does not produce a significant difference in the values of the entropy at $T=298.16°K$.

It is interesting to compare the lattice heat capacity of diamond with those of the other elements crystallizing in the diamond structure, namely Si, Ge, and grey tin. This is most conveniently done by plotting reduced $\theta_{\text{eff}}$ *versus* reduced absolute $T$, reduction in each case being accomplished by dividing by the appropriate $\theta_D(0)$. Such a plot, incorporating the results of several investigators, is shown in Fig. 6, $\theta_D(0)$ being calculated from the measured elastic constants except in the case of grey tin where it was estimated from the observed limiting values. It is seen that the reduction of the observations in this way causes essentially a superposition of the results for all of the materials except diamond up to about $\theta_D(0)/6$. The behavior of diamond over this temperature range is exceptional although qualitatively similar. A discrepancy of the type shown in Fig. 6 suggests that the dispersion of lattice vibrations is similar in silicon, germanium, and grey tin and stronger in these substances than in the lattice prototype diamond.

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# Coupling of Angular Momenta in Odd-Odd Nuclei*

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The coupling of the angular momenta of individual particle states in odd-odd nuclei is shown to be generally describable as spin-spin coupling if the asymptotic-quantum-number description of particle states is used for deformed nuclei. Coupling rules for these nuclei are given, and all available data are treated by them. The results are compared with results based upon a *j-j* coupling model for a spherical nucleus. A formula based upon the present coupling description is given for calculating magnetic moments of deformed nuclei, and magnetic moments calculated by it are compared to the experimental moments and to those calculated assuming the gyromagnetic ratios of the odd nucleons are those given by the Schmidt formulas. A qualitative theoretical discussion of the basic validity of the coupling rules is given.

## I. INTRODUCTION

THE ground states of odd-odd nuclei provide significant information regarding the effective forces in nuclear matter. This problem was first studied by Nordheim,[1] who pointed out that the ground state spins of a number of odd-odd nuclei could be accounted for on the basis of a *j-j* coupling model plus certain simple rules governing the angular momentum coupling of the last odd proton and neutron. These rules are the following:

$$I=j_p+j_n \quad \text{if} \quad j_p=l_p\pm\tfrac{1}{2} \quad \text{and} \quad j_n=l_n\pm\tfrac{1}{2}, \qquad (A1)$$

$$I=|j_p-j_n| \quad \text{if} \quad j_p=l_p\pm\tfrac{1}{2} \quad \text{and} \quad j_n=l_n\mp\tfrac{1}{2}, \quad \text{"strong" rule}, \qquad (A2)$$

where $j_p$, and $l_p$ (or $j_n$ and $l_n$) represent the total and orbital angular momenta of odd proton (or neutron) as deduced from a study of neighboring odd-$A$ nuclei in the light of the nuclear shell model, and, more specifically, the single-particle version of this model.[2,3] The rule ($A1$) is frequently given in the following less specific form[4]:

$$|j_p-j_n|<I\leqslant j_p+j_n \text{ "weak" rule.} \qquad (A1')$$

In the present article, however, this form of the rule is not used.[5]

* Supported in part by the U. S. Atomic Energy Commission and the Office of Ordnance Research.

[1] L. W. Nordheim, Phys. Rev. 78, 294 (1950).

[2] M. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, Inc., New York, 1955).

[3] For a comprehensive review of the nuclear shell model and its applications, see J. P. Elliott and A. M. Lane, *Handbuch der Physik*, edited by S. Flugge (Springer-Verlag, Berlin, 1957), Vol. 39.

[4] See for example the discussion and application of this form of the rule in Way, Kundu, McGinnis, and van Lieshout, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1956), Vol. 6, p. 129.

[5] The form $A1'$ is the form implied by Nordheim (reference 1). Although we have been somewhat arbitrary in restating it as $A1$, we have done so in order to compare it more closely with the analogous rule $B1$. Stated in this definite form, the weak Nordheim rule is not, of course, as generally applicable as it has shown itself to be in the form $A1'$.

The Nordheim rules can be justified on the assumption that the intrinsic spins of the last odd proton and neutron always tend to line up parallel.

The case of nuclei with one odd proton and neutron outside closed shells, interacting by means of a delta-function force, was first investigated theoretically by de-Shalit[6] who found a theoretical basis for the success of the Nordheim rules, at least for nuclei of this kind. Other special configurations were investigated by Kurath[7] and by Flowers.[8] The odd-group model with $j$-$j$ coupling has been studied using different nuclear interactions.[9] Calculations involving an arbitrary number of particles outside closed shells, but still assuming the validity of $j$-$j$ coupling and delta function interactions were made by Schwartz.[9] He found that in most cases involving more than two particles outside of closed shells, the Nordheim rules, especially the strong rule, ($A2$), would be expected to hold. However, for nuclei in which we have a proton (or neutron) outside of a closed shell and a neutron (or proton) missing from a closed shell, the resultant ground-state angular momentum may be given by

$$I=j_p+j_n-1 \qquad (A3)$$

in agreement with the experimental evidence for these cases. This rule we will call ($A3$). A qualification of it is that both $j_p$ and $j_n$ are $\geqslant\frac{3}{2}$, so that there is a real distinction between particles and holes.

Pandya[9] has derived some general results for the odd-group model. His results indicate that when forces of longer range than those used by Schwartz are used to calculate the energy levels of odd-odd nuclei they do not, in general, agree with experiment.

In cases with $N=Z$, the results of the coupling calculations are somewhat ambiguous, especially for $j_p=j_n=\frac{3}{2}$. However, there is expected to be a rather close competition between the $T=0$, $J=1$; $T=0$, $J=2j$; and $T=1$, $J=0$ states for the ground state. These configurations we shall refer to as $A4$.

While the $j$-$j$ coupling model seems to have some validity for a surprisingly large class of nuclei (at least for predicting their ground state spins and binding energies), effects of the correlations between particles outside closed shells are extremely important for those nuclei in which a sizeable fraction of the nucleons are outside of (or missing from) closed shells.[3] Thus there are three well-defined groups ($A\sim 25$, $150<A<190$, $200<A$) for which the so-called rotational model is applicable.[10] In the application of this model it is assumed that the nucleons move approximately independently, but that the average binding field is spheroidal,[11] rather than spherical as in the conventional $j$-$j$ coupling model. In this way a much larger fraction of the correlations is taken into account than in the spherical $j$-$j$ coupling model.[12]

In these spheroidal nuclei, the $j$ of each nucleon is no longer a good quantum number because the binding field no longer has spherical symmetry. However, the spheroidal nuclei do have axial symmetry, so that in this case the component of angular momentum along the symmetry axis is a constant of the motion, provided that the rotational frequency of the nucleus is sufficiently small.[13] The magnitude of the component, $\Omega$, the sum of the components of angular momenta of the two particles along the nuclear symmetry axis, is given by either $\Omega_p+\Omega_n$ or by $|\Omega_p-\Omega_n|$, because the particle orbits about the symmetry axis of the deformed nucleus have twofold degeneracy, corresponding to the two equal and oppositely-directed angular momenta. The ground-state spin, $I_0$, is equal to $\Omega$ if the interactions between different rotational bands can be neglected.

