Magnetization Mechanism and Domain Structure of Multidomain Particles*

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The free energy of a two-domain cube of iron is considered with and without an applied magnetic field. It is shown that the two-domain configuration may exist only beyond a certain critical size (200 A), that the wall characteristics are size dependent and that their values are substantially different from the values assumed in bulk material. Moreover, magnetization by wall motion is shown to be a "hard process."

I. INTRODUCTION

THE magnetic behavior of large multidomain particles is similar to that of bulk material. We therefore reserve the name of multidomain particles to those particles that are too large to be single-domain and too small to contain more than a few domains (i.e., two or three). The study of these particles raises some basic problems in ferromagnetic theory, and some of the questions long settled for bulk material reappear in a new and more complex form due to the greater role of the magnetostatic energy. Some of these questions are: What are the width and energy per unit area of the walls in the multidomain particles? How do these properties depend on particle size and shape? What is the magnetization mechanism in these particles, or more specifically, can wall motion still be considered as an "easy process"? Directly related to these questions, are the problems of inertia, viscosity, and motion of the wall in high-frequency fields.

We consider in this paper some of the questions raised above, confining ourselves to the two-domain cube. Brief preliminary investigations[1,2] have already suggested substantial differences between the wall properties in bulk material on one hand, and thin films and fine particles on the other hand. We undertake here a more systematic and more complete study. In part II, we outline the methods used in computing the magnetostatic energies involved in the subsequent parts. These methods are based on a paper by Wright,[3] extended and corrected by Rhodes and Rowlands,[4] hereafter referred to as RR. The essential results of the latter paper are summarized in Appendix A. We have recast and extended some of these results in a form more suitable for our needs. In part III we analyze the characteristics of a wall in a two-domain cube of iron, the size dependence of these characteristics, and the criteria for the existence of such a two-domain configuration.[5] In part IV we calculate the characteristics and the position of the wall in presence of an applied field.

II. MAGNETOSTATIC ENERGIES

We start out by defining an "effective demagnetization factor." Let E_x be the magnetostatic energy of a body of volume v of arbitrary shape uniformly magnetized in some direction x, the magnetization density being designated by I. We define the "effective demagnetizing factor" N_x by the equation

$$E_x = v(\tfrac{1}{2}N_x I^2), \quad \text{or} \quad N_x = 2E_x/vI^2. \tag{1}$$

This average energy definition disregards entirely the field distribution and reduces to the standard definition when the shape of the body is ellipsoidal. Moreover, the effective demagnetization factors along three mutually perpendicular directions satisfy the relationship

$$N_x + N_y + N_z = 4\pi, \tag{2}$$

just as in the classical case of the ellipsoid. The proof of this (or a similar) relation has been independently established by several authors (in particular, Brown and Morrish,[6] Rhodes,[7] and the author). It is to be noted however, that the only published proof, that of Brown and Morrish, applies only to cases where the demagnetizing field H is uniform, and this reduces considerably the generality of relation (2). The proof that we give in Appendix B makes absolutely no assumption on the demagnetizing field distribution.

Using the relations in Appendix A, we calculate the magnetostatic energy E_c of a rectangular parallelepiped of dimensions a, b, c uniformly magnetized along the c direction. This energy is made up of the self-energy of two "charged" rectangular areas and of their interaction energy. This leads us to the following result:

$$E_c = v2I^2 g(b/a, c/a) = 2I^2 v g(p,q), \tag{3}$$

and the corresponding demagnetizing factor:

$$N_c = 4g(b/a, c/a) = 4g(p,q), \tag{4}$$

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[5] The results of part III and their derivation were reported at the Washington Conference on Magnetism and Magnetic Materials, November, 1957 [H. Amar, J. Appl. Phys. **29**, 542 (1958)].

[6] W. F. Brown and A. H. Morrish, Phys. Rev. **105**, 1198 (1957).

[7] P. Rhodes (private communication).

TABLE I. The function $g(p,q)=[F(p,0)-F(p,q)]/pq$.

