New Approach to General Relativity

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A generally covariant scalar field theory of gravitation is presented. The principle of equivalence as well as the principle of general covariance are preserved. A functional solution of Einstein's field equations is obtained for the general time-independent case. The theory predicts correctly the results of the three crucial tests of general relativity. Implications concerning the self-energy of point particles is presented. A new theory of cosmology is given and its application to the time-scale problem and to the derivation of the Mach principle are discussed. A new principle called the principle of observation is introduced.

I. INTRODUCTION

IN this article an attempt is made to discuss the theory of gravitation as a generally covariant scalar field theory. The approach differs both from the Lorentz-invariant scalar field theories of gravitation and from the conventional theory of Einstein. In Sec. II the theory is presented in a way which appears to us physically most plausible and where strict rigor is not emphasized. In Sec. VII an alternative and intrinsically covariant formulation is given. In Secs. III, IV, and V the field equations are solved for an arbitrary static set of singularities and applications to standard problems of astronomy and cosmology are made. It is seen that the results agree well with the observational data so far available. In Sec. VIII the theory is compared with other theories of gravitation, especially with Einstein's theory to which the present approach bears a close relationship. To avoid misconception we would like to stress from the beginning that the two basic assumptions of Einstein's theory, namely, the principle of general covariance and the principle of equivalence, are both preserved. Furthermore the field equations of Einstein are still valid. The differences lie, essentially, in what constitutes the gravitational field and the stress-energy tensor. In this theory the stress-energy tensor is taken to be that of the scalar field generated by the matter singularities, whereas in Einstein's theory it is taken to be zero away from matter. However, it is felt, as was first argued in a different context by Schrödinger,[1] that the mere formulation of a covariant theory and its solution in general coordinate systems would not make a well-defined physical theory unless it is supplemented with an underlying philosophy as to what are the expressions for the measured physical quantities in those coordinate systems. Considerable conceptual simplification is achieved in this direction by the introduction of a new principle called the principle of observation, which establishes the relationships of general coordinate frames to locally special relativistic frames where physical interpretation is clear cut. This principle has been the guiding idea in the present investigations and directly suggested the rigorous solutions of field equations as discussed in this paper. As will be seen, it is a statement about the relations between measurements of locally special relativistic observers situated in arbitrary time-independent gravitational fields. It is in the spirit of the general idea of relativity and can be considered as a natural extension of special relativity. In Sec. VI the principle of observation is presented as motivated from the general static solutions of the field equations, although it was originally formulated by a direct examination of space-time relationships of multiple observers in a gravitational field. Unfortunately, the principle, as stated, holds only for time-independent situations and hence we will refrain from discussing time-dependent solutions in this paper.

II. COVARIANT SCALAR FIELD THEORY OF GRAVITATION

It is well known that by a particular choice of reference frame the metric of space-time geometry can be reduced *locally* to the Lorentz form. Therefore, if we imagine a locally distributed set of observers in the above sense, the Lorentz-invariant field theories must be valid. We want to consider a scalar field ϕ with vanishing rest mass for such a set of observers. We take as the Lagrangian of the field, the expression[2]

$$\mathfrak{L}=(1/8\pi)\partial_\mu\phi\partial^\mu\phi, \tag{1}$$

where $g_{11}=g_{22}=g_{33}=-1$, $g_{44}=1$, and $\partial_\mu\phi=\partial\phi/\partial x^\mu$ are adopted. Equations of motion for the field are then obtained from the principle of stationary action,

$$\delta\int\mathfrak{L}d^4x=0, \tag{2}$$

to be $\partial_\mu\partial^\mu\phi=0$. This is at a source-free region of space. To include the sources we take

$$\partial_\mu\partial^\mu\phi=-4\pi\textstyle\sum_j M_g{}^j\delta(x-x^j), \tag{3}$$

where $M_g{}^j$ are the strengths of the mass singularities at the points x^j, and $\delta(x-x^j)$ is the Dirac δ function. The subscript g indicates that M_g is the *gravitating mass*. For the sake of convenience ϕ is here taken to

[1] E. Schrödinger, *Space-Time Structure* (Cambridge University Press, London, England, 1950), Chap. 10, especially pp. 84–85.

[2] Units are chosen such that $c=G=1$.

be the negative of the usual Newtonian potential; $\phi=-\varphi$.

The stress-energy tensor is obtained as

$$T_\mu{}^\nu=\partial_\mu\phi\frac{\partial\mathfrak{L}}{\partial(\partial_\nu\phi)}-\delta_\mu{}^\nu\mathfrak{L}. \tag{4}$$

It is symmetric and satisfies the relations

$$T=T_\mu{}^\mu=-2\mathfrak{L}, \tag{5}$$

$$T_\mu{}^\sigma T_\sigma{}^\nu=\delta_\mu{}^\nu\mathfrak{L}^2. \tag{6}$$

Energy density is easily seen to be positive definite;

$$H=T^{44}\geqslant 0. \tag{7}$$

We define a new tensor, $R_\mu{}^\nu$, by

$$(1/8\pi)R_\mu{}^\nu=\partial_\mu\phi\frac{\partial\mathfrak{L}}{\partial(\partial_\nu\phi)}. \tag{8}$$

It has the property

$$R=R_\mu{}^\mu=16\pi\mathfrak{L}=-8\pi T. \tag{9}$$

Thus the expression (4) can be written as

$$8\pi T_\mu{}^\nu=R_\mu{}^\nu-\tfrac{1}{2}\delta_\mu{}^\nu R, \tag{10}$$

or equivalently

$$(1/8\pi)R_\mu{}^\nu=T_\mu{}^\nu-\tfrac{1}{2}\delta_\mu{}^\nu T. \tag{11}$$

By virtue of (3) the stress-energy tensor satisfies the conservation laws, namely,

$$\partial_\mu(8\pi T_\nu{}^\mu)=\partial_\mu(R_\nu{}^\mu-\tfrac{1}{2}\delta_\nu{}^\mu R)=0. \tag{12}$$

We also note that as a consequence of vanishing rest mass for the field ϕ, we have,[3] for a time-dependent case,

$$\mathfrak{L}=R=T=0 \quad \text{(gravitational waves)}. \tag{13}$$

Therefore the energy density is given by

$$H=T^{44}=(1/4\pi)(\nabla\phi)^2. \tag{14}$$

In a time-independent case, instead of (13), we have

$$\textstyle\sum_i T_{ii}=0;\quad R=2(\nabla\phi)^2 \quad \text{(stationary fields)}. \tag{15}$$

It is thus possible to characterize the gravitational *waves* by (13) and the *static* gravitational fields by (15).

