channels, Γ_i is the partial width, S is the usual statistical factor, Λ_i is the wavelength of relative motion in the initial channel, and E_i is the center-of-mass energy in the same channel. The resonance energy E_0 is used as a constant which is taken to be nearly equal to the experimental energy at which the maximum cross section occurs. The partial widths will be taken to be the product of an inherent channel width and a penetration parameter appropriate to the channel involved. In each case the inherent width is defined to be the factor by which the penetration parameters enumerated in the next sentence must be multiplied in order to give Γ. Fits have been attempted with the following penetration parameters: $k/(F_L^2+G_L^2)$, f_L^2/k, k/G_L^2, and F_L^2/k. The functions F_L and G_L are the regular and irregular solutions of the Schrödinger equation in the region beyond the range of the nuclear forces. If a Coulomb barrier is present, they are the functions tabulated by Bloch, Hull, Broyles, Bouricius, Freeman, and Breit.[22] The function f_L is the regular solution of the radial Schrödinger equation in the external region where an effective nuclear interaction at small distances has been used. In terms of F_L and G_L, the function f_L may be written

$$f_L=F_L\cos K_L+G_L\sin K_L. \quad (2.2)$$

The phase shift K_L produced by an average nuclear interaction may be calculated from a one-body potential model by the formula

$$\tan K_L=(F_L{}^iF_L{}'-F_L{}^{i'}F_L)/(F_L{}^{i'}G_L-F_L{}^iG_L'), \quad (r=b), \quad (2.3)$$

with $F_L{}^i$ denoting the regular solution of the radial Schrödinger equation in the interior of the assumed potential well and primes indicating differentiation with respect to kr. The radius of the channel involved is b, $k/2\pi=1/\Lambda$ is the wave number, and L is the relative orbital angular momentum expressed in terms of $\hbar$ as a unit.

The penetration parameter $k/(F_L^2+G_L^2)$ occurred first in the work of Breit and Johnson,[1] who used a complex potential well with strong absorption, and it appears under more general circumstances in the work of Wigner and Eisenbud[6] on the $\mathcal{R}$ matrix. Breit[5] has shown that f_L^2/k is the appropriate form for a strongly localized nuclear interaction with f_L evaluated at the distance r at which the interaction is localized, and he also obtained the form k/G_L^2 for a strong repulsive interaction in a large region. The form F_L^2/k is obtained[23] from a one-body specialization of a one-channel model with a second channel introduced to give a small cross section. In these two forms the value of the interparticle distance r is supposed to be that corresponding to the location of the interaction rather than the usual nuclear radius. For channel radii at which F_L^2 is small compared to G_L^2, the quantities $k/(F_L^2+G_L^2)$ and k/G_L^2 will be similar functions of energy; at other radii they may be quite different. The functions k/G_L^2 and K_L^2/k will usually have similar energy dependences because the barrier effects largely compensate in the combination F_LG_L/k.

The radii in the initial channels of both reactions were taken to be either 4.0×10^{-13} cm or 5.0×10^{-13} cm. The former value corresponds to taking the constant b_0 in the formula $b=b_0(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ to be 1.45×10^{-13} cm while the latter value is made larger in an attempt to allow for the relatively large size of the deuteron caused by its small separation energy. In the final channels three radii were tried: (2.82, 3.20, and 3.54)$\times10^{-13}$ cm, the latter value corresponding to a value of $b_0=1.40\times10^{-13}$ cm and the smaller values being used in order to show the effect of varying the channel radius. In their work on the scattering of protons by alpha particles,

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TABLE II. Fitting parameters and inherent widths for the form f_L^2/k.

(a) The fitting parameters obtained using a penetration parameter $[f_L^2/k_i]_{r_i=b_i}$ in both channels for each reaction. The values of $\mathcal{G}_{f1}=(k_i/f_L^2)_{i=1}\Gamma_1$ in parentheses were found using values of V_1 in parentheses to calculate f_0.

