Length Effect in the Heat Transport in Helium II†

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A length effect in the heat transport in helium II in small channels has been observed. Measurements were made using a column of packed jeweller's rouge for lengths of 3.179, 5.166, and 8.156 cm in the temperature range 1.70–2.17°K. The channel size was approximately 0.1 micron. The results verified the linear dependence of the heat current density on temperature difference and showed a decrease in the heat transport by approximately a factor of 2 as the rouge length was increased from 3.179 to 8.156 cm.

THE heat transport in helium II in small channels has been studied to determine whether the heat flow is dependent on the length of the channel. London and Zilsel[1] had pointed out, earlier, that the experimental heat conductivities in the Meyer and Mellink[2] data were consistently higher than the values obtained by Keesom and Duyckaerts.[3] The experimental arrangement for these groups of experimenters differed in that Meyer and Mellink used channels of shorter length. London and Zilsel reasoned that if this length effect was real, it could not be explained on the basis of end effects, since such an effect would destroy the linearity between heat flow and temperature difference. The reason for this is that they would appear as quadratic terms in the velocities of flow. The present experiment has attempted to investigate this length effect more closely.

The apparatus consisted of a column of packed jeweller's rouge (Fe_2O_3) mounted in an adiabatic jacket. The details of this apparatus are given in an earlier paper.[4] The size of the channels formed by the interstices of the packed powder was measured by a flow method (assuming Knudsen-type flow) and was approximately 0.1 micron. The error was estimated to be approximately 30%.

The initial measurements were made using a length, 3.179 cm, of packed powder. The second length, 5.166 cm, was obtained by adding sufficient powder to the first length. The last length, 8.156 cm, was obtained in a similar manner. In each case the added powder was packed into the tube in such a way that the size of the channel remained constant. This was checked by repeating the flow measurements after each filling.

Helium gas was condensed into the experimental apparatus, after liquid helium had been introduced into the helium Dewar. Sufficient helium was condensed so that a level could be observed in the glass tube above the rouge column. Power was introduced into the volume below the rouge column and the temperatures at the bottom and at the top of the column were measured by means of carbon resistance thermometers which had previously been calibrated against the vapor pressure of liquid helium. The current through and the voltage across the manganin wire heater (~400 ohms) were measured by a Leeds and Northrup type K-2 potentiometer to give the power values. The heat leaks, along the glass tubing and measuring leads, were calculated to be approximately 15 microwatts. Since the powers used were in the order of milliwatts, these heat leaks were neglected in the calculations. The results are given in Table I. The first column in the table gives the length of the rouge column; the second column gives the temperature at the "hot" end of the rouge column (corresponding to several bath temperatures); the third column gives the temperature difference between the ends of the rouge length; in the fourth column are listed the heat current densities; the fifth column gives the thermal conductivity as calculated from

$$\dot{Q}/A = K_{\text{exp}}\,\text{grad}T, \qquad (1)$$

where $\dot{Q}/A$ is the heat current density and K_{exp} is the experimental thermal conductivity. The sixth column

Table I. Heat transport results.

Length (cm)	T_1, °K (bottom)	ΔT, °K	$\dot{Q}/A$ (mw/cm²)	$\bar{K}_{\text{exp}}$ cal/deg-cm-sec	$\bar{K}_{\text{theo}}$ cal/deg-cm-sec	$\bar{K}_{\text{exp}}/\bar{K}_{\text{theo}}$	v_s (cm/sec)
3.179	1.863	0.016	6.14	0.30	0.027	11.1	$2.3\cdot10^{-2}$
	1.925	0.060	24.03	0.30	0.037	8.1	9.7
	2.028	0.023	18.81	0.62	0.068	9.1	9.2
	2.076	0.002	1.54	0.78	0.059	13.2	0.9
	2.091	0.012	13.77	0.91	0.060	15.2	9.5
	2.106	0.022	24.32	0.84	0.060	14.0	16.5
	2.150	0.022	27.50	0.93	0.104	8.9	24.4
5.166	1.743	0.056	4.82	0.11	0.009	12.2	1.6
	1.834	0.109	10.30	0.11	0.028	3.9	3.6
	1.942	0.030	6.54	0.27	0.029	9.3	2.6
	2.054	0.028	10.41	0.46	0.048	9.6	6.1
	2.108	0.025	13.51	0.66	0.055	12.0	8.7
	2.160	0.023	13.49	0.72	0.061	11.8	13.2
8.156	1.769	0.032	2.10	0.13	0.010	13.0	0.7
	1.803	0.066	4.72	0.14	0.012	11.7	1.6
	1.950	0.192	18.89	0.19	0.028	6.8	7.3
	2.080	0.024	4.79	0.39	0.048	8.1	3.0
	2.114	0.007	2.19	0.58	0.052	11.2	1.3
	2.126	0.008	2.40	0.55	0.053	10.4	0.9
	2.172	0.028	7.31	0.50	0.057	8.8	12.1

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[1] F. London and P. R. Zilsel, Phys. Rev. **74**, 1148 (1948).