We have found it is possible to determine $\Omega$ by assigning values to $\Omega_p$ and $\Omega_n$ using the Nilsson[11,14] classification of single-particle states in deformed nuclei. This is in agreement with the results of Bohr and Mottelson[13] and Peker,[15] who have shown that the ground state spins of a sizeable number of deformed odd-odd nuclei can indeed be accounted for on the basis of the coupling between the last odd proton and neutron. However, assuming such a coupling, there remains the problem of deciding whether the odd particles couple their angular momenta parallel or antiparallel.

The purpose of the present paper is to show that this question can be answered by the same considerations that led to the Nordheim rules in the case of $j$-$j$ coupling. We merely assume that the components $\Sigma_p$ and $\Sigma_n$ of proton and neutron spin along the nuclear symmetry axis always couple parallel. If the deformation is sufficiently large, then the orbital angular momentum $\Lambda$ and spin angular momentum $\Sigma$ of each single-particle state are separately good quantum numbers. For most strongly deformed nuclei, the separation of $\Omega$ into $\Lambda$ and $\Sigma$ is still expected to have approximate validity.

We then arrive at the following coupling rules:

$$I=\Omega_p+\Omega_n \quad \text{if} \quad \Omega_p=\Lambda_p\pm\tfrac{1}{2} \quad \text{and} \quad \Omega_n=\Lambda_n\pm\tfrac{1}{2}, \qquad (B1)$$

$$I=|\Omega_p-\Omega_n| \quad \text{if} \quad \Omega_p=\Lambda_p\pm\tfrac{1}{2} \quad \text{and} \quad \Omega_n=\Lambda_n\mp\tfrac{1}{2}. \qquad (B2)$$

The following section contains a tabulation of all measured (or well-established) ground-state spins and magnetic moments of odd-odd nuclei, and interpreta-

[6] A. de-Shalit, Phys. Rev. **91**, 1479 (1953).
[7] D. Kurath, Phys. Rev. **91**, 1430 (1953).
[8] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).
[9] C. Schwartz, Phys. Rev. **94**, 95 (1954); S. P. Pandya, Phys. Rev. **108**, 1312 (1957).
[10] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).
[11] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).
[12] S. A. Moszkowski, Phys. Rev. **110**, 403 (1958).
[13] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
[14] B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).
[15] L. K. Peker, Izvest. Akad. Nauk. S.S.S.R. Ser. Fiz. **31**, 1029 (1957).

tions on the basis of the above coupling rules. As will be seen, the coupling rules appropriate to deformed nuclei hold quite well not only for these nuclei, but also for some nuclei not far from closed shells. Indeed, in a number of nuclei, the spherical shell model and Nilsson model give the same result.

In Sec. III we make some remarks regarding the theoretical basis of the coupling rules in spherical and deformed nuclei. Section IV consists of a discussion of the empirical data.

## II. PRESENTATION OF DATA

In recent years the amount of experimental data on the ground-state spins of odd-odd nuclei deduced from nuclear spectroscopic studies has increased tremendously. Recently, in addition, the spins of a number of odd-odd nuclei have been measured directly, principally by the atomic-beam magnetic-resonance method.[16] In the second column of Table II we list all the odd-odd spins which have been measured directly (without parentheses), and those which have been deduced from spectroscopic data (in parentheses). Unless otherwise indicated, the data were taken from the new compilation of nuclear data by Strominger, Hollander, and Seaborg,[17] which contains the original references.

The definitions of the coupling rules $A1$, $A2$, $A3$, $A4$, and $B1$ and $B2$ have been given already. The designations $A1-$, $A2-$, $B1-$, and $B2-$ mean that the spins can be described by a coupling in the opposite sense of the rules, and hence constitute violations of them. $C$ indicates that a reasonable configuration cannot be given using the coupling rules. The $D$ and $E$ classifications (which apply only to spheroidal cases) apply to nuclei in which one or both of the two restrictions we imposed in selecting the odd-proton and odd-neutron states are violated.

These restrictions are that any odd-particle state used must appear experimentally (a) as the ground or excited state in an odd-$A$ nucleus with the same $Z$ or $N$ as those of the odd-odd nucleus in question, or (b) in the case where little experimental information is available, in nuclei with $Z\pm2$ or $N\pm2$. The $D$ classification is given nuclei where one or both of the odd-particle states is not experimentally observed but can be obtained from the Nilsson diagram for this $Z$ or $N$.

In a similar spirit the $E$ classification refers to the violation of the second restriction we imposed in selecting the data, namely, that the states used must also be obtainable from the Nilsson diagram for the $Z$ or $N$ in question. However, in all cases where the $E$ classification is used the particle states are observed experimentally in the odd-$A$ cases. We consider that this (which corresponds to a breakdown of the Nilsson description) is more serious than $D$.

In the cases where the data comply with the restrictions, the actual appearance of the state is listed in column 9. The designation 0-0 in column 9 means that the states coupled are both observed as ground states; 0-1 means that the proton state is observed as the ground state, the neutron state as the first excited state; and so on.

The assignment of the Nilsson states to odd particles was done using the ordinary Nilsson diagram,[11] except that the proton configurations for $Z=50$ and $Z=82$ are taken from a later modification.[18] A complete compilation and interpretation of odd-particle states has been given by Mottelson and Nilsson.[19]

We have listed both spherical and spheroidal coupling rules in Table I wherever applicable. Because the single-particle configurations are easily determined, they are not included in the table.

In the cases where more than one spheroidal configuration is possible, we have listed the one we prefer in the table; the other is listed in column 10. In the cases where no choice is possible we have listed both in the table. For example, $Eu^{152m}$ probably has spin 0, negative parity,[20] but because the spin has not been measured, we also list a possible $1-$ configuration.

The experimental magnetic moments listed in Table II are also taken from the compilation of Strominger, Hollander, and Seaborg.[17] The expressions used to calculate the theoretical values listed in columns 3 and 4 are discussed in Sec. IV.

## III. THEORETICAL CONSIDERATIONS

We shall regard the angular momentum of an odd-odd nucleus as a vector sum of that of the last odd proton and neutron. While such a description is not completely accurate, the essential idea that the last two particles move independently except for interactions between themselves can be justified to some extent empirically by considering an odd-odd nucleus as a kind of "superposition" of two odd-$A$ nuclei. In these nuclei, as is well known, the angular momentum (or its component along the nuclear symmetry axis in the case of a deformed nucleus) does seem to be carried largely by the last odd nucleon.[2,3] Thus a study of the angular momentum coupling in odd-odd nuclei should provide us with interesting information regarding the effective interactions between the last odd proton and neutron.[3,21] Let us now consider some features of this angular momentum coupling in the cases of spherical and spheroidal nuclei.

Basically, the nuclear forces are mainly attractive,

[16] See for example, the review article by W. A. Nierenberg, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 349.

[17] Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[18] Courtesy of Dr. S. G. Nilsson. This modification has been published, for example, by Cranston, Bunker, and Starner, Phys. Rev. **110**, 1427 (1958).

[19] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. (to be published).