	$b_1=5.0\times10^{-13}$ cm		$b_1=4.0\times10^{-13}$ cm	
	H^3	He^3	H^3	He^3
$\mathcal{G}_{f1}$ (10^{12} Mev cm^{-1})	0.82 (0.52)	0.84 (0.56)	1.6 (1.2)	1.7 (1.3)
Γ_2 (Mev)	0.013	0.014	0.013	0.014
E_0 (Mev)	0.066	0.255	0.066	0.255
V_1 (Mev)	−0.51 (−0.96)	−0.06 (−0.48)	−0.51 (−0.96)	−0.06 (−0.48)

(b) The quantity $\mathcal{G}_{f2}=(k_2/f_L^2)_{i=2}\Gamma_2$ at several exit channel radii.

b_2 (10^{-13} cm)	V_2 (Mev)		$\mathcal{G}_{f2}$ (10^{12} Mev cm^{-1})	
	H^3	He^3	H^3	He^3
2.82	−16.5	−14.0	0.18	0.21
	−22.1	−19.6	0.14	0.16
	−27.5	−25.0	0.10	0.12
3.20	−16.5	−14.0	0.092	0.11
	−22.1	−19.6	0.074	0.081
	−27.5	−25.0	0.071	0.079
3.54	−16.5	−14.0	0.076	0.079
	−22.1	−19.6	0.10	0.090
	−27.5	−25.0	0.18	0.15

Sack, Biedenharn, and Breit[24] were able to use a nuclear radius of 3.2×10^{-13} cm successfully, but the present compound nuclei are in a $^2D_{\frac{3}{2}}$ state which might possess a different radius than did their $^2P_{\frac{1}{2},\frac{3}{2}}$ states. The penetration parameters $k/(F_L^2+G_L^2)$, k/G_L^2 and F_L^2/k were calculated as functions of energy for each channel at the stated radii.

Such wells are intended to represent schematically conditions in the nuclear interior. They are applied as though the channel fragments were subject to the assumed potential, the justification for this procedure being found in the papers already quoted.[4,5,7] The principle of charge symmetry implies that the only difference between the two reactions is caused by the extra charge in the case of $He^3(d,p)He^4$. This charge causes additional Coulomb interactions in both the initial and final channels. It also may be expected to change some of the characteristics of the compound nucleus but these effects will not be considered here. A method of estimating this effect for the shallow wells which it was desired to use in the initial channels was the following.[25] The function $F_L{}^i$ in the $d+He^3$ channel was required to have the same homogeneous logarithmic derivative at the channel radius when in the field of the H^3 potential well plus a Coulomb potential due to a single charge as when in the field of a potential well alone. The value of the latter which was found to satisfy this requirement was the well depth used in the He^3 case. Since it seemed reasonable to use deeper wells in the final channels, the one-term perturbation formula of Breit, Condon, and Present[25] and Breit, Thaxton, and Eisenbud[26] was applicable because in this case the Coulomb correction is small. In both initial and final channels the well ranges were taken to be the same as the before-mentioned channel radii.

Functions having reference to the final channels are insensitive to the variation of the bombarding energy because of the high Q values of the reactions under consideration, the values being $Q[H^3(d,n)He^4]=17.58$ Mev, $Q[He^3(d,p)He^4]=18.34$ Mev. This situation also causes the final channel functions to have very nearly equal values. For example, at a radius of 3.2×10^{-13} cm, the quantity $k/(F_L^2+G_L^2)$ has the value 5.07×10^{12} cm^{-1} in the neutron channel and 5.12×10^{12} cm^{-1} in the proton channel. This near equality of the penetration parameters plus the near equality of the inherent transition probabilities due to charge symmetry implies that the fits must have approximately equal values of Γ_f.