$p \backslash q$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
0.0											
0.1	1.4989	1.0261	0.7868	0.6405	0.5410	0.4687	0.4139	0.3703	0.3352	0.3062	0.2818
0.2	1.9212	1.4321	1.1453	0.9554	0.8100	0.7184	0.6392	0.5758	0.5238	0.4804	0.4435
0.3	2.1300	1.6655	1.3700	1.1636	1.0109	0.8935	0.8002	0.7244	0.6616	0.6088	0.5636
0.4	2.2552	1.8172	1.5245	1.3125	1.1515	1.0251	0.9231	0.8393	0.7691	0.7097	0.6586
0.5	2.3385	1.9233	1.6370	1.4243	1.2593	1.1277	1.0204	0.9313	0.8560	0.7918	0.7363
0.6	2.3982	2.0016	1.7222	1.5108	1.3445	1.2101	1.0993	1.0065	0.9277	0.8600	0.8013
0.7	2.4430	2.0622	1.7992	1.5799	1.4134	1.2775	1.1646	1.0693	0.9880	0.9178	0.8565
0.8	2.4782	2.1100	1.8432	1.6362	1.4702	1.3337	1.2194	1.2135	1.0393	0.9672	0.9041
0.9	2.5065	2.1488	1.8873	1.6828	1.5178	1.3811	1.2661	1.1680	1.0835	1.0099	0.9453
1.0	2.5290	2.1810	1.9240	1.7223	1.5580	1.4215	1.3061	1.2072	1.1217	1.0472	0.9815
1.1	2.5476	2.2077	1.9551	1.7556	1.5926	1.4561	1.3409	1.2416	1.1555	1.0800	1.0134

where $v=abc$, $p=b/a$, $q=c/a$. Similar expressions for E_a, E_b may be obtained by cyclical permutation of a, b, c. We have introduced a new function $g(p,q)$ expressed in terms of RR functions by the equation

$$g(p,q)=[F(p,0)-F(p,q)]/pq. \tag{5}$$

The introduction of this function is justified by its important physical meaning as proportional to the magnetostatic energy and the demagnetization factor. Equation (A5) shows the elaborate computations involved in evaluating $g(p,q)$. These computations have been considerably reduced by using two properties of the function $g(p,q)$. These properties, obtained by simply transcribing Eqs. (A4) and (2), are, respectively,

$$g(p,q)=g(p^{-1},qp^{-1}) \tag{5a}$$

$$g(p,q)+g(q,p)+g(pq^{-1},q^{-1}). \tag{5b}$$

Inspection of these properties shows that if $g(p,q)$ is known over the "unit square": $0\leq p\leq 1$, $0\leq q\leq 1$, its value for any pair of positive values of p,q can be simply derived. Table I gives $g(p,q)$ over the "unit square."

The energy of a rectangular block uniformly magnetized in a direction defined by its direction cosines α, β, γ (with respect to axes parallel to a, b, c) can be

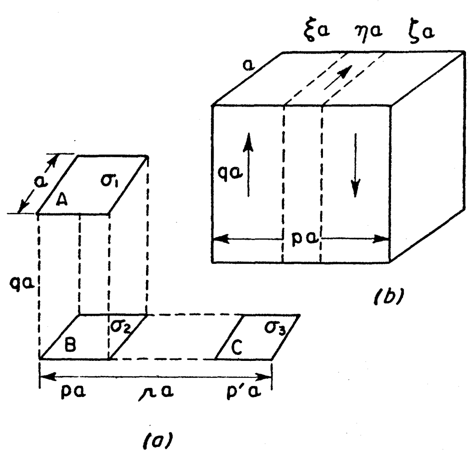

FIG. 1. (a) Charged rectangular areas, energies given in Appendix A; (b) parallelopiped partitioned in two antiparallel domains.

determined from considerations in the appendix and in this section. It is

$$E(\alpha,\beta,\gamma)=2vI^2\{\alpha^2g(p/q,1/q)+\beta^2g(q,p)+\gamma^2g(p,q)\}, \tag{6}$$

and the corresponding demagnetizing factor is $N=2E/vI^2$. For a cube ($p=q=1$);

$$E=2vI^2g(1,1)=\tfrac{2}{3}vI^2\pi,\quad N=\tfrac{4}{3}\pi, \tag{7}$$

which shows complete isotropy. For a parallelepiped with square cross section ($a=b$) uniformly magnetized in a direction making an angle ϑ with the c axis, the magnetostatic energy is

