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Total Cross Section of Hydrogen for 143- to 205-Mev Positive Pions*

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Using counter techniques, a higher-precision determination of the $\pi^{+}+p$ total cross section in the neighborhood of the $T=J=\frac{3}{2}$ resonance has been made. The energy width, statistical errors, and absolute errors were considerably reduced in an extension of our previously reported work. The eight total cross sections which were determined in the pion kinetic-energy range of 143 to 205 Mev are compared to other earlier work and their theoretical interpretation is discussed.

I. INTRODUCTION

IN previous publications[1,2] we have reported a series of measurements of $\pi^{\pm}+p$ total cross sections for 150- to 750-Mev positive and negative pions, using the counter telescope transmission method. As one phase of this program, fourteen $\pi^{+}+p$ total cross sections were determined in the energy interval of 146 to 335 Mev using a liquid hydrogen target. These prior results indicated a peak in the lab system cross-section curve at $\sim$175 Mev. One should note here that all energies refer to pion kinetic energy in the lab system unless otherwise stated.

The work of Ashkin *et al.*[3] at that time and the somewhat later work of Ignatenko *et al.*[4] indicated that the lab system peak may occur at 195 Mev. Furthermore, when the α_{33} phase shifts deduced from our total cross-section results were plotted on a Chew-Low or Serber-Lee plot, an appreciable change in slope from the straight line at lower energies seemed probably to occur in the neighborhood of the resonance. This also implied the possibility of a somewhat lower energy resonance than the Bethe–de-Hoffman *et al.*[5] value at $\sim$195 Mev.

When one considered the sum of the relative and absolute errors involved in the various measurements, there was not necessarily a discrepancy in any one cross section; however, the systematic differences noted above were sufficient to warrant further study. Therefore, we performed a new series of measurements of the $\pi^{+}+p$ total cross section in the region of 140–205 Mev in which momentum widths, statistical errors, and relative errors were considerably reduced. This work will here be described, analyzed, and compared to previous work.

II. EXPERIMENTAL ARRANGEMENT, AND PROCEDURE

The experimental arrangement and procedure were similar to those previously described.[2] A five-element fast (2–3 mμsec resolution) scintillation-counter telescope with a deflecting magnet between the second and third counters was employed to define the momentum interval of positive particles accepted. Time-of-flight rejection of protons was employed. The analysis of the π^{+} beam and the μ^{+} plus e^{+} contamination was determined by range-curve methods previously described. The values of the contamination and the energy dependence were similar to the previous results shown in Fig. 5 of reference 2.

A Styrofoam target and an identical empty dummy were used alternately between the fourth and fifth counters to determine the transmission of π^{+} in H_2.

The Cosmotron was operated in the same manner as previously described[2] such that 0.8-Bev protons struck a beryllium target located in the east straight section. A direct beam produced at 32° from the target was allowed to pass through collimators in the shield and then was incident on the first two counters.

The experimental arrangement was changed from the previous one only in the following essential respects:

a. The inner copper can of the hydrogen target which contains the liquid hydrogen was reduced in length from 12.5 in. to 6 in. in order to reduce the energy

TABLE I. Absolute and relative errors of the systematic type in the total cross-section determination of hydrogen for positive pions of 143–205 Mev.

Source of error	Absolute rms percentage error	Relative rms percentage error
(1) Muon and electron contamination correction (3–12%)	1%	$\frac{1}{2}$%
(2) Geometry correction (3–4%)	1%	$\frac{1}{4}$%
(3) Background correction (1–2%)	$\frac{1}{4}$%	$\frac{1}{8}$%
(4) Hydrogen content of target	$\frac{1}{4}$%	<0.1%
(5) Double scattering, changes in π-μ decay due to energy loss in H_2, additional Coulomb scattering in H_2, and other small effects	$\frac{3}{4}$%	$\frac{1}{4}$%
(a) Total rms absolute error $=[1^2+1^2+(\frac{1}{4})^2+(\frac{1}{4})^2+(\frac{3}{4})^2]^{\frac{1}{2}}=1.6\%$		
(b) Total rms relative error $=[(\frac{1}{2})^2+(\frac{1}{4})^2+(\frac{1}{10})^2+(\frac{1}{8})^2+(\frac{1}{4})^2]^{\frac{1}{2}}=0.6\%$		

* Work carried out under the auspices of the U. S. Atomic Energy Commission.