It was found to be impossible to obtain fits with the penetration parameter F_L^2/k which would satisfy charge symmetry. It may be that the specialization of f_L to F_L is too poor an approximation since the type f_L^2/k did allow fits. Since F_L^2 is small compared to G_L^2 in the initial channel, the fits obtained with k/G_L^2 are not given. The parameters are very nearly equal to those found using $k/(F_L^2+G_L^2)$. The fitting parameters using the penetration forms $k/(F_L^2+G_L^2)$ and f_L^2/k and giving satisfactory agreement with experiment are given in Tables I and II.

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B. One-Level Fits in the $\mathfrak{R}$-Matrix Formalism[27]

In the $\mathfrak{R}$-matrix formalism of Wigner and collaborators[6,20,21] the wave function in the interior of the compound nucleus is expanded in terms of a complete set of functions X_λ having proper energies E_λ. The boundary condition at the surface of the compound nucleus is used here in the form

$$\partial(r_p X_\lambda)/X_\lambda \partial r_p = l_p, \quad (r_p = b_p), \tag{2.4}$$

with p denoting any channel. The reduced partial widths are defined by

$$\gamma_{\lambda p}^2 = [(2/\hbar)^{\frac{1}{2}}(\hbar^2/2\mu_p)(X_\lambda, \Psi_p{}^{Jc})]^2, \tag{2.5}$$

where $\hbar$ is Planck's constant divided by 2π, μ_p is the reduced mass in channel p and $\Psi_p{}^{Jc}$ is the corresponding channel function representing a standing wave subject to the boundary conditions of the theory. The superscript c indicates its generic resemblance to a cosine type wave which represents $\Psi_p{}^{Jc}$ in the slow neutron case. The $\mathfrak{R}$ matrix has the elements

$$\mathfrak{R}_{pq} = \sum_\lambda \gamma_{\lambda p}\gamma_{\lambda q}/(E_\lambda - E), \tag{2.6}$$

the sum extending over all of the states of the compound nucleus. If only one level in the compound nucleus is assumed to be effective, then it proves possible to write the cross section for a two-channel reaction as

$$\pi\sigma(i\to f)/\Lambda_i^2 S = \frac{\Gamma_i \Gamma_f}{(E_\lambda - E + \Delta_\lambda)^2 + (\Gamma_i + \Gamma_f)^2}, \tag{2.7}$$

where the customary symbols have their usual definition and Δ_λ is the energy level shift. The partial widths and the level shift are defined as

$$\begin{aligned} &\Gamma_p = \gamma_{\lambda p}^2 B_p^2/(1+\mathfrak{C}_p^2), \\ &\Delta_\lambda = \sum_{p=i}^{f} \Delta_{\lambda p}, \quad \Delta_{\lambda p} = \mathfrak{C}_p \Gamma_p. \end{aligned} \tag{2.8}$$

The important functions $\mathfrak{C}_p^2$ and B_p^2 involve the boundary condition, Eq. (2.4), explicitly and in this work l_p will be taken to be

$$l_p = [\rho_p G_L'/G_L]_{E_i = E_0}, \quad (r_p = b_p). \tag{2.9}$$

With this condition $\mathfrak{C}_p^2$ and B_p^2 are given by

$$\begin{aligned} \mathfrak{C}_p = &[(\rho_p G_L'/G_L)_{E_0} - \rho_p G_L'/G_L] \\ &+ (F_L^2 + G_L^2)/\rho_p - F_L/G_L, \\ B_p^2 = k_p | i/G_L - [(\rho_p G_L'/G_L)_{E_0} - \rho_p G_L'/G_L] \\ &\times (G_L + iF_L)/\rho_p |^2, \end{aligned} \tag{2.10}$$

which hold at $r_p = b_p$. The energy E_0 may be any energy but was kept near the energy of the maximum cross section in the present work. It may be noted that if the quantity $(\rho_p G_L'/F_L)_{E_0} - \rho_p G_L'/G_L$ is considered to be zero over the resonance, then

$$\begin{aligned} &\Gamma_p = \gamma_{\lambda p}^2 k_p/(F_L^2 + G_L^2), \\ &\Delta_{\lambda p} = -(F_L/G_L)\Gamma_p. \end{aligned} \tag{2.11}$$

Calculation has shown that for the present work Eqs. (2.11) would be a poor approximation to Eqs. (2.10) and the latter were those used in the fitting.