$$E=2vI^2\{g(q,1)+\sin^2\vartheta[g(1,q)-g(q,1)]\}. \tag{8}$$

We conclude this section by considering a rectangular block of dimensions a, $b=pa$, $c=qa$ partitioned by two planes perpendicular to edge b in three uniformly magnetized sub-blocks of respective widths: ξa, ηa, ζa [Fig. 1(b)]. The energy is made up of the respective energies of the three blocks and of the interactions between blocks. The interaction between the middle block and an end block may be shown to be zero, due to obvious cancellations. The only interaction left is that between the two antiparallel blocks, and it can be calculated with the help of Eqs. (A3), (5). The resulting expression for the energy E_M is

$$\begin{aligned}E_M/2a^3I^2=q\{&-pg(p,q)-\eta g(\eta,q)+(\xi+\eta)g(\xi+\eta,q)\\&+(\eta+\zeta)g(\eta+\zeta,q)+\xi g(\xi,q)+\zeta g(\zeta,q)\\&+\eta g(\eta/q,1/q)\}.\end{aligned} \tag{9}$$

III. TWO-DOMAIN CUBE IN THE ABSENCE OF AN APPLIED FIELD

Consider a cubic single crystal of iron, whose edges, parallel to the easy axes of magnetization ([100] directions) have a common length a. The cube is assumed to composed of two ferromagnetic domains separated by a 180° wall of width $y=\eta a$. For symmetry reasons the two domains will be of equal width. Subsequent analysis (part IV) will confirm this fact. The total magnetic energy is made up of two parts, the magnetostatic energy E_M and the wall energy E_w (sum of magnetocrystalline and exchange energies). Using the same reducing factor $2a^3I^2$ as in the preceding section, we may write the total "reduced" energy as $e=e_M+e_w$. The term e_M is found to be

$$e_M=-g(1,1)+(1+\eta)g\left(\frac{1+\eta}{2},1\right)+(1-\eta)g\left(\frac{1-\eta}{2},1\right). \quad (10)$$

The derivation of this result and the detailed calculations have been published elsewhere.[5] The results are summarized in Table II and in Fig. 2. They indicate in particular that below the critical size of 200 A the single-domain behavior is energetically more favorable, and describe the size dependence of the wall characteristics.

IV. TWO-DOMAIN CUBE WITH AN APPLIED FIELD

We now proceed to analyze the effect of an applied field H on the width, energy, and displacement of the wall in a two-domain cubic particle in order to gain some understanding of the magnetization mechanism of multidomain particles. The field intensity adds to the variables of size, shape, wall width, etc. We shall therefore eliminate the shape and size factors by choosing a cube of edge $a=400$ A. The preceding analysis shows that in the absence of an applied field, the cube is divided into two equal antiparallel domains separated by a 180° wall of width 160 A approximately. The effect of an applied field will be to change the widths of the wall and of the domains. If one assumes the field H parallel to the magnetization density in one of the two domains, the width $x=\xi a$ of that domain will be larger than the width $z=\zeta a$ of the other domain. If $y=\eta a$ designates again the wall width, then $x+y+z=a$. The total energy may now be considered as a function of the three variables, x, y, H or ξ, η, H. For a given value of H the "reduced" energy will be made of two parts: (a) the sum e_M+e_w of the magnetostatic and wall energy which does not contain H explicitly, and (b) the energy e_H of interaction between the particle and the field. Thus,

$$e(\xi,\eta,H)=e_M(\xi,\eta)+e_w(\eta)+e_H(\xi,\eta,H). \quad (11)$$

The wall energy $e_w(\eta)$ is the same function as that used in the case $(H=0)$. The magnetostatic energy may be obtained from formula (9) by setting $p=q=1$. It is found to be:

$$e_M=-g(1,1)+(\xi+\eta)g(\xi+\eta,1)+(\eta+\zeta)g(\zeta+\eta,1)+\xi g(\xi,1)+\zeta g(\zeta,1), \quad (12)$$

with $\xi+\eta+\zeta=1$. The interaction energy between the wall and the applied field is zero provided the spin orientations are symmetrical about the median plane of the wall. Thus, the only interactions to be considered are those of the two domains with the field. The

TABLE II. Size dependence of the wall characteristics of a cubic two-domain iron particle. Columns 3, 4 give the width y_{min} and the energy σ_{min} (in erg cm^{-2}). Columns 5 and 7 give the fraction of the total energy taken by the wall and its relative width. Column 6 gives the ratio of the energies of the two-domain cube and of the same cube uniformly magnetized.