[1] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **92**, 1578 (1953) and **93**, 917 (1954).

[2] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **100**, 306 (1955).

[3] Ashkin, Blaser, Feiner, Gorman, and Stern, Phys. Rev. **96**, 1104 (1954).

[4] Ignatenko, Mukhin, Ozerov, and Pontekorvo, Zhur. Eksptl. i Teoret. Fiz. U.S.S.R. **30**, 7 (1956) [translation: Soviet Phys. JETP **3**, 10 (1956)].

[5] de-Hoffman, Metropolis, Alei, and Bethe, Phys. Rev. **95**, 1586 (1954); H. A. Bethe and F. de-Hoffman, *Mesons and Fields* (Row Peterson and Company, White Plains, 1956), Vol. 2.

losses in hydrogen and thereby reduce the energy widths associated with the cross section.

b. The first and fourth counters, which define the momentum width of the incident beam, were replaced by $1\frac{1}{2}$-in. diameter counters instead of the $2\frac{1}{2}$-in. diameter counters previously used. The half-width of the incident beam momentum range was determined by wire calibration to be $<1\%$.

c. The absolute error associated with the momentum determinations by wire measurement[2] was reduced to $\sim\frac{1}{2}\%$. The relative error in mean momentum determinations was measured to be $\sim$0.1–0.2%.

III. CORRECTIONS AND CALCULATIONS OF CROSS SECTION

a. Background.—The background corrections were of the same order of magnitude (1–2%) as in the previous runs[2] and the nature of this background and the method of correction are described in reference 2.

b. Contamination.—The percentage of the sum of μ^+ and e^+ contamination in the pion beam as a function of incident energy was determined by range curves and the values are similar to the previous results shown in Fig. 5 of reference 2. The absolute error of the determinations is estimated to be $\sim$1%. The relative errors are estimated to be $\sim\frac{1}{2}\%$.

c. The geometry corrections.—The mean angle of acceptance of the last counter was $\pm 6.8°$. The geometry corrections were made in a manner similar to that previously described[2] except for the fact that the recent experimentally determined angular distributions[6–8] were employed to obtain the variation of the differential cross section near 0°, instead of the function $1+3\cos^2\theta$ which was previously used. The changes thereby introduced are insignificant. The very small corrections to the cross sections due to the interference of Coulomb and nuclear scattering were also taken into account as was done previously.

d. Momentum widths.—The momentum widths of the pion beam, corresponding to the measurements, were computed from the estimates of the incident momentum spectrum obtained by wire measurement and also calculations which were then modified appropriately to include the effects of the known energy loss in the hydrogen target. The corrections to the measured transmissions, the various sources of error, and the resultant cross sections were determined in the manner previously described.[2]

The various absolute and relative errors of the systematic type are listed in Table I.

TABLE II. The total nuclear interaction cross section of hydrogen for positive pions, obtained by measuring the transmission of liquid hydrogen.

Positive pion kinetic energy in Mev in the lab system	Total cross section of hydrogen in millibarns
143 ±3	140.5±5
162 ±3.5	170.5±3.5
170 ±3.5	198 ±3.5
173.5±3.5	193.5±3.5
177 ±3.5	198 ±5
183.5±3.5	192 ±3.5
195 ±3.5	174 ±4
205 ±3.5	178 ±4.5