A radius of 5.0×10^{-13} cm was used in the initial channel of both reactions and a value of 3.2×10^{-13} cm was tried in the final channels. The quantities $\gamma_{\lambda p}^2$ are descriptive of the compound state and only fits which yielded similar reduced widths in corresponding channels of the two reactions were retained. The appropriate parameters are given in Table III.

TABLE III. The fitting parameters obtained using the one-level formula from the $\mathfrak{R}$-matrix formalism. The radius in the entrance channels was 5.0×10^{-13} cm and in the exit channels was 3.2×10^{-13} cm. As a boundary condition, the interior wave function was joined to the irregular channel function at the channel radius.

	H^3	He^3
$\gamma_{\lambda 1}^2$ (10^{-12} Mev cm)	0.100	0.100
$\gamma_{\lambda 2}^2$ (10^{-12} Mev cm)	0.014	0.015
E_λ (Mev)	0.172	0.205

[27] The authors are grateful to Professor G. Breit for permitting them to use the formulas given in Eq. (2.10) of this section and in Eqs. (3.7) through (3.9) of Part III, Sec. 2 in advance of their publication in the *Handbuch der Physik*.

3. FITS USING ONE-LEVEL-PLUS-BACKGROUND FORMULAS

A. Fits with the Schematic Treatment

The schematic treatment of Breit[5] deals with a nuclear reaction in a manner analogous to that used in treating coupled vibrating systems. A set of coupled equations is introduced and solved by means of the Green's function technique. The radial wave function describing the residual nucleus is expanded in terms of a complete set of eigenfunctions $u_n(r)$ which satisfy the Schrödinger equation in the absence of any interaction between the residual nucleus and the channel particle. In the channel region, functions $f_p(r)$ and $g_p(r)$ are introduced and an asymptotic solution of the form $f_p\delta_{pq} + A_q(g_q + if_q)$ is sought in the channel q. The function f_p is the regular solution of the radial equation of the pth channel in the presence of the potential which represents schematically the average action of the nucleus on the particle emitted in that channel. It is normalized so as to be a sine function of unit amplitude at infinite distance and g_p is a solution of the same differential equation as f_p, has unit amplitude at infinity and is the cosine of the phase of which f_p is the sine. The reaction cross section for transfer from channel p to channel q may be expressed in terms of the quantity A_q as

$$\pi\sigma(i\to f)/\Lambda_i^2 S = (w_i/w_f)(A_f)^2, \tag{3.1}$$

where i and f have been used in place of p and q to denote the initial and final channels, respectively, $w_i=2/\hbar v_i$, v_i= relative velocity. The remaining factors have their usual meaning.

Breit then considered the case of an isolated level of the compound nucleus and derived a formula for the cross section which contains a dispersion-type term plus a slowly varying background. Some effects of the background are actually contained in the dispersion type term but if the approximation is made of considering the entire background effect to be expressed by the slowly varying term, his formula may be written

$$\pi\sigma(i\rightarrow f)/\Lambda_i^2 S=\left|\frac{\Gamma_i^{\frac{1}{2}}\Gamma_f^{\frac{1}{2}}}{(E_0-E_i)-i(\Gamma_i+\Gamma_f)}+a(E_i)\right|^2, \quad (3.2)$$

where the partial widths are Γ_i and Γ_f and E_0 is given by

$$E_0=W_1-j_i-j_f. \quad (3.3)$$

Here j_i and j_f are the energy level shifts due to interactions of the residual nucleus with the respective channels. The quantity $a(E)$ represents the background due to other levels. The partial widths are given as radial integrals over products of wave functions $u(r)$ and $f(r)$ and multiplied by the interaction energy between the residual nucleus and the channel particle, denoted by $H_{i,f}(r)$. The indicated integration can be carried out simply if a localized type of interaction is assumed in the form