[2] L. Meyer and J. H. Mellink, Physica **13**, 197 (1947).

[3] W. H. Keesom and G. Duyckaerts, Physica **13**, 153 (1947).

[4] H. Forstat and C. A. Reynolds, Phys. Rev. **101**, 513 (1956).

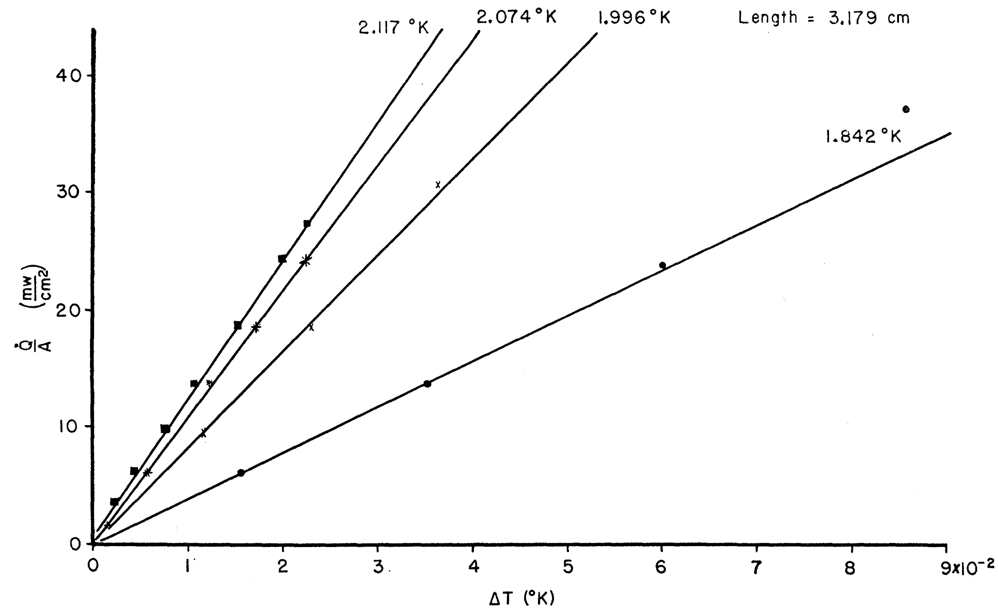

FIG. 1. Heat current density *vs* temperature difference.

lists the values of the thermal conductivity as obtained from the linear theory,

$$K_{\text{theo}}=2.61(R^2/\eta_n)T^{12.2}, \tag{2}$$

where R is the channel size and η_n is the viscosity of the normal fluid (a cylindrical channel is assumed). The values for η_n were taken from the data of Hollis-Hallet.[5] The seventh column gives the ratios of the experimental heat conductivity (column 5) to the theoretical heat conductivity (column 6). The last column lists the superfluid velocities.

The linearity in the heat flow was checked by plotting the heat current density *versus* the temperature difference for each of the experimental runs. In the temperature range used, *viz.*, 1.70°K to 2.17°K, no deviation from linearity was observed. Figure 1 shows a typical plot of the heat current density as a function of the temperature difference for the 3.179-cm length. The temperatures corresponding to each of the straight lines refer to the bath temperatures at which the data were taken. Similar curves were obtained for the other lengths.