Now, Eqs. (1) to (13) are tensor equations [and are also tensor density equations since $(-g)^{\frac{1}{2}}=(-\det g_{\mu\nu})^{\frac{1}{2}}=1$] in the locally special relativistic set of frames. Therefore, they retain their form and validity when expressed in a general coordinate system given by the line element

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu. \tag{16}$$

For this we must make the following change:

$$\int\mathfrak{L}d^4x\rightarrow\int\mathfrak{L}(-g)^{\frac{1}{2}}d^4x,$$

if we wish to use a scalar $\mathfrak{L}=(1/8\pi)\partial_\mu\phi\partial^\mu\phi$, a tensor $T_\mu{}^\nu$, and an invariant volume element $(-g)^{\frac{1}{2}}d^4x$. With this understanding, the tensor equations (10) are valid in general coordinate frames. Thus, in the general frame given by (16), we have the equations

$$8\pi T_\mu{}^\nu=R_\mu{}^\nu-\tfrac{1}{2}\delta_\mu{}^\nu R. \tag{17}$$

Likewise the conservation equations (12) are valid in (16) in the form of a covariant divergence as a consequence of the generally covariant form of the wave equation (3).

Since the stress-energy tensor is expressed in a similar form to (17) in Einstein's theory of general relativity, we now wonder what would happen if we interpreted the tensor $R_\mu{}^\nu$ as the Ricci tensor corresponding to the metric (16). Then, of course, the line elements (16) could be determined by solving (17) and one would be able to examine if these solutions make any physical sense. In the remainder of this article, we shall endeavor to show that this is indeed the case. Before we start discussing the solutions of the field equations and their applications we may try to clarify whether the above identification is plausible. The first thing which comes to mind is this: why does one have curvature quantities given by (11) while working with a locally special relativistic frame of reference to begin with? Although the answer to this can be given more satisfactorily after one discusses the principle of observation, we make here the following remark as pointed out by Schrödinger[1]: Gravitational field is depicted essentially not by the numerical values of $g_{\mu\nu}$ but by their first derivatives. Therefore, a locally special relativistic frame, that is, the reduction to $g_{11}=g_{22}=g_{33}=-1$ and $g_{44}=1$, does not necessarily mean the complete elimination of geometrical curvatures. Reduction to this form is, however, sufficient for local physical interpretations of the stress-energy tensor. A second and perhaps more immediate question is this: The stress-energy tensor (10) and its transform (17) contain only the first derivatives of ϕ. On the other hand, we are looking for a solution for the metric tensor $g_{\mu\nu}$ which functionally depends on ϕ as $g_{\mu\nu}(\phi)$. Then the right-hand side of (17), with the interpretation of $R_\mu{}^\nu$ as the Ricci tensor, would involve second derivatives of $\phi(x)$ as well as its first derivatives. But this only means that we should look for solutions for which the second-order terms in $R_\mu{}^\nu$ can be combined to form an expression of the form (3). Then this expression can be dropped everywhere except at the singularities where the field equations are not expected to hold anyway. A more convincing argument to this can be given on the basis of the Lagrangian method. The above identification of $R_\mu{}^\nu$ as the Ricci tensor would imply $\mathfrak{L}=(1/16\pi)R$, where

[3] This is because in the classical limit the action integral is $\int\mathfrak{L}d^4x\rightarrow\int mdt$. Since the field has no rest mass we have, for each harmonic mode, $m=\mathfrak{L}=0$.

$\mathcal{L}$ involves only the first derivatives of ϕ, whereas R, the curvature invariant, contains second derivatives as well. But it is well known that two Lagrange functions differing by a divergence describe the same physical system. Therefore if the second-order terms in R can be arranged as the general form of the wave equation (3), the divergence of a gradient, then they can be dropped. This actually is what happens in the present theory.

III. TIME-INDEPENDENT SOLUTION THE THREE EXPERIMENTAL TESTS

A functional solution, $g_{\mu\nu}(\phi)$, of the field equations (17) for a static set of singularities is proven to be[4]

$$ds^2 = e^{-2\phi}dt^2 - e^{2\phi}(dx^2+dy^2+dz^2), \qquad (18)$$

where ϕ satisfies the covariant equation

$$\phi^{\mu}{}_{;\mu} = e^{-2\phi}\nabla^2\phi = -4\pi e^{-2\phi}\sum_j M_g{}^j\delta(x-x^j), \qquad (19)$$

which is the corresponding static case of the wave equation (3) in the space (18). Indeed, when we calculate the stress-energy tensor corresponding to the line element (18) in the standard way,[4] we find

$$T_\mu{}^\nu = -(1/8\pi)e^{-2\phi} \times \begin{bmatrix} 2\alpha^2-\xi^2 & 2\alpha\beta & 2\alpha\gamma & 0 \\ 2\alpha\beta & 2\beta^2-\xi^2 & 2\beta\gamma & 0 \\ 2\alpha\gamma & 2\beta\gamma & 2\gamma^2-\xi^2 & 0 \\ 0 & 0 & 0 & -\xi^2-2\nabla^2\phi \end{bmatrix}, \qquad (17')$$

where $\alpha=\partial\phi/\partial x$, $\beta=\partial\phi/\partial y$, $\gamma=\partial\phi/\partial z, \xi^2=\alpha^2+\beta^2+\gamma^2$. In view of (19) this is exactly of the form (17) everywhere except at the singularities. Of course, as in all other field theories the field equations are not satisfied at the singularities. It should be emphasized that the functional line element (18) with the scalar field equation (19) [i.e., $\nabla^2\phi=(\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2)\phi=0$ away from matter] makes the Einstein equations an algebraic identity except at the singularities.

If there is only one mass singularity, we have a simple and spherically symmetric case with

$$\phi(r) = M_g/r. \qquad (20)$$

The line element (18) with this $\phi(r)$ corresponds to a central field problem. One can show by direct comparison that this line element agrees with the isotropic form of the Schwarzchild line element up to and including the order M^2/r^2 in $g_{44}=1-2M/r+2M^2/r^2+\cdots$ and up to and including order M/r in $g_{ii}=1+2M/r+\cdots$. Since it is well known that the so-called "three crucial tests of general relativity" are at the present status of the experimental accuracy insensitive to the next higher orders, we conclude that the theory here described accounts for these three crucial phenomena just as well as the conventional general relativity of Einstein.