$$H_i(r)=H_i^0\delta(r_i-b_i). \quad (3.4)$$

For this case the partial widths are expressible as

$$\Gamma_i=(2\mu_p/\hbar^2)H_i^{02}u_1^2f_i^2/k_i, \quad (r_i=b_i), \quad (3.5)$$

and the energy level shift is

$$j_i=(g_i/f_i)\Gamma_i, \quad (r_i=b_i), \quad (3.6)$$

and similarly in the final channel. Here, it may be seen that the quantity $(2\mu_p/\hbar^2)H_i^{02}u_1^2$, where u_1 is to be evaluated at $r_i=b_i$, plays the role of the inherent width for the channel i in this formalism. The implication of the principle of charge symmetry is that the above quantity should be approximately the same in the two initial and in the two final channels when fits are made.

The functions f_i and g_i were calculated from one-body square wells which are charge symmetric according to the first method outlined in Sec. 2 A, with $V_i=-1.60$ Mev for H^3+d and $V_i=-1.00$ Mev for He^3+d, these values applying in both cases for $r_i<5.0\times10^{-13}$ cm. At distances in the channel i greater than 5.0×10^{-13} cm, only the Coulomb potential was assumed to act.

It was found that the potential well used to calculate f_f and g_f could not be accurately determined from fitting the data because of the similar energies in the two final channels. However, for purposes of finding the inherent widths the interior final state potential V_f was used as having values -21.8 Mev and -19.6 for $n+He^4$ and $p+He^4$, respectively. In both cases these values were used for $r_f<3.2\times10^{-13}$ cm and the Coulomb potential energy was supposed to be absent inside the potential wells. At distances greater than 3.2×10^{-13} cm both the Coulomb and centrifugal barriers were taken into account. These potential wells had been previously employed in Sec. 2.

The distance at which the interaction between the channel particle and the residual nucleus occurs was taken to be at the edge of the square wells $b_i=5.0\times10^{-13}$ cm, $b_f=3.2\times10^{-13}$ cm.

It was not found possible to obtain fits employing an energy-independent $a(E)$. A background $a(E)\propto E_i$ was found to give the most satisfactory results and in the final fits the background accounted for about 20% of the total cross sections near the resonances. The parameters which were found to give the best agreement with the experimental data are shown in Table IV. The fits are illustrated in Fig. 3.

B. Fits with the $\mathfrak{R}$-Matrix Formalism[27]

The $\mathfrak{R}$-matrix formalism of Wigner and his collaborators[6,20,21] can be used in the approximation of one level plus a background by making the assumption that only one term in the sum occurring in each matrix element has a strong energy dependence around the observed resonance. The matrix element referring to a transition from the channel p into the channel q may then be written

$$R_{pq}(E)=\gamma_{\lambda p}\gamma_{\lambda q}/(E_\lambda-E)+R_{pq}^{(1)}, \quad (3.7)$$

where the subscript λ denotes the energy level in the compound state which is causing the resonance and the quantities $R_{pq}^{(1)}$ represent the slowly varying background. The cross section may now be written in the convenient manner

$$\pi\sigma(i\rightarrow f)/\Lambda_i^2 S=B_i^2B_f^2/|D|^4|R_{if}^{(1)}D+(A_i/B_i)\times(A_f/B_f)/(E_\lambda-E+\Delta_\lambda-i\Gamma_\lambda)|^2 \quad (3.8)$$

TABLE IV. The fitting parameters obtained by using the one-level-plus-background formula from the schematic treatment of Breit. The background parameter $a(E)$ was taken to be proportional to the energy and had the following values at the resonance energy: for $H^3(d,n)He^4$, $a(E_0)=0.26$, and for $He^3(d,p)He^4$, $a(E_0)=0.17$.

