Mobility of Electrons in Germanium-Silicon Alloys

Maurice Glicksman

RCA Laboratories, Princeton, New Jersey

(Received March 4, 1958)

The mobility of electrons in germanium-silicon alloys with compositions varying from 0 to 30 atomic percent silicon has been measured between 77 and 300°K and the contribution of ionized impurity scattering has been subtracted. The resulting mobility for electrons in the conduction band with (111) symmetry was found to vary with composition as a combination of a constant term and a term varying as $\{\alpha(1-\alpha)\}^{-1}$, where α is the mole-fraction of minority component in the alloy. In the region where either these (111) electrons or electrons in the (100) conduction band are dominating the conductivity, a subtraction of a constant mobility due to scattering by phonons leaves a mobility which varies with temperature as $T^{-0.7}$ to $T^{-0.8}$. Both of these observations are in qualitative agreement with what is to be expected on the basis of presently available theories for the scattering by disorder.

INTRODUCTION

OBSERVATIONS of mobility in germanium-silicon alloys reported by Levitas[1] and the author[2] showed a decrease in the mobility as the alloy content was increased. The results reported by Levitas are for p-type material and include single crystals only in the composition region 0–4 atomic percent silicon. The results reported previously by the author have included only measurements made at room temperature and have dealt exclusively with n-type material. Measurements have been made of the mobility in n-type germanium-silicon alloy single crystals in the region 0–30 atomic percent silicon and between 77 and 300°K. These results are reported below, and an explanation in terms of scattering by disorder is presented.

OBSERVATIONS

In Fig. 1 are plotted measured values of the Hall mobility (the product of the Hall coefficient and the conductivity) as a function of temperature for a number of alloy crystals. Further information about these samples is contained in Table I. As can be seen from the third column of the table, the crystals contained appreciable concentrations of ionized impurities. Estimates of these concentrations were made from a knowledge of the Hall effect in various slabs cut from each crystal. These have been used to apply corrections to the observed mobility to obtain the mobility due to scattering by the lattice alone. For the corrections, the ionized impurity scattering was assumed to follow the Brooks-Herring relation,[3] with an effective mass of $\frac{1}{4}m_0$, and a dielectric constant of 16. These values are those used in a fit to germanium experiments; the effective mass does not change much in the alloys, at least at low silicon compositions,[4,5] and the dielectric constant is also changed only a small amount in the

alloys of low silicon content, the estimate from transmission data[6] being a six percent decrease for the 30% silicon alloy compared to pure germanium. An analysis of such data yields as large a change[7] in dielectric constant when the temperature is lowered to 77°K. Both of these small effects have not been included. The last column of Table I lists the mobilities of the electrons in the alloys, corrected for the effects of ionized impurity scattering.

Figure 2 is a plot of these corrected mobilities as a function of alloy composition. Some points included in this plot are not shown in Table I but have been de-
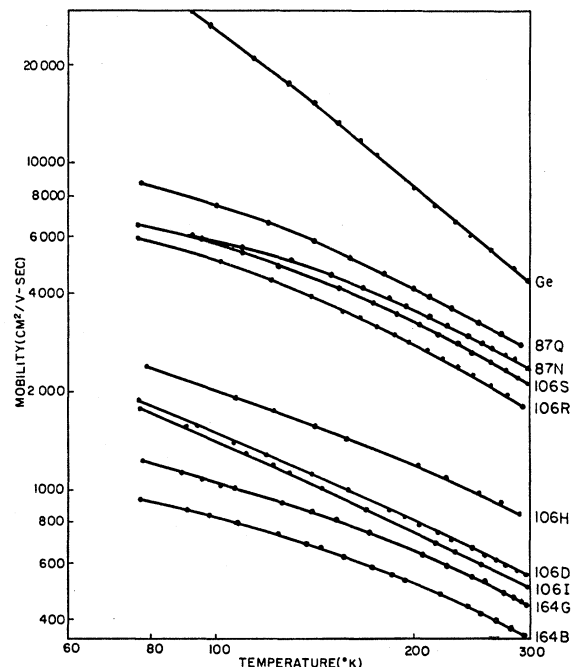

FIG. 1. The Hall mobility of germanium-silicon alloys as a function of temperature. The compositions are given in Table I.