IV. SELF-ENERGY PROBLEM

An interesting consequence of the above line element is that the self-energy of a point singularity does not diverge, but it is equal to M_gc^2 (units are $c=G=1$, but below we use them explicitly). The total energy of the field can be calculated from (14) and the solution associated to (20) and gives[5]

$$E=(1/4\pi)\int_0^\infty (GM_g{}^2/r^4)\exp(-GM_g/c^2r)dv = M_gc^2. \quad (21)$$

The exponential factor comes from the fact that, as in the usual general relativity, the energy is expressed as $\epsilon=\epsilon_0\sqrt{g_{44}}$, where ϵ_0 is the measure of energy in a special relativistic system.[5] We shall see in Sec. VI [Eqs. (37) and (38)] that this is actually a consequence of the observation principle.

Now due to the relation, $E=M_ic^2$, between energy and inertia we see that

$$M_g = M_i. \qquad (22)$$

This is the expression of the equality of the *gravitating* mass, M_g, of a point singularity to its *inertial* mass, M_i, namely, the principle of equivalence. This equivalence justifies also our using the geodesic equations of motion in finding the trajectories of material objects and of heavenly bodies:

$$\frac{d^2x^i}{ds^2}+\left\{ {i \atop \mu\nu} \right\}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}=0. \qquad (23)$$

[4] This can be shown by direct calculation or use can be made of the formulas of Dingle; R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford University Press, New York, 1949), pp. 254–257. In the latter case, note the difference in sign of $T_{\mu\nu}$ in Tolman and in our theory. The difference is due to the fundamental difference in interpretation of $T_{\mu\nu}$ in Einstein's theory and ours. We have made the choice of sign so that the field energy is positive definite. The solution (18) is unique for the static case when the condition that the principle of equivalence be valid is also demanded. This was established by consideration of the complete explicit solutions of the case of a single fixed source within the present approach by S. Schneider (to be published elsewhere). In connection with the above-mentioned sign convention it appears that if the scalar field is indeed the agent of the gravitational field as proposed in this theory, then in the presence of external (e.g., electromagnetic) influences, one might have to write

$$R_\mu{}^\nu - \tfrac{1}{2}\delta_\mu{}^\nu R = \{(T_\mu{}^\nu)_{\text{scalar}} - (T_\mu{}^\nu)_{\text{other}}\}.$$

[5] The effect of time dependence reveals itself in the quantized theory as standing waves and also as the emission and re-absorption of virtual photons. This brings a contribution to self-energy, which may be termed kinetic energy. (18) is the sum of the kinetic and static parts of self-energy. See also Eqs. (37) and (38) for the meaning of the exponential factor. This factor is discussed by A. Einstein (in first order) in connection with the gravitation of energy [Ann. Physik **35**, 898 (1911); translated in *The Principle of Relativity* (Dover Publications, New York, 1923), p. 102]. Most textbooks omit it and give instead the formula for the gravitational red shift, which in view of the relation $E=h\nu$ is the same thing. Another way of looking at it, within the present theory of stationary observers in the static field, is the spatially invariant expression

$$E=\int T_{44}(-g_{(3)})^{\frac{1}{2}}dx^1dx^2dx^3.$$

Note that T_{44} has a factor $e^{-4\phi}$, while $(-g_{(3)})^{\frac{1}{2}}$, the Jacobian of the space part, is $e^{3\phi}$. This way we get again the same expression as in (21).

In this way the ponderomotive equations (23) appear as an independent assumption from the field equations as originally postulated by Einstein. In order to see if the theory contains the ponderomotive equations, one usually carries out a successive approximation process developed by Einstein, Infeld, and Hoffmann and by Fock and Petrova. This is not done as yet in the present case. However, it is seen that a remark made by Infeld and Schild[6] applies and relying on their analysis we may conclude that (23) is a consequence of Eqs. (17).

Before we close this section it may be interesting to note that the result (21) is valid for any number of singularities. The total field energy is always given by $E_T=\sum M_g c^2$. This can be proved easily by writing the field energy in the metric (18) and using Gauss's theorem. In this way we get

$$E_T=(1/4\pi)\int(\nabla\phi)^2e^{-\phi}dxdydz=\sum M_g c^2. \quad (21')$$

The interaction energy is automatically included in this sum. We shall see in Sec. VI that with the help of the observation principle the M_g's can be calculated [see formula (37′)] in terms of their noninteracting values.

V. NEW LINE ELEMENT FOR COSMOLOGY

Next we want to discuss a line element which corresponds to a distribution of mass singularities such that a local observer finds, on the average, the same mass density, σ, everywhere in the universe.[7] Thus for a large sphere of radius r, we have

$$2\phi(r)=(2G/c^2)\sum_j(M_g{}^j/|r-r_j|)$$
$$=(8\pi\sigma G/3c^2)r^2=\alpha^2r^2, \quad (24)$$

$$ds^2=\exp(-\alpha^2r^2)c^2dt^2-\exp(\alpha^2r^2)(dx^2+dy^2+dz^2). \quad (25)$$

This line element is identical up to and including order α^4r^4 in g_{44} and up to and including order α^2r^2 in g_{ii} to the isotropic time-independent form [de Sitter form (31)] of the Bondi-Gold and Hoyle universe. As in their theory, although the density σ is assumed to be constant, (25) does not represent a static universe. It represents a steady-state universe. The geodesic equations of motion (23) show that a particle originally at rest in the vicinity of an observer will have a radial velocity, v, later at r. A simple calculation shows that up to fourth order in αr we have

$$v^2=(8\pi\sigma G/3)r^2. \quad (26)$$

This is a velocity-distance relation. It will cause a Doppler shift in the spectral lines of light received from distant stars,

$$\delta\lambda/\lambda=(8\pi\sigma G/3c^2)^{\frac{1}{2}}r, \quad (27)$$

which is accompanied by a positive second-order term $(\alpha^2r^2/2)$ due to the pure gravitational effect. The formula (27) relates the average density of matter in the universe to Hubble's constant of recession of nebulae. Since the latter is better known from observation, it predicts an average mass density of about 5×10^{-28} g/cm^3, which is the same as Hoyle's result.

The continual recession of the nebulae and the assumption of constant density can be reconciled only if the matter is continually created. The rate of creation of matter per unit volume per unit time is $3\alpha\sigma$ (the same as in the Bondi-Gold theory).