[20] L. Grodzins, Phys. Rev. **109**, 1014 (1958).

[21] E. Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, New Jersey, 1955).

TABLE I. The experimental spins of odd-odd nuclei and the coupling rules applicable assuming the spherical and spheroidal models (columns 4 and 5). Columns 6, 7, 8, and 9 refer to the spheroidal model only. Configurations are not listed for the spherical cases. The configurations listed for the spheroidal cases can be for either prolate (+) or oblate (−) deformations (column 6). The asymptotic quantum numbers, $(N,n_z,\Lambda,\Sigma)$, are deduced from the Nilsson diagram.[a] The experimental observation of the states is given in column 9.

| $_ZX^A$ | Ground state spin $I_0$ | Reference[b] | Coupling rule Spherical | Coupling rule Spheroidal | Deformation $\delta$ | Configuration Proton $Nn_z\Lambda\Sigma$ | Configuration Neutron $Nn_z\Lambda\Sigma$ | Exp. obs. of state | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| ${}_{1}H^{2}$ | 1+ | | $A4$ | $B1$ | + | 000↑ | 000↑ | 0-0 | Positive quadrupole moment due to tensor forces |
| ${}_{3}Li^{6}$ | 1+ | | $A4$ | $B1$ | + | 110↑ | 110↑ | 1-1 | |
| ${}_{3}Li^{8}$ | (2+) | | $A3$ | $B1$ | + | 110↑ | 101↑ | 1-0 | |
| ${}_{5}B^{10}$ | 3+ | | $A4$ | $B1$ | + | 101↑ | 101↑ | 0-0 | |
| ${}_{5}B^{12}$ | (1+) | | $A2$ | $B2$ | + | 101↑ | 101↓ | 0-0 | |
| ${}_{7}N^{12}$ | (1+) | c | $A2$ | $B2$ | + | 101↓ | 101↑ | 0-0 | |
| ${}_{7}N^{14}$ | 1+ | | $A4$ | $B1$ | + | 101↓ | 101↓ | 0-0 | |
| ${}_{7}N^{16}$ | (2−) | | $A2$ | $B1-$ | − | 110↑ | 202↑ | 0-0 | |
| ${}_{9}F^{18}$ | (1+) | | $A4$ | $B1$ | + | 220↑ | 220↑ | 0-0 | |
| ${}_{9}F^{20}$ | (2+) | d, e | $C$ | $B1$ | + | 220↑ | 211↑ | 0-0 | |
| ${}_{11}Na^{22}$ | 3+ | | $C$ | $B1$ | + | 211↑ | 211↑ | 0-0 | |
| ${}_{11}Na^{24}$ | 4+ | | $C$ | $B1$ | + | 211↑ | 202↑ | 0-0 | |
| ${}_{11}Na^{24m}$ | (1+) | | $C$ | $B2$ | + | 211↑ | 211↓ | 0-1 | Can also be described as $B1-$, 211↑, 202↑ |
| ${}_{13}Al^{24}$ | (4+) | f | $C$ | $B1$ | + | 202↑ | 211↑ | 0-0 | |
| ${}_{13}Al^{26}$ | (5+) | | $A4$ | $B1$ | + | 202↑ | 202↑ | 0-0 | |
| ${}_{13}Al^{26m}$ | (0+) | | $A4$ | $B1-$ | + | 202↑ | 202↑ | 0-0 | |
| ${}_{13}Al^{28}$ | (3+) | f | $A1$ | $B2-$ | + | 202↑ | 211↓ | 0-0 | |
| ${}_{15}P^{30}$ | (1+) | f | $A4$ | $B1$ | ± | 211↓ | 211↓ | 0-0 | |
| ${}_{15}P^{32}$ | 1+ | | $A2$ | $B2$ | − | 211↑ | 211↓ | 0-0 | |
| ${}_{15}P^{34}$ | (1+) | | $A2$ | $B2$ | + | 202↑ | 202↓ | 2-0 | |
| ${}_{17}Cl^{34}$ | (0+) | | $A4$ | $B1-$ | − | 211↑ | 211↑ | 0-0 | |
| ${}_{17}Cl^{34m}$ | (3+) | | $A4$ | $B1$ | − | 211↑ | 211↑ | 0-0 | |
| ${}_{17}Cl^{36}$ | 2+ | | $A3$ | $B1$ | − | 211↑ | 220↑ | 0-[t] | |
| ${}_{17}Cl^{38}$ | (2−) | | $A2$ | $B1-$ | − | 211↑ | 303↑ | 0-0 | |
| ${}_{17}Cl^{40}$ | (2−) | f | $A2$ | $C$ | | | | | |
| ${}_{19}K^{38}$ | (3+) | f | $A4$ | $B1$ | + | 202↓ | 202↓ | 0-0 | |
| ${}_{19}K^{38m}$ | (0+) | f | $A4$ | $B1-$ | + | 202↓ | 202↓ | 0-0 | |
| ${}_{19}K^{40}$ | 4− | | $A3$ | $(B1)$ | − | 220↑ | 303↑ | $D$ | |
| ${}_{19}K^{42}$ | 2− | | $A2$ | $(B2)$ | − | 220↑ | 303↓ | $D$ | |
| ${}_{21}Sc^{42}$ | (0+) | f | $A4$ | $B1-$ | − | 303↑ | 303↑ | 0-0 | |
| ${}_{21}Sc^{44}$ | 2+ | | $C$ | $(B1)$ | − | 330↑ | 321↑ | $D$ | Can also be described as $D(B2-)$, 303↑, 321↑ |
| ${}_{21}Sc^{46}$ | (4+) | | $C$ | $(B1)$ | + | 321↑ | 312↑ | $D$ | Can also be described as $D(B1)$, 330↑, 303↑ |
| ${}_{21}Sc^{46m}$ | (7+) | | $C$ | $B1$ | ~0 | 303↑ | 303↑ | 0-0 | |
| ${}_{23}V^{46}$ | (0+) | g | $A4$ | $(B1-)$ | + | 321↑ | 321↑ | $D$ | |
| ${}_{23}V^{48}$ | (4+) | | $C$ | $B1$ | + | 321↑ | 312↑ | 2-1 | |
| ${}_{23}V^{50}$ | 6+ | | $A3$ | $E(B1)$ | + | 312↑ | 303↑ | 1-0 | |
| ${}_{23}V^{52}$ | (2+) | | $A1-$ | $C$ | | | | | |
| ${}_{25}Mn^{50}$ | (0+) | g | $A4$ | $B1-$ | + | 312↑ | 312↑ | 0-0 | |
| ${}_{25}Mn^{52}$ | (6+) | | $C$ | $B1$ | + | 312↑ | 303↑ | 0-0 | |
| ${}_{25}Mn^{52m}$ | (2+) | | $C$ | $B2$ | + | 312↑ | 321↓ | 0-2 | |
| ${}_{25}Mn^{54}$ | 2+ | h | $A1-$ | $B2$ | + | 312↑ | 321↓ | 0-1 | |
| ${}_{25}Mn^{56}$ | 3 | i | $C$ | $B1$ | + | 312↑ | 310↑ | 0-0 | Can also be described as $B2$, 303↑, 321↓ |
| ${}_{27}Co^{54}$ | (0+) | g | $A4$ | $B1-$ | + | 303↑ | 303↑ | 0-0 | |
| ${}_{27}Co^{56}$ | 4+ | | $A3$ | $B2-$ | + | 303↑ | 321↓ | 0-1 | |
| ${}_{27}Co^{58}$ | (2+) | | $A1-$ | $B2$ | + | 303↑ | 312↓ | 0-0 | |
| ${}_{27}Co^{58m}$ | (5+) | j | $A1$ | $B2-$ | + | 303↑ | 312↓ | 0-0 | |
| ${}_{27}Co^{60}$ | 5+ | | $A1$ | $B2-$ | + | 303↑ | 312↓ | 0-0 | |
| ${}_{27}Co^{60m}$ | (2+) | | $A1-$ | $B2$ | + | 303↑ | 312↓ | 0-0 | |
| ${}_{29}Cu^{60}$ | 2+ | h | $A3$ | $B2-$ | − | 312↓ | 310↑ | 0-0 | |
| ${}_{29}Cu^{62}$ | (1+) | c | $A2$ | $B2$ | − | 312↓ | 312↑ | 0-1 | |
| ${}_{29}Cu^{64}$ | 1+ | | $A2$ | $E(B2)$ | − | 312↓ | 312↑ | 0-0 | Can also be described as $B2$, 312↓, 310↑ |
| ${}_{29}Cu^{66}$ | (1+) | | $A2$ | $E(B2)$ | − | 312↓ | 312↑ | 0-0 | |
| ${}_{31}Ga^{64}$ | (0+) | k | $A1-$ | $B1-$ | + | 312↓ | 312↓ | 0-0 | |
| ${}_{31}Ga^{66}$ | 0(+) | | $A1-$ | $B1-$ | + | 312↓ | 312↓ | 0-[u] | Can also be described as $B2$, 312↓, 301↑ |
| ${}_{31}Ga^{68}$ | 1+ | h | $A2$ | $B1-$ | + | 312↓ | 303↓ | 0-0 | |
| ${}_{31}Ga^{70}$ | (1+) | | $A2$ | $B1-$ | + | 312↓ | 303↓ | 0-0 | |
| ${}_{31}Ga^{72}$ | 3 | l | $A1-$ | $B2$ | − | 312↓ | 404↑ | 0-0 | |
| ${}_{33}As^{72}$ | (2−) | j | $A2$ | $C$ | | | | | |
| ${}_{33}As^{74}$ | (2−) | | $A2$ | $B2$ | + | 303↓ | 404↑ | 1-0 | |
| ${}_{33}As^{76}$ | 2− | | $A2$ | $E(B2)$ | + | 303↓ | 404↑ | 1-1 | |
| ${}_{35}Br^{80}$ | (1+) | | $C$ | $B2$ | + | 301↑ | 301↓ | 0-1 | |