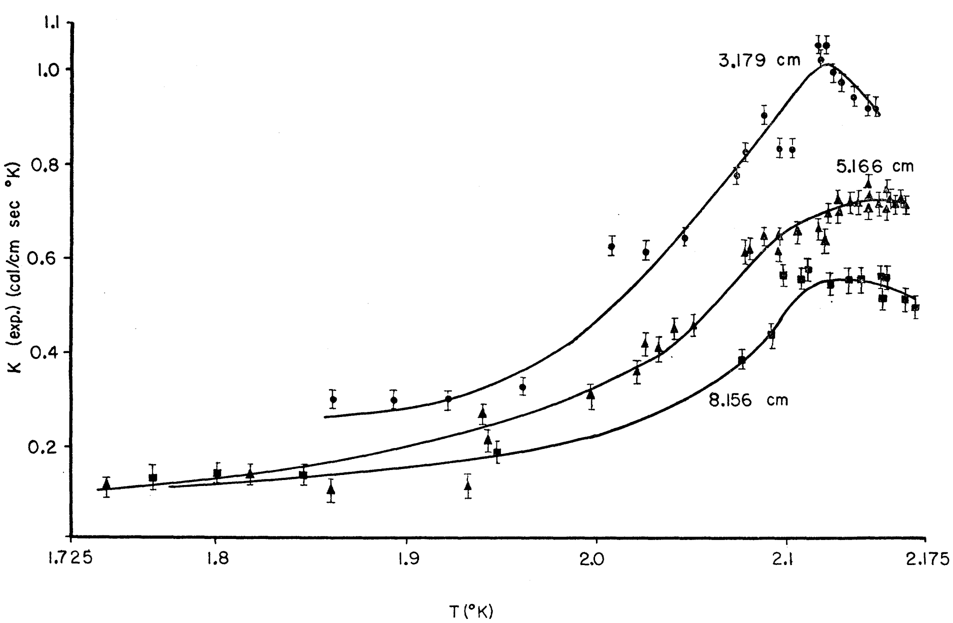

FIG. 2. Variation of the thermal conductivity as a function of the "hot" temperature for each of the three lengths 3.179 cm, 5.166 cm, and 8.156 cm.

[5] A. C. Hollis-Hallet, Proc. Roy. Soc. (London) **A210**, 404 (1952).

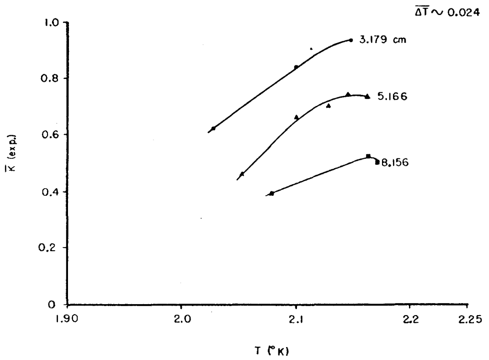

FIG. 3. Variation of the thermal conductivity for a fixed temperature difference, $\Delta T \sim 0.024$°K.

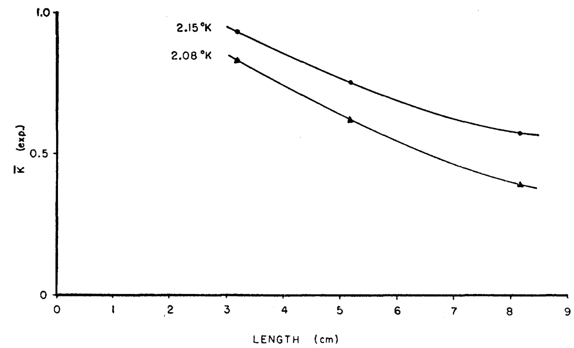

FIG. 4. Thermal conductivity as a function of length, for $\Delta T \sim 0.024$°K.

By plotting the thermal conductivity as a function of temperature, it was possible to reproduce the shape of the temperature dependence curve for a particular length observed by Keesom and Duyckaerts. Figure 2 shows these curves for each of the lengths tried where the values of the abscissa correspond to the "hot" temperature end of the rouge column. These curves were drawn without accounting for the variation of the thermal conductivity with temperature difference. In order to separate out this effect, some of the values were replotted for a fixed temperature difference ($\Delta T \sim 0.024$°K). This is shown in Fig. 3. The dependence of the thermal conductivity on length is shown in Fig. 4, where the data are plotted for a fixed temperature difference.

A comparison of the experimental and theoretical heat conductivities (column 7 in Table I) shows that the experimental values are larger than the theoretical values by approximately a factor of 10. This is not too surprising in the light of the uncertainty of the channel size.

A calculation of the flow velocities (column 8 in Table I) to further verify the existence of linear heat flow in this experiment showed them to be very much smaller than any critical velocity for the channel size used. Allen and Misener[6] have obtained the value 13 cm/sec for a slit width of 0.2 micron.

It has not been possible, at the present time, to account for this length effect on the basis of the current theory on heat transport in small channels. End effects, as was pointed out earlier, usually introduce nonlinear terms which destroy the proportionality between heat current density and temperature difference. Zilsel[7] has also remarked that if it were possible to represent end effects as being linear in the velocities of flow, then one would expect the thermal conductivity to increase with increasing length of the channel rather than decrease, as the present experiment shows. In order to determine whether one would expect mean-free-path effects to be important, a calculation was made for the mean free path of a roton in the temperature region used (the chief contribution to the thermal excitation in this region is due to rotons). This turned out to be less than 10^{-7} cm, several orders of magnitude smaller than the channel size.

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[6] J. F. Allen and A. D. Misener, Nature **141**, 75 (1939).
[7] P. R. Zilsel (private communication).