It is interesting to note that according to the present theory, the universe will appear to be finite to an observer situated at any point in the universe. For, due to the invariance of line element (25) the length measure of a local observer, dl, will appear to 0 as $dl'=e^{-\phi}dl$. (The proof is similar to the shortening of meter sticks laid in a gravitational field in the usual theory of relativity.) Thus, according to (25), the effective operational radius of the universe is

$$R=\int_0^\infty\exp(-\alpha^2r^2/2)dr=\left(\frac{\pi}{2}\right)^{\frac{1}{2}}R_0, \quad (28)$$

where $R_0=(1/\alpha)$ is the Hubble radius. Similarly, for the effective cosmic time we have

$$T=(\pi/2)^{\frac{1}{2}}T_0, \quad (29)$$

where $T_0=(R_0/c)$. In the earlier cosmological theories R_0 and T_0 were considered to be the radius and the age of the universe, respectively. In our theory, R and T define the extensions of that portion of the whole of existence which can have any appreciable physical influence upon us, namely, our operationally defined universe. As in the Bondi-Gold theory, here there is no beginning and there is no end to the universe. It is a perpetually existing self-creating and expanding universe. The occurrence of a nebula with a certain age anywhere in the universe is a purely statistical attribute. The average age of a nebula is $\frac{1}{3}T=7.5\times10^8$ years, but any particular nebula may be arbitrarily old. For example, our galaxy may well be 4 to 6 billion years old or older, as already indicated by various independent data.

Finally, it is perhaps interesting to emphasize that the requirement of a time-independent solution with a universally constant density seems to lead, automatically, to continual creation, although the meaning of creation is now, largely, a matter of interpretation. For, if the steady-state solution (25) is to represent our universe in an operational sense, then, operationally, the total matter in the universe is constant. In this

[6] L. Infeld and A. Schild, Revs. Modern Phys. **21**, 408 (1949). See also M. F. Shirokov and V. B. Boradovskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 1027 (1956); translation: Soviet Phys. JETP **31**, 904 (1957). These authors show that the post-Newtonian equations of planetary motion are contained in second order in the solutions of the equations (17). This remark also applies to the present theory as our solutions are identical in second order with Einstein's theory.

[7] The motivations and plausibility of constant mass density is related to the "perfect cosmological principle" of Bondi and Gold. See H. Bondi, *Cosmology* (Cambridge University Press, London, England, 1952). This book contains also a complete discussion of the problem of inertia and Mach's principle in Chap. IV.

sense, the total matter in the universe is conserved. On the other hand, due to recessions of nebulae, matter is leaving continually our operationally defined universe in all directions. Then, to keep the solution time-independent, we must keep the density constant. This leads to the idea of continual creation as discussed by Bondi-Gold and Hoyle.

We now turn to a derivation of Mach's principle. Take a test particle of gravitational mass m_g. The potential energy of this particle in the presence of all other masses in the universe is $m_g(4\pi G\sigma/3)\bar{R}^2$ where $\bar{R}^2$ is the operational radius square for the universe. For this potential energy, we obtain

$$E_p = m_g \int_0^\infty (4\pi\sigma G/3)\exp(-\alpha^2 r^2/2) 2r dr = m_g c^2. \quad (30)$$

Since from special relativity any kind of energy possesses inertial mass, we have $E_p = m_i c^2$. Hence the result just obtained means that the inertia of a body and therefore the local inertial frame is determined by the existence of all the other bodies in the universe. This latter statement is known as Mach's principle.[7] In this connection we call attention to a curious feature of the present theory: Eq. (30) regards the test body as a particle, while the derivation given by (21) is purely a field-theory calculation.

We may note that by a method of substitution[8] known in the usual general relativity theory, the line element (25) can be transformed up to and including order $\alpha^4 r^4$ in g_{44} and up to and including order $\alpha^2 r^2$ in g_{11} into the de Sitter line element,

$$ds^2 = (1-\alpha^2 r^2)c^2 dt^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 - (1-\alpha^2 r^2)^{-1} dr^2. \quad (31)$$

In order to investigate the behavior of mass particles as a whole, we may choose, if we wish, a new set of coordinates, in which the particles appear constantly on the coordinate surfaces. Such a transformation of (31) is well known to lead to

$$ds^2 = c^2 dt^2 - e^{2\alpha t}(dx^2 + dy^2 + dz^2). \quad (32)$$

This is indeed the standard Bondi-Gold form of the line element representing an expanding but steady-state universe. With Hoyle's theory, which is arranged to give (32) exactly by adding an additional hypothesis (a creation term) to the general relativistic field equations in order to satisfy the Bondi-Gold principle, we differ only at third order in $\alpha^2 r^2$ (e.g., the cosmological red shift is a first-order effect). Thus, for most practical purposes and for most parts of the universe thus far explored, our theory gives essentially the same results as the Bondi-Gold and Hoyle theory. The *time-scale difficulty* which arises in the first order is eliminated in the same manner as in the Bondi-Gold theory. An important difference, however, exists between the two theories in the case of the soft-photon (radio-frequency) content of the universe. To make this clear, we will call attention to the fact that (*i*) the operational four-dimensional volume of our universe is $(\pi/2)^2$ times larger than the Bondi-Gold universe, and (*ii*) the factor $\exp(-\alpha^2 r^2)$ is a softer cutoff than the factor $(1-\alpha^2 r^2)$ in (31). Due to these, the radiation received from the most distant parts of the universe (shifted to radio-frequency region) is considerably greater than what will be expected according to the Bondi-Gold theory. This fact is in qualitative agreement with the observation. Radio-astronomers have found that the universe has 10 to 100 times more radiation in the soft-photon region than is predicted by the existing theories of the universe and of radiation.

From a conceptual point of view the original Bondi-Gold theory was incomplete because it did not have field equations. Hoyle modified the Einstein field equations by adding a new term so that the Bondi-Gold line element becomes a solution of these modified field equations. Unfortunately the new term destroys the general covariance of the whole theory (e.g., it implies fundamentally preferred coordinate systems). Our theory on the other hand gives the satisfactory Bondi-Gold theory as an approximation (and more) without any additional hypothesis and further without destroying the general covariance of the field equations. This, of course, is a highly desirable feature.

VI. PRINCIPLE OF OBSERVATION

In this section we will examine the solution (18) and from it motivate a new principle which will be called the principle of observation.[9] We start with the remark that the validity of the line element (18) as a solution of Einstein's field equations (17) is unaltered if we change ϕ by an additive constant ($\phi \rightarrow \phi + K$, in a sense a gauge invariance). We could, of course, say that the constant is not physically meaningful and fix it once and for all, say by putting it equal to zero. However, there is another possibility, namely that the constant is not trivial, but rather that it is indicative of the position of the particular stationary observer in the gravitational field. But how do we prescribe a definite constant for a definite stationary observer? To do this, we introduce a new principle which we call the principle of observation. The principle states: *Any observer will observe light local to himself to propagate with the same velocity in all directions regardless of his position in the gravitational field.* Thus, for a stationary observer at x' the constant should be chosen to be $-\phi(x')$ so that

$$ds^2 = e^{-2[\phi(x)-\phi(x')]}dt^2 - e^{2[\phi(x)-\phi(x')]}(dx^2 + dy^2 + dz^2), \quad (33)$$

[8] Reference 4, p. 240.