TABLE I.—*Continued.*

| $_ZX^A$ | Ground state spin $I_0$ | Reference[b] | Coupling rule Spherical | Coupling rule Spheroidal | Deformation $\delta$ | Configuration Proton $Nn_z\Lambda\Sigma$ | Configuration Neutron $Nn_z\Lambda\Sigma$ | Exp. obs. of state | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| $_{35}Br^{80m1}$ | (2−) | | $A2$ | $B1-$ | + | 301↑ | 413↑ | 0-0 | |
| $_{35}Br^{80m2}$ | (5−) | | $C$ | $B1$ | + | 301↑ | 413↑ | 0-0 | |
| $_{35}Br^{82}$ | 5− | m | $C$ | $B1$ | + | 301↑ | 413↑ | 0-1 | |
| $_{37}Rb^{82m}$ | 5− | | $C$ | $E(B1)$ | + | 301↑ | 413↑ | 0-0 | |
| $_{37}Rb^{84}$ | 2− | | $A2$ | $E(B2)$ | + | 303↓ | 404↑ | 0-0 | |
| $_{37}Rb^{86}$ | 2− | | $A2$ | $B2$ | + | 303↓ | 404↑ | 0-0 | |
| $_{37}Rb^{88}$ | (2−) | | $A2$ | $B2$ | + | 303↓ | 404↑[v] | 0-2 | |
| $_{39}Y^{88}$ | (4−) | | $A2$ | $B1-$ | + | 330↑ | 404↑ | 0-0 | |
| $_{39}Y^{90}$ | (2−) | | $A2$ | $B2$ | − | 330↑ | 413↓ | 0-0 | |
| $_{39}Y^{92}$ | (2−) | | $A2$ | $B2$ | − | 330↑ | 413↓ | 0-0 | |
| $_{45}Rh^{98}$ | (2+) | g | $A1-$ | $E(B2)$ | + | 404↑ | 413↓ | 2-0 | |
| $_{45}Rh^{100}$ | (2−) | c | $C$ | $B2$ | + | 550↑ | 413↓ | 0-0 | These states can also be described as $B1$, 301↓, 422↓ if the obs. ½− state is not 550↑ |
| $_{45}Rh^{102}$ | (2−) | c | $C$ | $B2$ | + | 550↑ | 413↓ | 0-0 | |
| $_{45}Rh^{104}$ | (1+) | | $A2$ | $B2$ | + | 413↑ | 413↓ | 0-0 | |
| $_{45}Rh^{104m}$ | (5−) | | $C$ | $(B1)$ | + | 413↑ | 541↑ | $D$ | |
| $_{45}Rh^{106}$ | (1+) | | $A2$ | $B2$ | + | 413↑ | 413↓ | 0-0 | |
| $_{47}Ag^{104}$ | 2(−) | | $A1-$ | $B2$ | + | 550↑ | 413↓ | 0-0 | |
| $_{47}Ag^{106}$ | 6+ | | $A2-$ | $B2-$ | + | 413↑ | 413↓ | 1-0 | |
| $_{47}Ag^{106}$ | 1+ | | $A2$ | $B2$ | + | 413↑ | 413↓ | 1-0 | $Ag^{106,108,110}$ can also be described $B2$, 404↑, 404↓ |
| $_{47}Ag^{108}$ | (1+) | | $A2$ | $B2$ | + | 413↑ | 413↓ | 1-[w] | |
| $_{47}Ag^{110}$ | (1+) | c | $A2$ | $B2$ | + | 413↑ | 413↓ | 1-1 | |
| $_{47}Ag^{110m}$ | 6+ | | $A2-$ | $B2-$ | + | 413↑ | 413↓ | 1-1 | |
| $_{47}Ag^{112}$ | (2−) | c | $C$ | $B2$ | + | 550↑ | 413↓ | 0-1 | Can also be described as $B2-$, 413↑, 541↑ |
| $_{49}In^{110}$ | (2+) | c | $A1-$ | $B2$ | + | 404↑ | 413↓ | 0-[t] | |
| $_{49}In^{110m}$ | 7(+) | h | $A1$ | $B2-$ | + | 404↑ | 413↓ | 0-[t] | |
| $_{49}In^{114}$ | (1+) | | $A2$ | $(B2)$ | + | 404↑ | 404↓ | $D$ | $In^{114}$, $In^{116}$ can also be described as $B2$, 413↑, 413↓ |
| $_{49}In^{114m}$ | 5+ | | $A1$ | $B1$ | + | 413↑ | 411↑ | [x]-2 | |
| $_{49}In^{116}$ | (1+) | | $A2$ | $(B2)$ | + | 404↑ | 404↓ | $D$ | |
| $_{49}In^{116m}$ | 5+ | | $A1$ | $B1$ | + | 413↑ | 411↑ | [x]-2 | |
| $_{51}Sb^{120}$ | (1+) | c | $A2$ | $B2$ | − | 413↓ | 431↑ | 0-1 | |
| $_{51}Sb^{122}$ | (2−) | | $A2$ | $B1-$ | − | 413↑ | 505↑ | 0-2 | Can also be described as $D(B1-)$, 413↓, 505↓ with negative $\delta$ |
| $_{53}I^{124}$ | 2(−) | n | $A2$ | $B1-$ | − | 413↑ | 505↑ | 0-2 | |
| $_{53}I^{126}$ | (2−) | | $A2$ | $B1-$ | − | 413↑ | 505↑ | 0-1 | |
| $_{53}I^{128}$ | 1+ | | $A2$ | $B2$ | − | 413↓ | 431↑ | 0-0 | |
| $_{55}Cs^{126}$ | (1+) | | $A2$ | $B2$ | − | 413↓ | 431↑ | 0-0 | Can also be described as $B2(\delta=+)$, 420↑, 400↑ $B2(\delta=-)$, 411↓, 431↑ |
| $_{55}Cs^{128}$ | (1+) | | $A2$ | $B2$ | − | 413↓ | 431↑ | 0-0 | |
| $_{55}Cs^{130}$ | 1+ | | $A2$ | $B2$ | − | 413↓ | 431↑ | 0-0 | |
| $_{55}Cs^{132}$ | 2+ | | $A1-$ | $B2$ | + | 413↓ | 400↑ | 0-0 | |
| $_{55}Cs^{134}$ | 4+ | | $A2-$ | $B1$ | + | 413↓ | 402↓ | 0-0 | |