1	2	3	4	5	6	7
a (A)	e_{min}	y_{min} (A)	σ_{min}	$\frac{E_w}{E_{min}}$	$\frac{E_{min}}{E_s}$	$\frac{y_{min}}{a}$
300	0.840	145	3.521	0.238	0.802	0.483
400	0.740	160	3.205	0.188	0.707	0.400
500	0.690	175	2.945	0.145	0.659	0.350
600	0.655	190	2.777	0.122	0.625	0.333
800	0.620	205	2.542	0.094	0.592	0.256
1000	0.595	225	2.335	0.067	0.568	0.225

interaction energy density is $\mp\frac{1}{2}IH$ in the "x domain" and "z domain," respectively. One thus obtains for $e_H=E_H/2a^3I^2$:

$$e_H=(1-2\xi-\eta)H/4I=1.705\times10^{-4}(1-2\xi-\eta)H. \quad (13)$$

Since $0\leq\xi$, η, $\xi+\eta\leq1$, the absolute value of $(1-2\xi-\eta)$ cannot exceed unity. In tabulating e or e_H for a constant value of H, one must confine oneself to values of ξ, η, satisfying $\xi+\eta\leq1$. A simple computation gives e_M, e_w, e_H for every set of values of ξ, η (ξ, $\eta=0, 0.1, 0.2, \cdots, 1$). Thus, the total energy $e(H_0,\xi,\eta)$ is calculated in tables with two entries, and its minimum approximately located. It is found that for relatively small values of H (of the order of 100 oersteds) the "static minimum" remains practically

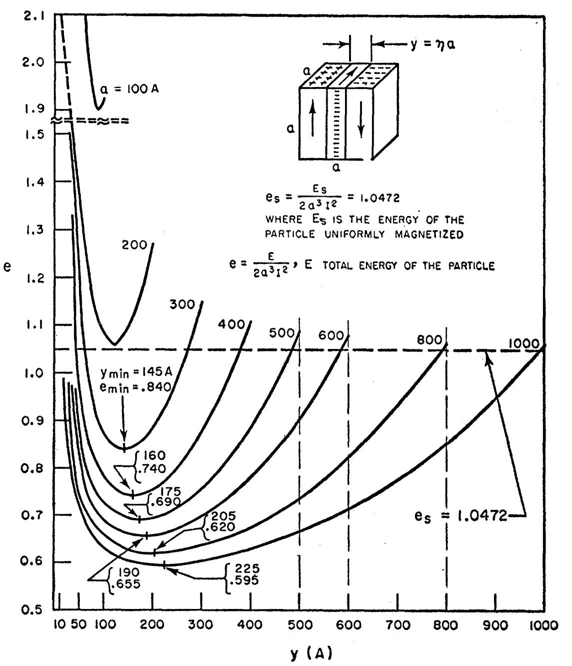

FIG. 2. Reduced energy e per unit volume of a cubic two-domain particle as a function of the wall width $y(=\eta a)$ for various values of the size parameter a.

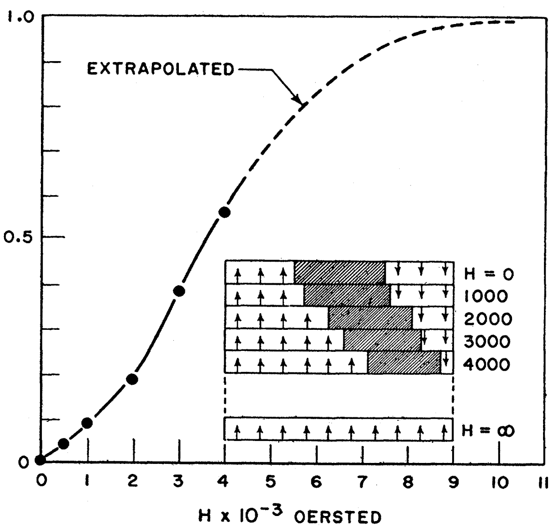

FIG. 3. Initial magnetization curve of a two-domain cube based on six calculated points and extrapolated for $H>4000$ oersteds. The illustration shows the position and thickness of the wall for increasing values of the applied field H.

unchanged. The tables have been computed for the following values of H: 500, 1000, 2000, 3000, and 4000 oersteds.

Table III summarizes the results of these computations giving for each H value, the position and width of the wall, the relative size of the two domains, and the specific magnetization $i_H = I_H/I = \xi - \zeta$.