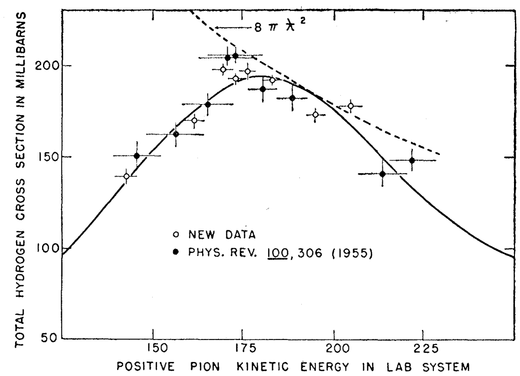

FIG. 1. The total cross-section values of hydrogen for positive pions of 143–205 Mev determined in the present work, plotted as a function of pion kinetic energy in the laboratory system. Our earlier measurements in and near this energy interval are also plotted for comparison. The solid line represents the contribution of α_{33} alone, taken from Fig. 3 according to the procedure described in the text in Sec. V.

IV. RESULTS AND COMPARISON WITH OTHER EXPERIMENTS

The resultant cross sections are shown in Table II. The errors (rms) listed are predominantly statistical although they also include the relative errors of the systematic type. The energy errors represent the half-width of the energy distribution of the interacting pions.

The new cross-section data are shown in Fig. 1 and compared to our previous results. It is clear that the general agreement between the new and old determinations is quite good considering the errors in both measurements. Although the new points near the peak (170,173,177) are slightly lower, the peak in the lab system probably occurs at $\sim$180 Mev, which is reasonably consistent within the errors with our previous observation[2] that the lab system peak occurs at $\sim$175 Mev. In particular, the values of the total $\pi^+ + p$ cross section in the region of 170–180 Mev seem to be significantly higher than the values at and near 195 Mev.

In Fig. 2, our results are compared to the latest results of other laboratories. The general agreement of all of the data is quite good. One should note that only relative errors are shown in most of the data and

[6] Mukhin, Ozerov, and Pontekorvo, Zhur. Eksptl. i Teoret. Fiz. U.S.S.R. **31**, 371 (1956) [translation: Soviet Phys. JETP **4**, 237 (1957)].

[7] Ashkin, Blaser, Feiner, and Stein, Phys. Rev. **101**, 1149 (1956); **105**, 724 (1957).

[8] H. L. Anderson and M. Glicksman, Phys. Rev. **100**, 268 (1955); Anderson, Davidon, Glicksman, and Kruse, Phys. Rev. **100**, 279 (1955).

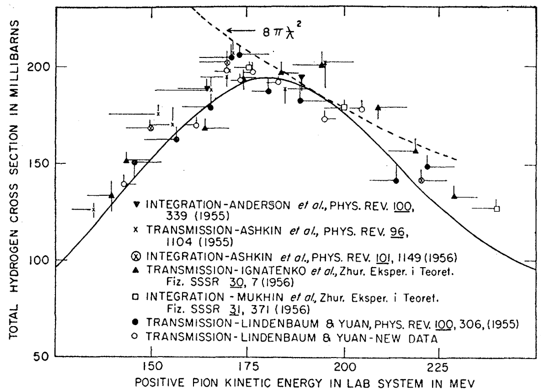

FIG. 2. A comparison of the recent $\pi^{+}+p$ total cross-section data in the energy region of 125 to 250 Mev, determined by various groups. The solid line represents the contribution of α_{33} alone, taken from Fig. 3 according to the procedure described in the text in Sec. V.

absolute scale factor errors, which are comparable in many cases to the errors shown, are not included.

When one considers our new results taken together with our previous results, it appears that the peak of the $\pi^{+}+p$ total cross-section curve in the lab system occurs at ~ 180 Mev in the lab system. This is consistent with the recent total cross sections at 176 and 200 Mev determined via integration obtained by Mukhin *et al.*[6] The cross-section value at 200 Mev is much less than that at 176 Mev. However, the earlier results[4] obtained by this group by transmission indicated a relatively high value of total cross section at 195 Mev, but the errors were larger than in the latest determination.