| $_{55}Cs^{134m}$ | 8− | | $A1$ | $B2-$ | + | 413↓ | 505↑ | 0-1 | |
| $_{57}La^{138}$ | 5− | | $A1$ | $B1$ | + | 404↓ | 402↓ | 0-0 | 5+ state predicted |
| $_{59}Pr^{140}$ | (1, 0+) | | $A2$ | $C$ | | | | | |
| $_{59}Pr^{142}$ | (2−) | | $A2$ | $B2$ | − | 422↑ | 514↓ | 0-0 | |
| $_{59}Pr^{144}$ | 0− | | $A2$ | $B1-$ | − | 422↑ | 512↑ | 0-[w] | |
| $_{63}Eu^{152}$ | 3± | | | $B1$ | + | 411↑ | 521↑ | [y]-[w] | 3− state predicted |
| | | | | $B1$ | + | 411↑ | 651↑ | [y]-[w] | 3+ state predicted |
| $_{63}Eu^{152m}$ | (0, 1−) | | | $B2$ | + | 413↓ | 512↑ | 0-[t] | 0− state predicted |
| | | | | $B2$ | + | 413↓ | 521↑ | 0-[y] | 1− state predicted |
| $_{63}Eu^{154}$ | 3± | | | $B1$ | + | 411↑ | 521↑ | [y]-0 | 3− state predicted |
| | | | | $B1$ | + | 411↑ | 651↑ | [y]-0 | 3+ state predicted |
| $_{67}Ho^{166}$ | (0−) | | | $B1-$ | + | 523↑ | 633↑ | 0-0 | |
| $_{69}Tm^{170}$ | (1−) | | | $B1$ | + | 411↓ | 521↓ | 0-0 | |
| $_{71}Lu^{176}$ | (⩾7±) | | | $B1$ | + | 404↑ | 514↑ | 0-0 | 7− state predicted |
| $_{71}Lu^{176m}$ | (1±) | | | $B2$ | + | 402↑ | 514↓ | 1-0 | 1− state predicted |
| $_{73}Ta^{178}$ | (8,9) | o | | $B1$ | + | 514↑ | 624↑ | 1-1 | 9− state predicted |
| $_{73}Ta^{178}$ | (1±) | o | | $B2$ | + | 402↑ | 514↓ | 1-0 | 1− state predicted |
| $_{73}Ta^{180m}$ | (1−) | c | | $B2$ | + | 402↑ | 514↓ | 1-[z] | 1− state predicted |
| $_{73}Ta^{182}$ | (3±) | d | | $B2$ | + | 404↓ | 510↑ | 0-0 | 3− state predicted |
| $_{75}Re^{182}$ | (3−) | p | | $B1$ | + | 402↑ | 510↑ | 0-2 | |
| $_{75}Re^{182}$ | (7−) | p | | $B1$ | + | 404↓ | 514↓ | [x]-[w] | |
| $_{75}Re^{184}$ | (3−) | p | | $B1$ | + | 402↑ | 510↑ | 0-0 | |
| $_{75}Re^{186}$ | (1−) | | | $B2$ | + | 402↑ | 512↓ | 0-[w] | |
| $_{75}Re^{188}$ | (1−) | | | $B2$ | + | 402↑ | 512↓ | 0-0 | |
| $_{77}Ir^{194}$ | (1−) | c | | $B2$ | + | 402↓ | 510↑ | 0-0 | |
| $_{79}Au^{192}$ | 1(−) | q | $A1-$ | $B2$ | + | 402↓ | 510↑ | 1-0 | |

TABLE I.—*Continued.*

| $_ZX^A$ | Ground state spin $I_0$ | Reference[b] | Coupling rule Spherical | Coupling rule Spheroidal | Deformation $\delta$ | Configuration Proton $Nn_z\Lambda\Sigma$ | Configuration Neutron $Nn_z\Lambda\Sigma$ | Exp. obs. of state | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| $_{79}Au^{194}$ | 1− | | $A1-$ | $B2$ | + | 402↓ | 510↑ | 1-0 | |
| $_{79}Au^{196}$ | 2− | h | $A2$ | $B2$ | + | 400↑ | 503↓ | 1-0 | |
| $_{79}Au^{198}$ | 2− | | $A2$ | $B2$ | + | 400↑ | 503↓ | 1-0 | |
| $_{81}Tl^{198}$ | (2−) | | $A2$ | $B2$ | + | 400↑ | 503↓ | 0-2 | |
| $_{81}Tl^{198m}$ | 7+ | | $A1$ | $B1$ | + | 400↑ | 606↑ | 0-0 | |
| $_{81}Tl^{200}$ | 2− | h | $A2$ | $B2$ | + | 400↑ | 503↓ | 0-0 | |
| $_{81}Tl^{202}$ | (2−) | r | $A2$ | $B2$ | + | 400↑ | 503↓ | 0-0 | |
| $_{81}Tl^{204}$ | 2− | | $A2$ | $B2$ | + | 400↑ | 503↓ | 0-0 | |
| $_{81}Tl^{208}$ | (5+) | | $A1$ | ($B1$) | − | 440↑ | 624↑ | $D$ | |
| $_{83}Bi^{204}$ | 6+ | h | $C$ | $B1$ | + | 514↑ | 541↑ | 0-0 | |
| $_{83}Bi^{206}$ | 6+ | s | $A3$ | $B1$ | + | 514↑ | 541↑ | 0-[t] | |
| $_{83}Bi^{210}$ | 1− | | $A1-$ | ($B2$) | − | 514↓ | 624↑ | $D$ | |
| $_{83}Bi^{212}$ | (1−) | | $A1-$ | | | | | $D$ | |
| $_{93}Np^{236}$ | (1+) | | | $B2$ | + | 532↑ | 514↓ | 1-0 | |
| $_{93}Np^{238}$ | 2(±) | h | | $B2$ | + | 642↑ | 631↓ | 0-0 | 2+ state predicted |
| | | | | $B2$ | + | 532↑ | 631↓ | 1-0 | 2− state predicted |
| $_{95}Am^{242m}$ | 0− | h | | $B2$ | + | 523↓ | 622↑ | 0-0 | |