[9] The principle was first discussed at Washington Meeting of the American Physical Society, April, 1955, under the name of the "Principle of Superposition." Also, a lecture was given at the Theoretical Physics Colloquium, National Research Council, Ottawa, Canada, 1955, on its application to gravitation, cosmology, and field theory.

and local to himself ($x=x'$) one sees

$$ds^2=dt^2-dx^2-dy^2-dz^2$$

independent of where x' is. For an observer at a large distance from a finite body $\phi(x')\rightarrow 0$.

Whether or not the principle is true will be determined by future observations. There has never been an experiment to test the validity of the principle, and indeed it is seen that it would be quite difficult to perform such an experiment. It should be emphasized that the validity of the scalar field theory of gravity presented in this paper is independent of the validity of the principle. The principle of observation is an additional physical statement arising from a possible interpretation of a degree of freedom due to the ambiguity of ϕ to within an additive constant.

Now in view of (33) consider the direct transformation,

$$\begin{pmatrix} d\mathbf{r}' \\ dt' \end{pmatrix}=\begin{pmatrix} e^{\phi} & 0 \\ 0 & e^{-\phi} \end{pmatrix}\begin{pmatrix} d\mathbf{r} \\ dt \end{pmatrix}, \qquad (34)$$

between the covariant components of the displacement vector, dx_μ, where $\phi=\phi(x,x')=\phi(x)-\phi(x')$. This is a transformation between two observers O and O' situated at the points x and x', respectively, and ϕ is the potential difference between the two observers.

Equation (34) is to be understood as follows: the observer O at x measures, say, a local displacement dx. The observer O' situated at x' sees the components of the same displacement as different depending upon the potential difference between the two observers.

If the two observers are at the same point, then $\phi(x,x')=\phi(x)-\phi(x')=0$, and the line element (33) reduces locally to the Lorentz form. If we denote the 2×2 transformation matrix in (34) by $(B|A)$ we see that the transformation is transitive, that is

$$(B|A)=(B|S)(S|A). \qquad (35)$$

We also note that these matrices, when understood in the sense of (34), are unimodular:

$$\det(B|A)=1. \qquad (36)$$

We shall now examine what physical meanings we can attach to formulas like (21), (28), and (29). We know that energy and time have the same transformation properties. Therefore, if an observer situated at x' observes the energy of a body at x in the gravitational field, we shall have

$$t'=te^{-\phi(x,x')}; \quad E'=Ee^{-\phi(x,x')}, \qquad (37)$$

where $\phi(x,x')=\phi(x)-\phi(x')$. Using $E=mc^2$, we have in first order the expected result,

$$\Delta t/t=(G/c^2)\left(\frac{M}{r}-\frac{M}{r'}\right); \quad \Delta E=m\left(\frac{GM}{r}-\frac{GM}{r'}\right). \qquad (38)$$

The first of these corresponds to time dilation in the gravitational field (red shift). The second is the familiar expression of the gravitational potential energy difference for a mass m.

Let us now consider (21) in the same sense. We write it as

$$E=(1/4\pi)\int_0^\infty (GM_g^2/r^4)e^{-\phi(r,r')}dv. \qquad (39)$$

Clearly this formula means that at each point the local observer measures the special relativistic result $(1/4\pi)(GM_g^2/r^4)dv$, but when it is referred to the observer O' we transform it with the transformation function given by the exponential. This way we get the interesting result

$$E'=M_gc^2e^{\phi(r')}. \qquad (40)$$

The formula given by (21) holds for an observer at infinity (observation from fixed stars). Thus, in view of these and (28) and (29) we see that for physical quantities of these kinds we have, in general, the expression

$$\xi_B=\int e^{\phi(B)-\phi(A)}d\xi_A. \qquad (41)$$

The relations given by (34), (35), (36), and (41) and their generalization to other quantities is in general what we want to refer to as the *principle of observation*. We notice that similar relations are also valid for the Lorentz transformations. Therefore, the principle of observation, in this form, can be regarded as a natural generalization of the special theory of relativity.

It is interesting to note also that the observation principle provides a means to calculate M^i's in formula (21′). For example, if we denote the noninteracting value of M^i as M^{0i}, we have for an observer at infinity

$$M^i=M^{0i}\exp\Big(-\sum_{k\neq j}GM^k/c^2|r_k-r_j|\Big). \qquad (37')$$

More general cases can be calculated similarly.

The principle of observation can be formulated in an abstract and postulational manner and leads to many interesting consequences. But here we do not want to go into it any further. We only mention that the theory presented in this paper was developed on the basis of this principle and the line elements corresponding to (20) and (24) were discussed earlier (1955) in connection with the central-field-problem cosmology and field theory. But it was only recently that the author has realized that the general line element (18) which was obtained from the principle of observation is a rigorous solution of the field equations (17). It is now clear, by reversing the argument, that the principle of observation implies to a certain extent a *field theory* derivable from a Lagrangian of the form (1). Once the Lagrangian is fixed, the stress-energy distribution in (17) is uniquely determined. This distribution is left arbitrary in

Einstein's theory, in which one can assign the stress-energy tensor in any way one wishes but one does not know whether such choices are *actually* realized in nature.

The principle of observation brings along with the possibility of distant observation the concept of the *relativity of geometry*.[10] For as we have seen by actual examples, the geometrical attributes as well as the material content of the world depend upon the position of the observer.