Figure 3 illustrates the results of the table, showing how the wall position and width vary with the applied field. In the same figure an "initial" magnetization curve is sketched, using the data in the table, and is extrapolated for $H>4000$ oersteds. (For $H \geqslant 4000$ oersteds the second domain vanishes and the assumptions underlying our computation break down.)

The most striking result is that it takes a relatively strong field, $H \geqq 500$ oersteds, to move the wall even by a small amount and a very strong field, $H>4000$ oersteds, to get rid of the wall and saturate the particle. This implies that in these particles, magnetization by wall motion is a *hard process* in contrast with the wall behavior in bulk ferromagnetic samples. In this particular case the contrast is even stronger. Equations (6) and (7) show that the energy of a uniformly magnetized cube is independent of the magnetization direction and magnetization by rotation is a perfectly *easy* process. But this latter result holds only when $N_x = N_y = N_z$.

CONCLUSION

The analysis of the energy and domain structure of a two-domain single-crystal cube of iron has been used to show, in small multidomain particles:

(a) The wall characteristics may assume values remarkably different from those assumed in bulk material, and dependent on the size and shape of the particles.

(b) Wall motion is not necessarily an "easy" process of magnetization. A single-domain criterion for cubic particles has been derived. An approximate method of evaluation of the magnetostatic energy of multidomain rectangular blocks has been described. It should be pointed out that the magnetocrystalline part of the energy has been evaluated indirectly (as part of the wall energy) and only for uniaxial anisotropy.

TABLE III. Field dependence of the position ξ and width η of the wall. The corresponding values of $i_H[=(I_H/I)=\xi-\zeta]$ give the initial magnetization curve.

H (oersteds)	ξ	η	ζ	i_H
0	0.30	0.40	0.30	0.00
500	0.32	0.40	0.28	0.04
1000	0.35	0.39	0.26	0.09
2000	0.46	0.36	0.18	0.19
3000	0.52	0.34	0.14	0.38
4000	0.62	0.32	0.06	0.56
6000	0.70	0.30	0	0.70 ?

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APPENDIX A

Consider the three rectangular surfaces A, B, C [Fig. 1(a)] with uniform charge densities $\sigma_1\sigma_2\sigma_3$, respectively, and let (AA), (BB), (CC) denote the self-energies and (AB), (BC), (AC) denote the mutual energies. These energies can be evaluated in terms of a function $F(p,q)$ of two variables or of a derived function $Z(p,q)=p^{-2}F(p,q)$. The results are

$$(AA)=\sigma_1^2a^3F(p,0)=\sigma_1^2a^3p^2Z(p,0), \tag{A1}$$

$$(AB)=2\sigma_1\sigma_2a^3F(p,q)=2\sigma_1\sigma_2a^3p^2Z(p,q), \tag{A2}$$

$$(AC)=\sigma_1\sigma_3\{F(p+p'+r,\,q)+F(r,q)-F(p'+r,\,q) -F(p+r,\,q)\}. \tag{A3}$$

Function $Z(p,q)$ is shown to have the following property:

$$Z(p,q)=p^{-1}Z(p^{-1},qp^{-1}). \tag{A4}$$

The complete expression for $F(p,q)$ as given by Rhodes and Rowlands is

$$\begin{aligned}F(p,q)=&(p^2-q^2)\sinh^{-1}[1/(p^2+q^2)^{\frac{1}{2}}]\\&+p(1-q^2)\sinh^{-1}[p/(1+q^2)^{\frac{1}{2}}]+pq^2\sinh^{-1}(p/q)\\&+q^2\sinh^{-1}(1/q)+2pq\tan^{-1}[(q/p)(1+p^2+q^2)^{\frac{1}{2}}]\\&-\pi pq-\tfrac{1}{3}(1+p^2-2q^2)(1+p^2+q^2)^{\frac{1}{2}}+\tfrac{1}{3}(1-2q^2)\\&\times(1+q^2)^{\frac{1}{2}}+\tfrac{1}{3}(p^2-2q^2)(p^2+q^2)^{\frac{1}{2}}+\tfrac{2}{3}q^3.\end{aligned} \tag{A5}$$

A table of numerical values of $Z(p,q)$ is given by Rhodes and Rowlands in the range $0\leq p\leq 1.1$, $0\leq q\leq 1.6$.