[a] See reference 11.
[b] Unless specifically stated, references for all data will be found in Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).
[c] B. S. Dzhelepov and L. K. Peker, *Decay Schemes of Radioactive Isotopes* (Academy of Sciences of the U.S.S.R., 1957) (in Russian).
[d] Assignment by authors deduced from experimental data in literature.
[e] The previous 1+ assignment of this level was based on stripping data which have recently been reinterpreted by the authors of the assignment [Bull. Am. Phys. Soc. Ser. II, 2, 52 (1957)].
[f] P. M. Endt and C. M. Braams, Revs. Modern Phys. **29**, 683 (1957).
[g] Way, Kundu, McGinnis, and van Lieshout, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1956), Vol. 6, p. 129.
[h] W. Nierenberg (private communication, March 1958).
[i] W. J. Childs and L. S. Goodman, Bull. Am. Phys. Soc. Ser. II, **3**, 21 (1958).
[j] *Nuclear Level Schemes, A=40—A=92*, compiled by Way, King, McGinnis, and van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).
[k] T. Jacobi (private communication, 1958).
[l] L. S. Goodman and W. J. Childs, Bull. Am. Phys. Soc. Ser. II, **3**, 21 (1958).
[m] Garvin, Green, and Lipworth, Bull. Am. Phys. Soc. Ser. II, **2**, 344 (1957).
[n] Garvin, Green, and Lipworth, Bull. Am. Phys. Soc. Ser. II, **2**, 383 (1957).
[o] F. F. Felber, Jr., thesis, University of California Radiation Laboratory Report UCRL-3618, September, 1956 (unpublished).
[p] C. J. Gallagher, Jr., thesis, University of California Radiation Laboratory Report UCRL-3928, September, 1957 (unpublished).
[q] Ewbank, Marino, Shugart, and Silsbee, Bull. Am. Phys. Soc. Ser. II, **2**, 383 (1957).
[r] B. Aström, Arkiv Fysik **12**, 237 (1957).
[s] Marino, Ewbank, Nierenberg, Shugart, and Silsbee, Bull. Am. Phys. Soc. Ser. II, **2**, 383 (1957).
[t] Observed $N-2$.
[u] $\frac{3}{2}$ states observed $N\pm2$.
[v] It is very peculiar that this is the ground state.
[w] Observed $N+2$.
[x] Observed $Z-2$.
[y] Observed $Z+2$.
[z] Observed $N\pm2$.

and, of course, short ranged; thus they tend to maximize the overlap between the wave functions of interacting particles. As is well known from the properties of the deuteron, the *n-p* forces are such as to make it favorable for their intrinsic spins to add.

In spherical nuclei the overlap of the wave functions is maximum if the two particles tend to couple their orbital angular momenta antiparallel. (This can be seen mathematically from the fact that the matrix element for the contact interaction is proportional to the square of a Clebsch-Gordan coefficient, $\langle l_1l_200|l0\rangle^2$, which has its largest value when $l_1=l_2$.) Then, taking into account the spin dependence of nuclear forces, we can see that if there were no spin-orbit coupling, and if the $l$ of each nucleon were a good quantum number, all spherical odd-odd nuclei would have $L=l_p-l_n$, $S=1$.

This result is changed considerably if strong spin-orbit coupling is present. Strong spin-orbit coupling is empirically manifest in the observed *j-j* coupling; indeed, for a *j-j* coupling description to hold, it is necessary for the spin-orbit coupling to be strong compared to the effect of the residual two-particle forces. For the case of a *n-p* configuration with $j_p=l_p\pm\frac{1}{2}$ and $j_n=l_n\mp\frac{1}{2}$ the tendency to couple the spins parallel and that to couple the orbital angular momenta antiparallel are not opposed by the spin orbit coupling if the total $J=|j_p-j_n|$. This is the theoretical basis of the strong Nordheim rule.

In the case of *n-p* configurations with $j_p=l_p\pm\frac{1}{2}$ and $j_n=l_n\pm\frac{1}{2}$ the spin-orbit coupling introduces some difficulty, because when the two particles coupled by the spin-orbit force try to combine, the spin-spin coupling and orbital-angular-momentum coupling tend to oppose each other. Which of the possible configurations will actually be lowest depends in this case on further details of the nuclear forces.[3,21] The weak Nordheim rule applies if the coupling of the intrinsic spins is more important than that of the orbital angular momenta. This would, for example, occur for long-range attractive forces because in this case the forces will depend only on the spin-dependent term. Furthermore, the results of deShalit[6] and Schwartz[9] imply that the spin-spin coupling dominates (although only slightly) for contact interactions if an exchange mixture of the form $[(1-\alpha)+\alpha(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]$ with $\alpha\geqslant\frac{1}{4}$, i.e., no attraction in odd states, is used.

TABLE II. Comparison of experimental and calculated magnetic moments. $\mu_{(\text{spherical})}$ is calculated assuming the gyromagnetic ratios of the odd nucleons are those given by the Schmidt formulas (with no quenching). $\mu_{(\text{spheroidal})}$ is calculated using the expression given in the text.