In the previous work of Ashkin *et al.*,[3] a relatively high value of the cross section at 195 Mev was also indicated; however, as they pointed out, the associated error on this point was relatively larger than on their other points.

V. ANALYSIS OF RESULTS

The $8\pi\lambda\!\!\!^{-2}$ curve in Figs. 1 and 2 shows the contribution to the $\pi^{+}+p$ total cross section at the resonance energy due to a p-wave resonance in the state of isotopic spin (T) and angular momentum (J) equal to $\frac{3}{2}$. It is clear that within the errors the total $\pi^{+}+p$ cross-section curve appears to be either tangent to or larger than the $8\pi\lambda\!\!\!^{-2}$ line somewhere in or near the region of $\sim$180–200 Mev. Hence these data are consistent with a resonance in the $T=J=\frac{3}{2}$ state in this energy region.

The Fermi set of phase shifts for S-P wave analysis with a resonance in α_{33} [9] near 190 Mev seems fairly well established when the latest analyses of the differential $\pi^{\pm}+p$ scattering experiments[5–8,10] are considered in conjunction with the requirement that the dispersion relations for the spin-flip scattering amplitudes are satisfied.[11,12] Two groups[11] have recently shown that the Yang sets do not satisfy these dispersion relations. Furthermore, it has also recently been shown[12] that the two sets of phase shifts generated from the conventional Fermi and Yang sets via use of the Minami ambiguity[13] both violate the predictions of the dispersion relations for the spin-flip scattering amplitude.[12] Therefore, the Fermi set is the only one that satisfactorily describes the pion-nucleon scattering below 300 Mev. This set for the $\pi^{+}+p$ scattering contains three phase shifts for the assumed S and P analysis, namely α_{33}, α_{31}, and α_{3}.

For the Fermi set, it appears from recent analyses[6–8,10,14–17] that only α_{33} is large and exhibits a resonance near 190 Mev. α_{3} is consistent with the Orear prescription $[\alpha_{3}=-0.11\eta]$ below 200 Mev. This prescription would contribute an approximately constant value of 3 mb to the total cross section in the energy region under consideration. The α_{31} phase shift is less well determined but appears to be [6–8,10,7–14] less than $10°$ and probably negative below 200 Mev. This will also contribute $\lesssim 3$ mb in the resonance region. Hence, $\gtrsim 97\%$ of the total $\pi^{+}+p$ cross section in the resonance region is expected to come from the contribution of the α_{33} phase shift. That this is indeed so is clear from Fig. 1 in which the solid curve represents the calculated contribution to the total $\pi^{+}+p$ cross section from the α_{33} phase shift.

A Chew-Low plot using the best values of α_{33} obtained from the phase-shift analyses of the pion-nucleon scattering[6–8,14–17] is shown in Fig. 3. A Serber-Lee plot using the same data has been published.[10] There is also in reference 10 a general discussion of the Chew-Low and Serber-Lee equations.

For both the Chew-Low plot (Fig. 3) and the Serber-Lee plot it is apparent that the data can be reasonably fitted by one straight line below resonance and another straight line of considerably different slope above resonance. This change of slope in these plots had been previously suggested[2] by the analysis of our earlier $\pi^{+}+p$ total cross-section measurements in the resonance region. The change of slope in these effective range plots is not too surprising, since they are only expected to be straight lines for energies low compared with the

[9] Charge independence is assumed in the phase-shift analysis. The first index equals twice the isotopic spin and the second index equals twice the angular momentum of the state, except for the case of an S wave for which the second index is omitted.

[10] S. J. Lindenbaum, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 317.

[11] W. C. Davidon and M. L. Goldberger, Phys. Rev. **104**, 1119 (1956); W. Gilbert and G. R. Screaton, Phys. Rev. **104**, 1758 (1956).

[12] S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **110**, 1174 (1958).

[13] S. Minami, Progr. Theoret. Phys. Japan **11**, 213 (1954).

[14] Ferrari, Ferretti, Gessaroli, and Manaresi, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. 2, p. 230.