	H^3	He^3
$H_1^0u_i(b_1)$ (Mev cm$^{\frac{1}{2}}$)	0.011	0.012
$H_2^0u_i(b_2)$ (Mev cm$^{\frac{1}{2}}$)	0.18	0.18
W_1 (Mev)	0.230	0.380
V_1 (Mev)	−1.60	−1.00
V_2 (Mev)	−19.6	−22.1
b_1 (10^{-13} cm)	5.00	5.00
b_2 (10^{-13} cm)	3.54	3.54

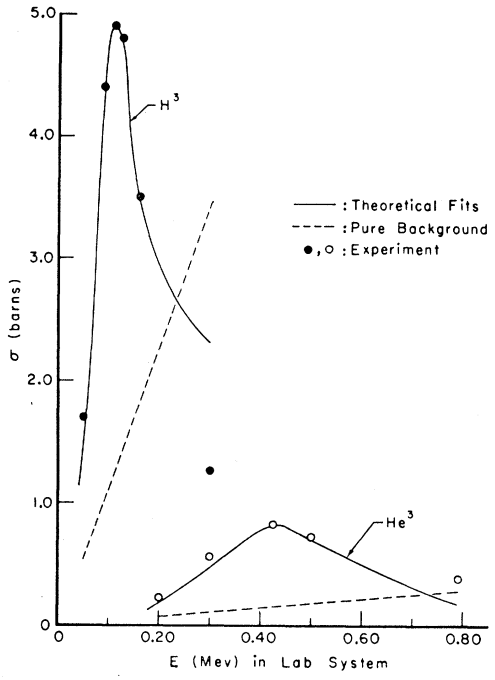

FIG. 3. Fits to both reactions made with the isolated-level formula obtained from the schematic treatment of Breit. In the entrance channels the interaction occurs at $r_1=5.0\times10^{-13}$ cm, while in the exit channels it occurs at $r_2=3.2\times10^{-13}$ cm. The background parameter, a, was taken to be proportional to the energy of the incident deuteron. The purely background contributions to the cross sections are shown as dashed lines.

where

$$A_i/B_i=(i+\mathcal{C}_f+B_f^2R_{ff}^{(1)})\gamma_{\lambda i}-B_f^2R_{if}^{(1)}\gamma_{\lambda f},$$
$$D=-1(\mathcal{C}_i+B_i^2R_{ii}^{(1)})(\mathcal{C}_f+B_f^2R_{ff}^{(1)})-B_i^2B_f^2R_{if}^{(1)2}+i(\mathcal{C}_i+B_i^2R_{ii}^{(1)}+\mathcal{C}_f+B_f^2R_{ff}^{(1)}), \quad (3.9)$$
$$\Delta_\lambda-i\Gamma_\lambda=D^{-1}\{(i+\mathcal{C}_f)B_i^2\gamma_{\lambda i}^2+(i+\mathcal{C}_i)B_f^2\gamma_{\lambda f}^2+B_i^2B_f^2[R_{ff}^{(1)}\gamma_{\lambda i}^2-2R_{if}^{(1)}\gamma_{\lambda i}\gamma_{\lambda f}+R_{ii}^{(1)}\gamma_{\lambda f}^2]\}.$$

These relations may be regarded as an extension of the one-level relations of the $\mathfrak{R}$-matrix formalism given in Sec. 2 B. It should be pointed out that if the quantities $R_{pq}^{(1)}$ are considered to be arbitrary functions of the energy, then no approximations are involved in the above equations.