| $_ZX^A$ | Ground state spin $I_0$ | $\mu_{(\text{exp})}$ | $\mu_{(\text{spherical})}$ | $\mu_{(\text{spheroidal})}$ |
|---|---|---|---|---|
| $_1H^2$ | 1+ | 0.857 | 0.88 | |
| $_3Li^6$ | 1+ | 0.822 | 0.63 | 0.70 |
| $_5B^{10}$ | 3+ | 1.80 | 1.88 | 1.80 |
| $_7N^{14}$ | 1+ | 0.403 | 0.37 | 0.30 |
| $_{11}Na^{22}$ | 3+ | 1.746 | | 1.80 |
| $_{11}Na^{24}$ | 4+ | 1.69 | | 1.92 |
| $_{17}Cl^{36}$ | 2+ | 1.2839 | 0.85 | 1.60 |
| $_{19}K^{40}$ | 4− | −1.2964 | −1.68 | 1.28 |
| $_{19}K^{42}$ | 2− | −1.137 | −1.73 | −0.27 |
| $_{23}V^{50}$ | 6+ | 3.3414 | 3.30 | 2.83 |
| $_{25}Mn^{54}$ | 2+ | ±5.1 | 6.23 | 2.20 |
| $_{27}Co^{56}$ | 4+ | ±3.86 | 4.28 | 6.48 |
| $_{27}Co^{58}$ | 2+ | ±3.5 | 6.23 | 2.87 |
| $_{27}Co^{60}$ | 5+ | ±3.6 | 3.88 | 6.75 |
| $_{29}Cu^{64}$ | 1+ | ±0.40 | −0.93 | −0.35 |
| $_{31}Ga^{66}$ | 0 | $<4\times10^{-5}$ | 0 | 0 |
| $_{31}Ga^{68}$ | 1+ | ±0.05[a] | −0.94 | 1.55 |
| $_{31}Ga^{72}$ | 3 | ±0.12 | −4.58 | −0.50 |
| $_{33}As^{76}$ | 2− | −0.906 | −2.13 | −1.14 |
| $_{35}Br^{82}$ | 5− | ±1.67[a] | | 1.92 |
| $_{37}Rb^{82m}$ | 5− | 1.50 | | 1.92 |
| $_{37}Rb^{84}$ | 2− | −1.32 | −2.13 | −1.14 |
| $_{37}Rb^{86}$ | 2− | −1.69 | −2.13 | −1.14 |
| $_{49}In^{114m}$ | 5+ | 4.7 | 4.88 | 3.58 |
| $_{49}In^{116m}$ | 5+ | 4.4 | 4.88 | 3.58 |
| $_{55}Cs^{130}$ | 1+ | 1.32 | 2.78 | 1.25 |
| $_{55}Cs^{132}$ | 2+ | 2.20 | 5.77 | 1.67 |
| $_{55}Cs^{134}$ | 4+ | 2.973 | 5.96 | 2.00 |
| $_{55}Cs^{134m}$ | 8− | 1.10 | 2.88 | −1.16 |
| $_{57}La^{138}$ | 5− | 3.685 | 2.87 | 2.92 |
| $_{63}Eu^{152}$ | 3 | 2.0 | | 1.73 |
| $_{63}Eu^{154}$ | 3 | 2.1 | | 1.73 |
| $_{71}Lu^{176}$ | ⩾7 | 4.2 | | 3.06 |
| $_{79}Au^{192}$ | 1− | ±0.008[a] | 0.90 | 0.75 |
| $_{79}Au^{198}$ | 2− | ±0.50 | −0.58 | −0.33 |
| $_{81}Tl^{204}$ | 2− | ±0.89<br>±0.62 | −0.58 | −0.33 |
| $_{83}Bi^{204}$ | 6+ | ±7.0[a] | | 4.52 |
| $_{83}Bi^{206}$ | 6+ | ±4.5[a] | 3.42 | 4.52 |

[a] W. A. Nierenberg (private communication, 1958).

As can be seen from the spherical cases in Table I, the empirical coupling schemes for nuclei with $j_p=l_p\pm\frac{1}{2}$, $j_n=l_n\pm\frac{1}{2}$ are somewhat ambiguous. Apart from nuclei with $N=Z$ and those clearly involving particle-hole configurations, we find that the weak Nordheim rule (both the $A1$ and $A1'$ forms) is violated, i.e., $I=|j_p-j_n|$ in the majority of cases. It appears from these results that the tendency to couple the orbital angular momenta antiparallel is actually somewhat stronger than would be expected on the basis of short-range attractive forces in even states only. It is true that by reducing the assumed amount of exchange interaction, i.e., by having some attraction in both even and odd states, it is still possible to obtain $I=|j_p-j_n|$ for such cases as $Co^{58}$ and $Ga^{66}$ where the weak Nordheim rule appears to fail.[6] However, the interactions between free nucleons are believed to be rather weak in odd states, and, if anything, they tend to be repulsive.[22] Thus it appears difficult to understand how the effective interactions in odd states could be attractive.

As will be discussed in a forthcoming article by one of us (S.A.M.), the effective interactions in the interior of the nucleus appear to act mainly between nucleons near the top of the Fermi sea, and with equal and opposite momenta. This momentum dependence of the interactions has essentially the same effect as shortening the range, thus enhancing the tendency for coupling the orbital angular momenta antiparallel. Such a tendency is in excellent agreement with the observed breakdown of the weak Nordheim rule. More detailed studies using this kind of momentum-dependent interaction are now in progress. It should be noted that configuration interaction, i.e., deviations from $j$-$j$ coupling description, also tends to energetically favor states of small spin over those of large spin, because of the statistically larger number of states with small spin.

The situation in strongly deformed nuclei is considerably simpler than in spherical nuclei. First of all, because each orbit is only twofold degenerate, there is no longer any difference between particles and holes. Secondly, preliminary calculations show that the tendency to couple the orbital angular momenta antiparallel is much weaker here than in spherical nuclei. Thus, for a delta-function interaction, the energy would, in fact, be the same for $\Lambda_p+\Lambda_n$ and $|\Lambda_p-\Lambda_n|$, because the overlap of the wave functions would be exactly equal in these two cases. The momentum dependence of the effective forces is expected to introduce some favoring of antiparallel coupling, but much less than in the spherical case. Consequently, in deformed odd-odd nuclei, the coupling of angular momenta is expected to be determined largely by the criterion that the intrinsic spins line up parallel. In this way we obtain coupling rules $B$, the analogues of the Nordheim rules for deformed nuclei. As is seen in the next section, these rules are satisfied surprisingly well in deformed nuclei, and on the whole they work considerably better than the corresponding coupling rules $A$ for nuclei near closed shells.

## IV. DISCUSSION OF EMPIRICAL DATA

In order to contrast the general applicability of the spheroidal coupling rules with the rather specific nature of the $j$-$j$ coupling rules we can mention the following statistics: of 139 ground or isomeric states of odd-odd nuclei that have been measured, 11 obey rule $A1$, 46 obey $A2$, 7 obey $A3$, and 16 obey $A4$. In 59 cases the $j$-$j$ coupling rules $A$ break down or are ambiguous. As mentioned previously, rule $A2$ works well, but in general can be applied only in regions near closed shells. $A3$ and $A4$ are very specific and apply only to a small class of nuclei. Rule $A1$ is very much weaker than previously

[22] See, for example, Gammel, Christian, and Thaler, Phys. Rev. **105**, 311 (1957); C. De Dominicis P. C. Martin, Phys. Rev. **105**, 1418 (1957).

thought, because in 15 cases out of 26 where it can be applied, the coupling is $I=|j_p-j_n|$ rather than the predicted $I=j_p+j_n$ (which is also a violation of the $A1'$ form). In fact, the only nuclei in which rule $A1$ is observed to apply at all are nuclei with one particle less than a complete shell or subshell. And in over half of these cases we are dealing with a metastable state, rather than the ground state.

The use of the $B$ coupling rules in determining the ground state spins of odd-odd nuclei has, rather surprisingly, shown itself to be generally applicable in almost all regions of the periodic table. The general success in applying this model suggests that we should consider seriously the cases in which the model breaks down.