[15] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1954).

[16] H. Taft, Phys. Rev. **101**, 1116 (1956).

[17] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).

effective cutoff energy which would correspond to $\omega_t^* \sim 6$.

One might note that the resonance energy is slightly above the threshold ($\sim$170 Mev) for producing an additional pion, and hence two meson states may begin to contribute appreciably. Furthermore, even a small inelastic cross section may slightly affect the values of α_{33} deduced from a real phase-shift analysis which is the only method that has been used by the various authors.

Due to the change in slope of the Chew-Low and Serber-Lee plots near the resonance, the determination of a resonance energy is uncertain. The resonance energy for the Chew-Low plot is determined by the point where the ordinate $[\eta^3 \cot\alpha_{33}/\omega_t^*]$ is zero, which is the intercept on the ω_t^* axis. The best guess for which the rate of change of slope of the plot would not be too rapid is that the intersection of the broken line (solution above resonance) with the ω_t^* axis is the resonant point. A resonable estimate from Fig. 2 is that the resonant energy is $\sim 190_{-10}^{+20}$ where the estimation of errors is somewhat uncertain. One should note here that if the low-energy straight line had been used to determine the resonance without taking into account the subsequent change of slope in the region just below the resonance, the resonant energy would be at 215 Mev. The same values for the resonance energy $[\sim 190_{-10}^{+20}]$ and the associated errors were previously[10] deduced from the Serber-Lee plot.

A reasonable functional variation of α_{33} with energy as deduced from the Chew-Low plot would appear to be obtained by following the solid line to 150–170 Mev and making a smooth transition to the broken line in this region and following the broken line thereafter. The same procedure was previously used for the Serber-Lee plot. The contribution to the $\pi^+ + p$ total cross section by the α_{33} phase shift was computed as a function of energy by following the above procedure and is shown as the solid line in Figs. 1 and 2.

The agreement of the data with the solid line is excellent when one considers that, as previously discussed, $\sim$3 mb independent of energy should be added

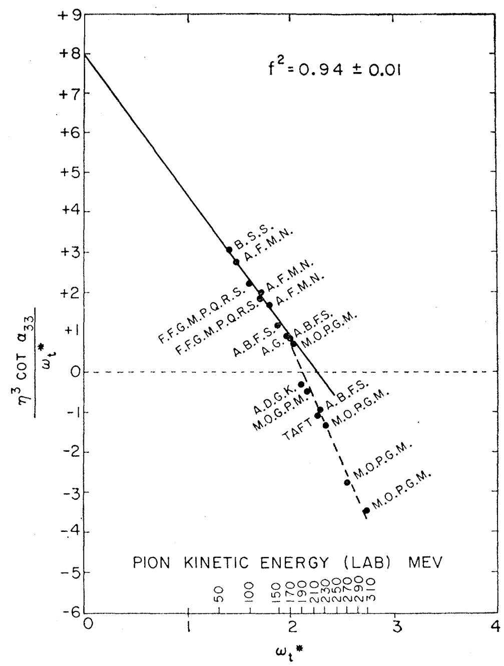

FIG. 3. A Chew-Low plot. The initials of authors' last names are used for identification of points (see references 6, 7, 8, 14, 15, 16, and 17).

for the contribution of α_3 to the total cross section and $\sim$3 mb should be added for the contribution of α_{31}.

The renormalized and unrationalized coupling constant[18] f^2 can be determined by the usual extrapolation from the Chew-Low and Serber-Lee plots. The values obtained are $f^2 \approx 0.094 \pm 0.01$ from the Chew-Low plot, and $f^2 \approx 0.107 \pm 0.01$ from the Serber-Lee plot. The experimental errors are at present larger than the difference between the two methods. If one considered primarily the α_{33} data in the resonance region, a considerably smaller f^2 would be obtained. This is the reason for earlier[2] lower determinations of f^2.

[18] A summary of the various recent coupling-constant determinations and an interpretation of the results is given in reference 10 on p. 326.