APPENDIX B

Consider an arbitrarily shaped body uniformly magnetized in the x direction. Let dS_1 dS_2 be two elements of area about points P_1 P_2 on its surface, and $d\mathbf{S}_1$ $d\mathbf{S}_2$ the corresponding vector areas. The elementary "magnetic charges" about P_1, P_2 are $\mathbf{I}\cdot d\mathbf{S}_1=IdS_{1x}$, $\mathbf{I}\cdot d\mathbf{S}_2=IdS_{2x}$, respectively. Denoting the distance $|P_1P_2|$ by $r_{12}=r_{21}$, we may express the total magnetostatic energy as

$$W_x=\tfrac{1}{2}I^2\oint\frac{dS_{1x}dS_{2x}}{r_{12}},\quad \text{i.e.,}\quad N_x=\frac{1}{v}\oint\frac{dS_{1x}dS_{2x}}{r_{12}}.$$

If x, y, z are three mutually perpendicular axes, we have

$$N_x+N_y+N_z=\frac{1}{v}\oint\frac{d\mathbf{S}_1\cdot d\mathbf{S}_2}{r_{12}}. \tag{B1}$$

We shall now use the vector identity[8]

$$\int d\mathbf{S}\star\Phi=\frac{1}{v}\int d\tau\nabla\star\Phi,$$

where Φ may be a scalar, a vector, or a tensor (dyadic) and where the star stands for a dot product, a cross product, or an ordinary multiplication. Taking the origin at P_1 and allowing P_2 to be any point of the body, we have

$$\int\frac{d\mathbf{S}_2}{r_{12}}=\int d\tau_2\nabla_2\left(\frac{1}{r_{12}}\right)=-\int d\tau_2\frac{\mathbf{r}_{12}}{r_{12}^3}=\int d\tau_2\frac{\mathbf{r}_{21}}{r_{12}^3}.$$

The integral in (B1) may thus be rewritten as

$$\int\frac{d\mathbf{S}_1\cdot d\mathbf{S}_2}{r_{12}}=\int d\tau_2\int\frac{d\mathbf{S}_1\cdot\mathbf{r}_{21}}{r_{21}^3}=\int d\tau_2(4\pi)=4\pi v,$$

and (B1) leads to Eq. (2) of the text, namely, $N_x+N_y+N_z=4\pi$.

[8] H. B. Phillips, *Vector Analysis* (John Wiley and Sons, Inc., New York, 1933), p. 72, Eq. (127).

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Large-Signal Surface Photovoltage Studies with Germanium

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Studies of the surface photovoltage of germanium were carried out over a considerably wider range of excess carrier densities than previously reported. Ambient induced inversion and accumulation layer surfaces were studied on p- and n-type Ge with resistivities ranging from 1 to 15 ohm-cm. The photovoltage was measured with ac methods and the excess carrier density was monitored by changes in the specimen conductance. The observed dependence of the photovoltage on the excess carrier density agreed quite well with theory that considers the surface space charge, but neglects charge changes in fast surface states. Comparison of the observed and theoretical curves is believed to give the surface potential to within about one kT/e unit for potentials less than about $8kT/e$ units, even if the effect of previously reported fast states is neglected. Excursions of the surface potential over the ambient cycle were found to be about the same as those reported for other types of surface measurements.

The large signal photovoltage, in the range of surface potentials covered in the present work, is insensitive to fast states having the range of parameters extant in the literature: sensitivity is largely restricted to unreported parameter values. Since no evidence for fast states was observed in the present experiments, it is concluded that the present results are at least consistent with previously reported fast-state parameter values.

I. INTRODUCTION

THE model shown in Fig. 1 is the presently accepted one for semiconductor surfaces.[1] Electric charge represented by Σ_{ss} and Σ_{fs} is immobilized at the surface in two different types of surface states. The first type of state, called "slow," is located on or within the surface oxide layer. These states are affected by the ambient atmosphere and usually contain a relatively large amount of charge. Charge exchange with the bulk occurs slowly, with time constants of the order of seconds. The model in the figure applies for n-type material where the slow-state charge Σ_{ss} is negative and the whole system is in thermal equilibrium with no injected carriers present. The second type of trap state is considered as existing at the interface between the oxide and the bulk material. These states, with a charge Σ_{fs}, are called "fast" because the charge transfer between them and the bulk is measured in times of the order of microseconds, or less. These states are thought to be relatively independent of ambient changes and also to be associated with the surface recombination of holes and electrons.

[1] R. H. Kingston, J. Appl. Phys. 27, 101 (1956).