Let us first consider the statistics. Of the 139 cases, 90 can be described by $B1$ and $B2$, and 27 by $B1-$ and $B2-$. Of the 19 cases where $B1$ is violated, every case corresponds to a nucleus in one of three categories: (1) with $Z=N$; (2) near a closed shell and describable in terms of the strong Nordheim rule; (3) near a closed shell and describable in terms of a violation of the weak Nordheim rule. That is, almost all of these 19 cases correspond to cases where the asymptotic quantum numbers assign parallel orbital angular momentum and spin, whereas the $j$-$j$ coupling description assigns $j_p=l_p\pm\frac{1}{2}$, $j_n=l_n\pm\frac{1}{2}$. In the remaining cases, to which the $j$-$j$ coupling model assigns $j_p=l_p\pm\frac{1}{2}$, $j_n=l_n\mp\frac{1}{2}$, the orbital angular momenta appear to be coupled antiparallel. Furthermore, 10 of these 19 nuclei have spin 0. These facts seem to indicate that, in general, the orbital angular momentum coupling is more important than the spin-spin coupling in spherical odd-odd nuclei. The cases where $B2$ is violated are near closed shells where the $j$-$j$ coupling model leads to rule $A1$ or $A3$. The most serious breakdown seems to be $Al^{28}$, where a 2+ state is predicted, and a 3+ state is observed. It is somewhat disturbing to find this breakdown occurring in what is, at present, the only odd-odd nucleus for which detailed information on ground and excited states is available.[23]

In general the coupling of odd-odd nuclei seems to be describeable in the following manner. In regions with several particles outside closed shells, the coupling can be described usually by either the $A$ or $B$ coupling rules, but tends to favor the rule which couples the orbital angular momenta antiparallel. In general in regions where there are three or more particles outside both proton and neutron closed shells the $B$ coupling rules hold. In regions where there is a definite particle-hole configuration, the coupling is well described by rule $A3$. The spins of all $N=Z$ nuclei listed can be accounted for on the basis of the spheroidal shell model, but in the nuclei with $A\geqq 34$, the 0+ state is, in general, lower, i.e., the antiparallel coupling of orbital angular momenta predominates.

[23] R. K. Sheline, Nuclear Phys. **2**, 382 (1956/57).

The tendency for the ground state spins to increase monotonically as the $1p^{\frac{3}{2}}$, $1d^{\frac{5}{2}}$, and $1f^{7/2}$ shells are being filled has already been noted by King and Peaslee,[24] and can readily be explained on the basis of the spheroidal model with prolate deformations.[14,25]

The surprising validity of the asymptotic-quantum-number description for nuclei in almost all regions of the periodic table suggests the possible applicability of this description for the calculation of other nuclear properties which are more sensitive to the mixing of the states, i.e., magnetic moments† and nuclear level spectra.

In Table II we compare the magnetic moments calculated assuming the validity of the asymptotic-quantum-number description, using the simple expression derived by Bohr and Mottelson[14] for deformed nuclei. This expression was used in the form:

$$\mu=(g_\Omega\Omega+g_R)I/(I+1),$$
$$g_R\cong Z/A,$$
$$g_\Omega\Omega=[\pm(\Lambda_p+5.6\Sigma_p)\mp3.8\Sigma_n],$$

where the signs of the two terms of the expression are the same as the signs of $\Omega_p$ and $\Omega_n$ appearing in the coupling rule, $\Lambda_p$ is the asymptotic quantum number $\Lambda$ of the proton, and the signs of $\Sigma_p$ and $\Sigma_n$ are plus or minus depending on whether the particle intrinsic spins are up (+) or down (−).

The expression used to calculate the magnetic moments assuming spherical nuclei is well known and is given, for example, by Feenberg.[26] We have assumed no quenching, i.e., we have used the Schmidt single-particle limit for the gyromagnetic ratios of each of the odd nucleons.

From Table II it is evident that the spheroidal model gives in general better agreement with experiment than does the $j$-$j$ coupling model. However, it is also seen that in about half of those cases where both spherical and spheroidal coupling rules hold, the magnetic moments lie somewhere between the values given by the two models. An interesting example of this is provided by the three nuclei $As^{76}$, $Rb^{84}$, and $Rb^{86}$. In all three cases both the $A$ and $B$ rules can be applied successfully. The value of the magnetic moment given by the spheroidal description is −1.14; that by the $j$-$j$ coupling description is −2.13. The observed magnetic moments of the three nuclei are −0.906, −1.32, and −1.69, respectively. This is just the sort of trend that might be expected if $As^{76}$ with 43 neutrons can be described by the spheroidal model, which becomes less applicable as the closed shell is approached, until in $Rb^{86}$ with 49 neutrons the agreement is closest of any of the sequence

[24] R. W. King and D. C. Peaslee, Phys. Rev. **90**, 1001 (1953).
[25] S. A. Moszkowski and C. H. Townes, Phys. Rev. **93**, 306 (1954).
† *Note added in proof.*—The calculation of magnetic moments in several odd-odd nuclei using Nilsson wave functions has recently been discussed by W. M. Hooke, Bull. Am. Phys. Soc. Ser. II, **3**, 186 (1958).
[26] See reference 17, Eq. III, p. 43.

to the $j$-$j$ coupling model value. The obvious disagreement of the experimental and spheroidal magnetic moments of $K^{40}$ is consistent with the reasonable expectation that $K^{40}$ is describable by $j$-$j$ coupling[27]; on the other hand, the rather good agreement in $Cs^{130}$ indicates that the spheroidal model is applicable in this case.

The excited states of odd-odd nuclei can be expected to serve as another guide in deciding the validity of the spheroidal description. The validity of the asymptotic quantum number implies the presence of rotational spectra. On the other hand, in regions where the Nilsson description is inadequate one might expect a rather complicated spectrum. Furthermore, as pointed out by Nordheim,[1] there will exist a pronounced competition between excited states involving recoupling of the angular momenta, and those involving particle excitations. Because the experimental data are limited, we will not attempt to discuss this problem in more detail here. However, it is interesting to note that it is possible to describe the two isomeric states of $Br^{80}$ as the $\Omega_p+\Omega_n$ and $|\Omega_p-\Omega_n|$ doublet of a configuration different from the ground state. It would be interesting to determine whether this nucleus has rotational states.

[27] In support of the applicability of a $j$-$j$ coupling model to $K^{40}$, see, for example, S. Goldstein and I. Talmi, Phys. Rev. **102**, 589 (1956).

## CONCLUSIONS

The present formulation of the angular momentum coupling rules suggest that the $j$-$j$ coupling model and spheroidal, or spin-spin, coupling model represent two extremes of behavior in odd-odd nuclei. In general we may conclude that the ground state spins of odd-odd nuclei will be given by one or both of the models, with spin-spin coupling largely predominating, while the features which are more sensitive to mixing of states, such as magnetic moments, will essentially define the extent to which the models are applicable. It is also noteworthy that the tendency of the orbital angular momenta of the odd particles to couple antiparallel seems to be stronger than would be expected on the basis of the conventional shell model.

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