VII. A MANIFESTLY COVARIANT FORMULATION OF THE THEORY

In this section we present a generally covariant formulation of the theory. Let us consider a four-dimensional space-time continuum given by the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{42}$$

When this continuum contains no stress energies and momenta let it be understood that it is flat. On the other hand, if it contains stress energies and momenta, following the Einsteinian ideas we assume that it will be curved. Now let a scalar field $\phi(x)$ be defined in this continuum. Physically, to the field ϕ there corresponds a certain stress-energy-momentum distribution which in turn contributes to the curvature of the space-time geometry defined by (42). In particular if there is nothing else contained in the geometry, its curvature will be completely determined by the field ϕ. Mathematically, this means that the metric tensor $g_{\mu\nu}$ will functionally depend on ϕ as $g_{\mu\nu}(\phi)$. In order to formulate the theory in a covariant manner, it is convenient to use the Lagrangian procedure. Let the Lagrangian density, $\mathfrak{L}$, of the field $\phi(x)$ be

$$\mathfrak{L} = \mathfrak{L}(\phi, \phi_\mu), \tag{43}$$

where $\phi_\mu = \partial_\mu \phi = \partial\phi/\partial x^\mu$. The equations of motion of the field $\phi(x)$ are then obtained from the principle of stationary action,

$$\delta \int \mathfrak{L}(\phi,\phi_\mu)(-g)^{\frac{1}{2}} d\Omega = 0, \tag{44}$$

where ϕ and ϕ_μ are the quantities to be varied, and $(-g)^{\frac{1}{2}} d\Omega$ is the four-dimensional invariant volume element. Following the usual rules of variational calculus in general coordinate systems, we obtain the equation of motion of the field as

$$\frac{\partial \mathfrak{L}}{\partial \phi} - \left(\frac{\partial \mathfrak{L}}{\partial \phi_\mu} \right)_{;\mu} = 0, \tag{45}$$

where the semicolon means covariant derivative.

The stress-energy tensor is obtained from the Lagrangian density as

$$T_\mu{}^\nu = \phi_\mu \frac{\partial \mathfrak{L}}{\partial \phi_\nu} - \delta_\mu{}^\nu \mathfrak{L}. \tag{46}$$

The covariant divergence of this tensor vanishes as a consequence of the equation (45):

$$(T_\mu{}^\nu)_{;\nu} = \phi_\mu \left[\frac{\partial \mathfrak{L}}{\partial \phi} - \left(\frac{\partial \mathfrak{L}}{\partial \phi_\nu} \right)_{;\nu} \right] = 0. \tag{47}$$

Of course, these equations are valid for any scalar, $\mathfrak{L}$, and therefore the actual dependence of $\mathfrak{L}$ on ϕ and ϕ_μ must depend on other considerations. Two plausible requirements are: (a) The equation of motion (45) of the field ϕ must be identical to the d'Alembert equation in the geometry defined by (42),

$$\phi^\mu{}_{;\mu} = (-g)^{-\frac{1}{2}} \frac{\partial}{\partial x^\mu} \left((-g)^{\frac{1}{2}} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = 0, \tag{48}$$

from which, to within a divergence, $\mathfrak{L}$ can be chosen to be

$$\mathfrak{L} = (1/8\pi) \partial_\mu \phi \partial^\mu \phi. \tag{49}$$

(b) The stress-energy tensor, $T_\mu{}^\nu$, must be identical to some geometrical tensor of second rank which has zero covariant divergence. This condition leads to the identification of $T_\mu{}^\nu$ as the Einsteinian tensor, $G_\mu{}^\nu = R_\mu{}^\nu - \frac{1}{2}\delta_\mu{}^\nu R$, to within a constant factor depending on the choice of the units. Thus, requiring positive definiteness for the energy, T^{44}, of the field and choosing the units suitably, we have

$$8\pi T_\mu{}^\nu = R_\mu{}^\nu - \tfrac{1}{2}\delta_\mu{}^\nu R. \tag{50}$$

Now substituting (49) into (46) and then contracting both sides of (50), we get

$$R = 16\pi \mathfrak{L} = 2\partial_\mu \phi \partial^\mu \phi, \tag{51}$$

where R is the curvature invariant.

The next question is this. R depends on the second-order derivatives of $g_{\mu\nu}(\phi)$ and consequently on the second-order derivatives of $\phi(x)$, whereas the right-hand side of (51) contains only the first derivatives. This problem is resolved if we remember that two integrands of the action integral, $\mathfrak{L}\sqrt{-g}$, differing by at most an ordinary divergence describes the same

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[10] Locally, i.e., when $x = x'$, we get $\phi(x,x') = 0$, so that the metric tensor reduces to $g_{11} = g_{22} = g_{33} = -1$; $g_{44} = 1$. But the observer can observe the phenomena at x when he himself is situated at x'. Or conversely the motion of an object at x can be referred to a distant reference *frame* just as well as a local reference frame when $\phi(x,x')$ is properly introduced. This throws new light upon the much-discussed coordinate issue in general relativity. For example, the present theory tells us that the usual general relativity does not contain the position of the observer, and therefore $(M_g/r') \to 0$; $r' = \infty$, i.e., it implies observation from fixed stars. This answers Wigner's question on the coordinate problem of general relativity; E. P. Wigner, Revs. Modern Phys. **29**, 255 (1957). For instance, an observer who moves with the planet Mercury would never be able to find out the perihelion motion of that planet unless he refers the motion to other objects, say, to fixed stars.

physical system. Therefore the addition to $\mathcal{L}$ of, say

$$\phi^{\mu}{}_{;\mu}=(-g)^{-\frac{1}{2}}\frac{\partial}{\partial x^{\mu}}\left((-g)^{\frac{1}{2}}g^{\mu\nu}\frac{\partial\phi}{\partial x^{\nu}}\right),$$

changes nothing and supplies the second-order derivatives sought.

In any case, this term is zero, due to (48), everywhere except at the singularities of the field. In no field theory are the field equations expected to hold at the singularities. As it is well known from classical field theories, this does not prevent us from including the effect of the singularities as the sources of the field. When sources are included, Eq. (48) takes the form

$$\phi^{\mu}{}_{;\mu}=(-g)^{-\frac{1}{2}}\frac{\partial}{\partial x^{\mu}}\left((-g)^{\frac{1}{2}}g^{\mu\nu}\frac{\partial\phi}{\partial x^{\nu}}\right)$$
$$=-4\pi(-g)^{-\frac{1}{2}}\sum_{j} M_g{}^j\delta(x-x^j), \qquad (52)$$

where $M_g{}^j$ are the strengths of the mass singularities at the points x^j, and $\delta(x-x^j)$ is the δ function. The subscript g is to indicate that M_g is the gravitating mass.