The quantities $\mathcal{C}_p$ and B_p^2 have been defined previously and the same boundary condition shall be used here as in Sec. 2 B. The functions $\mathcal{C}_p$ and B_p^2 were calculated at the initial channel radii $(4.0 \text{ or } 5.0)\times10^{-13}$ cm and at the final channel radius 3.2×10^{-13} cm. The quantities $R_{pq}^{(1)}$ were first taken to be constants and when fits had been obtained, it was found that they were not appreciably affected if the $R_{pq}^{(1)}$ were given a slight energy variation. The background term $R_{pq}^{(1)}$ was taken to be zero for both reactions since this introduced some simplification in the fitting procedure. If many higher levels contributed to the sum in $R_{if}^{(1)}$, the latter would be expected to be at least quite small since the reduced width amplitudes $\gamma_{\lambda i}$ and $\gamma_{\lambda f}$ are expected to have random signs. When fits had been obtained the effect of taking $R_{if}^{(1)}$ into account was investigated and it was found that $R_{if}^{(1)}$ could be made 30 percent of the quantities $R_{ii}^{(1)}$ or $R_{ff}^{(1)}$ without noticeably altering the fits. The formal resonance energy E_λ was kept near the observed resonance energy throughout the calculation and the only fits which were retained were those indicating the possibility of satisfying the principle of charge symmetry. As has been pointed out this principle requires that the reduced widths $\gamma_{\lambda i}^2$ and $\gamma_{\lambda f}^2$ in corresponding channels of both reactions be approximately equal. The parameters which were found to give acceptable agreement with experiment are given in Table V.

4. DISCUSSION

Inspection of Tables I and II shows that it has been possible to obtain fits to the data using a one-level Breit-Wigner formula and satisfying the requirements of the principle of charge symmetry within the approximations made. Three types of penetration parameters have been employed and evaluation of these has been carried out at several radii in both channels, all allowing good fits. This work has thus been unable to determine closely an effective nuclear radius from the fits and although one form of penetration parameter was ruled out, considerable uncertainty remains as to the best manner of describing the barrier effects in a nuclear reaction.

It has also proved possible to fit the present reactions satisfying charge symmetry using the one-level formula obtained from the $\mathfrak{R}$-matrix formalism. The boundary condition employed was that of joining the wave function in the interior to the irregular function in the channel region. The effect of the estimated experimental uncertainty of 8% in the data on both reactions is to give rise to approximately the same uncertainty in the fitting parameters. Consequently, it would seem that a lack of agreement of inherent or reduced widths in corresponding channels beyond this uncertainty indicates either a failure to satisfy the principle of charge symmetry or perhaps insufficient flexibility of the one-

TABLE V. The fitting parameters obtained by using the one-level-plus-background formula from the $\mathfrak{R}$-matrix formalism. The background parameters were taken to be constants and the radius in both exit channels was $b_2=3.2\times10^{-13}$ cm.

	$b_1=5.0\times10^{-13}$ cm		$b_2=4.0\times10^{-13}$ cm	
	H^3	He^3	H^3	He^3
$\gamma_{\lambda1}^2$ (10^{-12} Mev cm)	0.17	0.18	0.22	0.23
$\gamma_{\lambda2}^2$ (10^{-12} Mev cm)	0.0080	0.0080	0.0070	0.0068
E_λ (Mev)	0.063	0.240	0.040	0.240
$R_{11}^{(1)}$ (10^{-12} cm)	0.10	0.30	0.15	0.25
$R_{22}^{(1)}$ (10^{-12} cm)	0.10	0.30	0.15	0.25
$R_{12}^{(1)}$ (10^{-12} cm)	0.00	0.00	0.00	0.00

level or one-level-plus-background approximations used in these $\mathfrak{R}$-matrix theory fits. The influence of experimental errors appears to be too complicated to discuss.

The fits made with the schematic treatment are satisfying from the point of view of charge symmetry. They are in good agreement with experiment below and around the peak cross section but the fit to the H^3 reaction is too high at the higher energies while that to the He^3 reaction falls too low at the higher end. These discrepancies may perhaps be due to the fact that the model has not been used in its most general form but has been specialized to the isolated-level case. The form which has been chosen for the interaction energies may also have a bearing on the relative failure of the fits near the higher energies.

The fits made with the one-level-plus-background formula obtained from the $\mathfrak{R}$-matrix formalism using the same boundary condition as was mentioned above are also in accord with charge symmetry. These fits are an improvement over those made with the one-level $\mathfrak{R}$-matrix formula at the lower energies but are somewhat low near the higher energy end of the resonance. Again, this situation may be attributed to the approximations which were made in applying the formalism to experiment.