VIII. DISCUSSION

In this section we would like to discuss the differences and interrelationships of the present approach to other theories of gravitation with special emphasis on Einstein's theory. As we have seen, the two basic principles of Einstein's theory, namely the principle of general covariance and the principle of equivalence, are both preserved. Also, the geometrical identification of the stress-energy tensor $T_{\mu\nu}$ as

$$8\pi T_{\mu}{}^{\nu}=R_{\mu}{}^{\nu}-\tfrac{1}{2}\delta_{\mu}{}^{\nu}R, \qquad (53)$$

is not altered. But the interpretation of $T_{\mu}{}^{\nu}$ is completely different. In Einstein's theory, it is the stress-energy tensor of matter $T_{\mu\nu}=\rho v_{\mu}v_{\nu}$ alone. The gravitational field is not included in it. In the present approach it is the stress-energy tensor of the gravitational field $T_{\mu\nu}=(1/8\pi)(2\phi_{\mu}\phi_{\nu}-g_{\mu\nu}\phi_{\sigma}\phi^{\sigma})$ that is employed. The effect of the sources of the field $\phi(x)$ are taken into account in Eq. (51). These two interpretations are completely different from each other. Yet they lead to solutions which are so close to each other that they predict the same numerical results with regard to the so-called three experimental tests of general relativity. Differences become significant only at extremely close distances to a point singularity and also for extremely large systems such as the cosmos itself.

The next and perhaps the most important difference is the interpretation of what constitutes the gravitational field. In Einstein's theory, the gravitational field is represented by the components of the metric tensor, $g_{\mu\nu}$. In the present theory, the gravitational field is $\phi(x)$. The metric tensor is functionally dependent on this field as $g_{\mu\nu}(\phi)$. As a result the meaning of the Einstein equations are different in the two theories. In Einstein's theory they are the field equations for $g_{\mu\nu}(x)$. In the present theory the Einstein equations are algebraic identities (away from the singularities) expressing the equivalence between the geometric [right side of Eq. (53)] and the field theoretic [left side of Eq. (53)] descriptions of gravitation. The actual field equation is the wave equation

$$\phi^{\mu}{}_{;\mu}=-4\pi(-g)^{-\frac{1}{2}}\textstyle\sum_j M_g{}^j\delta(x-x^j).$$

In Einstein's theory, the law of gravitation in a matter-free part of space is $R_{\mu\nu}=0$. In our theory, the corresponding law in empty space is expressed as

$$R_{\mu\nu}=2\phi_{\mu}\phi_{\nu};\quad \phi^{\mu}{}_{;\mu}=0.$$

Consequently, in our theory, the equations $R_{\mu\nu}=0$ leads to $\phi=\text{constant}$, hence, imply $g_{\mu\nu}(\phi)=\text{constant}$, and therefore a completely flat space-time where gravitational effects vanish.

Our theory is a scalar theory in the usual sense of the word, but it is a *generally covariant* theory in contrast to scalar theories which are only Lorentz-covariant.* It is well known that the Lorentz-invariant scalar theories of gravitation lead to only half of the observed deflection of light and to a wrong prediction for the perihelion of Mercury.[11] The general covariance and the principle of equivalence seem to be essential in these small effects.

As we have emphasized, our theory preserves both the principle of general covariance and the principle of equivalence, although, in a sense, the principle of equivalence is not an independent postulate (we have seen in Sec. IV that the principle of equivalence may be considered as a derived consequence of the present theory). Thus, our theory differs radically from the theories which assume the principle of general covariance but reject the principle of equivalence. One such scalar theory was recently proposed by Dicke.[12] His ideas are somewhat similar to ours, especially in his assumption of local Lorentz invariance and his use of isotropic line elements. But, unfortunately, this theory contains many *ad hoc* assumptions and leads to a

* *Note added in proof.*—It has recently been called to the author's attention by S. Schneider that the Einstein field equations with the $T_{\mu\nu}$ of a spherically symmetric static scalar field was considered by O. Bergmann and R. Leipnik, Phys. Rev. **107**, 1157 (1957). Although conjecture was made there of the possible need for a covariant scalar field to account for the Mach principle no definite identification of the field ϕ and no concrete consequences of it were presented. In particular since they did not use an isotropic coordinate system and they did not obtain explicit solutions for $k\neq 0$ (k is the coupling constant between $T_{\mu}{}^{\nu}(\phi)$ and geometry) it appears that they did not realize $\phi=M/r=M/(x^2+y^2+z^2)^{\frac{1}{2}}$ is indeed meaningful even in the curved space

$$ds^2=e^{-2\phi}dt^2-e^{2\phi}(dx^2+dy^2+dz^2).$$

[11] See O. Bergmann, Am. J. Phys. **24**, 38 (1956), as an example of such theories.

[12] R. H. Dicke, Revs. Modern Phys. **29**, 363 (1957).

wrong numerical value ($\frac{5}{6}$ of the observed value) for the perihelion motion of the planet Mercury.[13]

Also there are some similarities between our theory and the ideas of Fock.[14] He feels that Einstein's theory is too general and some restrictions must be imposed on the solutions of the field equations. He imposes the coordinate conditions

$$\frac{\partial}{\partial x^\nu}[(-g)^{\frac{1}{2}}g^{\mu\nu}]=0,$$

and shows that under these conditions space-time accepts a Lorentz group of transformations. He calls the coordinates which satisfy these conditions "harmonic coordinates." However, this condition is not restrictive enough to lead to unique physical situations, nor is it necessary for the derivation of equations of motion as originally claimed by Fock. This latter point is discussed by Infeld.[15] But it is extremely interesting that our static solution (18) satisfies the conditions of Fock and therefore our coordinates are harmonic. In spite of this interesting relation our theory differs fundamentally from Fock's ideas because Fock adheres to the conventional interpretation of the stress-energy tensor as "matter tensor." Nevertheless, we feel that Fock's interpretation of general covariance and of Lorentz invariance seems to be closely realized by the present approach.

An essential difference of our theory from all other theories of gravitation is the existence of the principle of observation. By virtue of this principle the position of an observer is incorporated into the structure of the theory. This is done in such a manner that the special relativistic interpretations of physical quantities are valid locally. With the help of this principle one treats the interrelationships of observers in a gravitational field by a method similar to Lorentz transformations. The principle provides transitive and unimodular transformations. These properties are shared also by the Lorentz transformations. In this sense the principle is consistent with the original idea of relativity as expressed by the Lorentz transformations and therefore it can be considered as a natural extension of the Lorentz transformations. It is perhaps interesting to emphasize again in this connection that according to the principle of observation the velocity of light as measured locally is a universal constant.

Another important difference between our theory and all other theories of gravitation is that we give functional solutions as $g_{\mu\nu}(\phi)$, where ϕ is any function satisfying the Laplace's equation $\nabla^2\phi=0$ except at the singularities. This way the static problem is solved generally. In earlier approaches, for every new distribution of matter one had a new problem to solve in the form of $g_{\mu\nu}(x)$. Since the equations are coupled nonlinear partial differential equations, to attempt to solve them for every new situation was an "impossible" task.