Conner, Bonner, and Smith,[13] Bonner, Conner and Lillie,[14] and Kunz[19] have reported fits to the reactions under discussion employing a one-level formula obtained from the $\mathfrak{R}$-matrix formalism by using the boundary condition of making the interior wave function horizontal at the channel radius. An inspection of their work shows that while the fits to experiment are good, charge symmetry is not very well satisfied. Kunz[19] reports some 50% discrepancy between his initial channel reduced widths for the He^3 reaction and those of Conner, Bonner, and Smith[13] for the H^3 reaction. This seems to be well outside the limits allowed by the experimental data. A property of their fits is that the energy level shifts are large and strongly energy dependent, the formal resonance level being considerably removed from the observed resonance energy. This situation may be at least partly due to the boundary condition which was used by them. The level shifts which were found in the present fits both with the schematic treatment and with the $\mathfrak{R}$-matrix formalism were small and their energy variation was not very significant. A comparison is given in Table VI where the level shift of Conner, Bonner, and Smith[13] and that of the present one-level-plus-background $\mathfrak{R}$-matrix fit are shown. The level shifts of Conner, Bonner, and Smith have been recalculated for the purpose of the present comparison.

TABLE VI. Comparison of level shifts used by Conner *et al.*[a] (CBS) with those used here in the one-level-plus-background formula of the $\mathfrak{R}$-matrix formalism. In the CBS fit to $H^3(d,n)He^4$, the parameters were $\gamma_{\lambda 1}^2=1.0\times10^{-12}$ Mev cm, $\gamma_{\lambda 2}^2=0.028\times10^{-12}$ Mev cm, $E_\lambda=-0.464$ Mev, and $b_1=b_2=5.0\times10^{-13}$ cm. Parameters for the fit used here are given in Table V.

E (Mev)	0.078	0.110	0.124	0 198	0.314
$(\Delta_\lambda)_{CBS}$ (Mev)	0.698	0.611	0.583	0.484	0.378
$(\Delta_\lambda)_{here}$ (Mev)	0.0091	0.0135	0.0236	0.0320	0.0615

[a] See reference 13.

A striking feature found by the above workers and also in the present $\mathfrak{R}$-matrix fits is the smallness of the reduced widths in the final channels compared to those in the initial channels. The implication is that it is more probable to find a deuteron at the nuclear surface than to find a single neutron or proton. The compound nucleus He^5 seems to be composed of 2 neutrons and 1 proton in the S shell and 1 neutron and 1 proton in the P shell. The Li^5 nucleus would be similar except for having 2 protons and 1 neutron in the S shell.

Bonner, Prosser, and Slattery[28,29] have recently measured the $n+He^4$ scattering cross section which manifests a resonance near the energy of the final channel of the H^3 reaction. Breit[30] and Breit and Bloch[31] have discussed the analysis of scattering by formulas assuming one channel and one level plus background. A brief analysis of the cross-section curve of Bonner *et al.*[28] with the formula studied by Breit and Bloch[3] shows that the scattering width, corresponding to the present Γ_f, is approximately 0.060 Mev in the center-of-mass system of the final reaction channel. The present fits using the schematic treatment give the value $\Gamma_f(H^3)=0.065$ Mev, while the $\mathfrak{R}$-matrix fits, with and without background, give the value $\Gamma_f(H^3)=0.060$. This agreement, while probably partly fortuitous, appears to be significant.

It is concluded that it is possible to fit the $H^3(d,n)He^4$ and $He^3(d,p)He^4$ reactions with one-level and one-level-plus-background dispersion formulas without obvious conflict with the principle of charge symmetry.

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[29] The authors are grateful to Professor G. Breit for allowing them to see a preprint of the results of reference 28 and for pointing out the approximate circumstances.

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