Also, we note that the linear combination $a\phi_1+b\phi_2$ of two solutions ϕ_1 and ϕ_2 corresponds to another solution of the field equations. Thus the theory is, in this sense, *linear*. The nonlinearity of the original Einstein's theory caused insurmountable difficulties both in applications and the quantization of the theory. One sees already from the above-mentioned linearity how easily the present theory can be applied to various physical problems and how easily the quantization of $\phi(x)$ may proceed by taking over the standard Lorentz-invariant field quantization methods into general coordinate systems.

Finally, we may discuss here the question of what are the basic assumptions and the derived consequences of the theory. As we have presented it in this paper, the two basic assumptions of the theory are (1) the principle of general covariance and (2) the choice of the Lagrangian as a scalar field Lagrangian. Thus in a sense the theory of gravitation presented here is a synthesis of the curved space concept of Einstein and the Newtonian scalar gravitational field. The principle of equivalence appears as a derived result of the line element (18). There are not many differences in the process of identification of $T_\mu{}^\nu$ as a divergenceless geometric tensor in our theory and in Einstein's theory. Therefore, the only other conceptual change is the introduction of the principle of observation. In a sense this principle, too, may be considered as already contained in the theory. For, as we have seen in Sec. VI, it can be introduced as a property of the solutions of the equations (17). In another sense, however, it is a new and independent additional statement since it incorporates the position of the observers and implies further relativity requirements on physics and geometry. It is perhaps best to consider it at present as a correspondence principle which helps to understand observational relations between various observers in terms of a local set of Lorentz frames.

The principle of observation can in principle be tested experimentally by measuring the local velocity of light in a strong gravitational field. If the principle is right, the local measurement must always give the same value, c, with or without the gravitational field. If it is wrong, there will be a discrepancy amounting to $\delta c=-(2GM/cr)$. However, it should be stressed again that even if the principle is wrong, this does not invalidate the idea of a generally covariant scalar field theory of gravity, as presented in this paper. The principle of observation is an additional statement beyond the scalar field concept.

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[13] R. H. Dicke (private communication).
[14] V. Fock, Revs. Modern Phys. **29**, 325 (1957).
[15] L. Infeld, Revs. Modern Phys. **29**, 398 (1957).

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Possible Experimental Test of Universal Fermi Interaction

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The decay modes K_{e3} and $K_{\mu 3}$ of charged and neutral K particles are discussed with the aim of deriving, in the Feynman-Gell-Mann-Marshak-Sudarshan theory, possible experimental tests of the hypothesis of universal Fermi interaction, which is already apparently contradicted by the present data on the ratio of $\pi \to e+\nu$ to $\pi \to \mu+\nu$. Measurement of the K_3 spectra would already provide a test of the hypothesis, and measurements of the polarizations would give further confirmation. Unique forms of the spectra of the charged leptons are predicted on the basis of the universality hypothesis and of particular assumptions.

INTRODUCTION

FEYNMAN and Gell-Mann[1] and, independently, Marshak and Sudarshan[2] have recently proposed a theory of the weak interactions (to which we shall briefly refer to as the FGMS theory) based upon the assumption that the different spinor fields ψ are weakly coupled only in the projection $\frac{1}{2}(1+\gamma_5)\psi$. This theory seems to account successfully for most of the established experimental evidence on weak interactions. The total weak interaction is assumed to arise from the coupling of a current J_λ with itself; J_λ is the sum of bilinear covariants $[\bar{\psi}_a\gamma_\lambda(1+\gamma_5)\psi_b]$ over certain pairs of fermions a, b that satisfy particular requirements [for instance, (a,b) must be a single charged pair]. The current J_λ will in particular contain a part $[\bar{\psi}_\nu\gamma_\lambda(1+\gamma_5)\psi_e]+[\bar{\psi}_\nu\gamma_\lambda(1+\gamma_5)\psi_\mu]$ thus implying that μ and e have exactly the same weak interactions (conservation of leptons requires μ^- and e^- to be both particles).

Apparently both μ and e have no strong couplings. Under such conditions, from the hypothesis of minimal electromagnetic interaction, μ and e would also have exactly the same electromagnetic couplings, and the notion of relative parity between μ and e would lose any meaning. Measurements of the magnetic moment of the muon[3] do not give evidence so far for a complicated structure of the muon, as would be expected if the muon possessed strong interactions. The only difference between the two particles would then be due to the remarkably large difference of their masses. Such a situation seems rather peculiar but, if definitely established, may turn out to be very suggestive.

On the other hand, the only available experimental evidence, that is directly related to the problem, is the experimental upper limit for the ratio of $\pi \to e+\nu$ to $\pi \to \mu+\nu$. Anderson and Lattes find only a 1% probability that this ratio could be greater than 2.1×10^{-5}.[4] The value predicted by the FGMS theory is 13.6×10^{-5}, as can be shown independently of perturbation theory for the strong interactions.[5] The discrepancy may indicate either an intrinsic difference in the interactions of μ and e or a more complicated structure of the weak universal interaction.[6]

We want here to examine the possibility of an independent test of the hypothesis of identical interaction of μ and of e through a study of the decay modes $K\to\mu+\nu+\pi$ and $K\to e+\nu+\pi$ [namely: $K^\pm\to\mu^\pm$ or $e^\pm)+\nu+\pi^0$, $K_L{}^0\to\mu^+$ (or e^+)$+\nu+\pi^-$ or $\to\mu^-$ (or e^-)

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] E. C. G. Sudarshan and R. E. Marshak, *Proceedings of Padua-Venice Conference on Mesons and Newly Discovered Particles*, September, 1957 [Suppl. Nuovo cimento (to be published)].
[3] Coffin, Garwin, Penman, Lederman, and Sachs, Phys. Rev. **109**, 973 (1958).
[4] H. L. Anderson and C. M. G. Lattes, Nuovo cimento **6**, 1356 (1957).
[5] M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).
[6] It was proposed [R. Gatto, Nuclear Phys. **5**, 530 (1958)] that departures from locality, as introduced by Lee and Yang for μ decay [T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957)] could account for the $\pi\to e+\nu$ to $\pi\to\mu+\nu$ ratio. However the nonlocality required in this case would no longer be compatible with a form of the weak interaction as a coupling of the current J_λ with itself, even if such coupling is propagated through some finite space-time distance. (This model would instead be sufficient to explain in the FGMS theory the deviations of ρ from $\frac{3}{4}$, provided one removes from the theory the hypothesis of nonrenormalizability of the vector coupling.)