[1] A. Levitas, Phys. Rev. **99**, 1810 (1955).
[2] M. Glicksman, Phys. Rev. **100**, 1146 (1955).
[3] Quoted by P. P. Debye and E. M. Conwell, Phys. Rev. **93**, 693 (1954).
[4] Dresselhaus, Kip, Ku, Wagoner, and Christian, Phys. Rev. **100**, 1218 (1955).
[5] M. Glicksman, Phys. Rev. **102**, 1496 (1956).
[6] Braunstein, Moore, and Herman, Phys. Rev. **109**, 695 (1958); R. Braunstein (private communication).
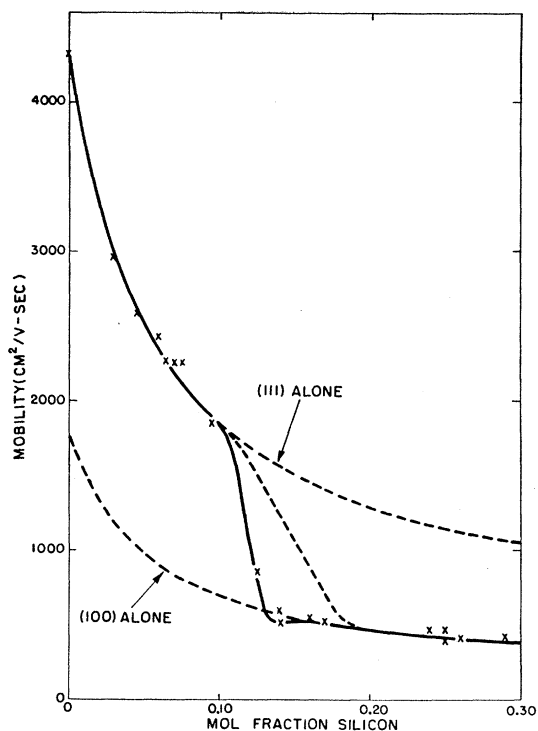[7] G. G. Macfarlane and V. Roberts, Phys. Rev. **97**, 1714 (1955).

FIG. 2. The Hall mobility of germanium-silicon alloys as a function of alloy composition. The curves are calculated; the unlabelled dashed curve includes both (111) and (100) bands without intervalley scattering, and the solid curve has included an arbitrary form of intervalley scattering.

scribed before.[2,8] The curves shown are calculated under the assumption that the scattering is given by a phonon term and a contribution due to the disorder in the lattice. This latter term should be proportional to the number of disordered atoms and their scattering factors[9,10] and should lead to a mobility proportional to $\{\alpha(1-\alpha)\}^{-1}$, where α is the mole fraction of minority component in the alloy. As is the case in scattering by acoustical phonons, the mean free path for the scattering should be independent of the carrier energy. If this is assumed, the two scattering processes then having the same dependence on energy, the resulting mobility may be expressed as

$$\frac{1}{\mu} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{dis}}, \qquad (1)$$

where μ_{ph} is the contribution from the scattering by phonons, and μ_{dis} is the contribution from the scattering by the lattice disorder.

Earlier work[2,5,8] has shown that at room temperature the conduction band minima are of the L_1 type [(111) minima, as in germanium] for $0 < \alpha < 0.10$, and of the

[8] M. Glicksman and S. M. Christian, Phys. Rev. 104, 1278 (1956).
[9] L. Nordheim, Ann. Physik 9, 607, 641 (1931).
[10] H. Brooks (private communication).

Δ_1 type [(100) minima as in silicon] for $0.20 < \alpha < 1.00$. For compositions between these two ranges, both conduction bands are populated and contribute to conduction. The curve labeled "(111) alone" is the result of assuming $\mu_{ph} = 4330$ cm²/v-sec, and fitting the expression at $\alpha = 0.065$. The fit in the range 0 to 10 percent silicon is good. For the curve "(100) alone" μ_{ph} was assumed to be 1800 cm²/v-sec, in agreement with measurements of Cronemeyer,[11] and fitted at $\alpha = 0.26$. Here the fit is good from 14% on to 29%, the highest composition of single crystal available for these studies. Knowing these mobilities and the variation of the energy of the two conduction bands with composition,[6] one can calculate what the observed product of Hall coefficient and conductivity should be. Expressions for these two quantities, assuming that the relaxation time (and mobility) in one band is independent of the position of the other, have been given.[5] The results of a calculation using the "(111)" curve are shown in Fig. 2 as a dashed curve. The combination does not give a good fit, the calculated mobilities being as much as a factor of two too high in the composition range 12–17%.

It might appear from an observation of Fig. 2 that a translation of the values in this region of composition to higher values of α would improve the fit. Since the band-gap determinations were not made on the same samples, some error in the knowledge of the composition of these crystals might be involved. However, there is available additional information which relates the samples measured directly to the conduction band configuration, and relative spacing of the (100) and (111) bands: the magnetoresistance. If these measurements are compared, it is evident that these samples agree in this detail with the observed band-gap variation. Indeed the form of disagreement in the mobility is evident, since the points which have mobilities of the (100) band arise from crystals which show in their magnetoresistance evidence that both bands contribute to the conduction. It was suggested[5] that interband

TABLE I. Data on n-type germanium-silicon alloy crystals.

Crystal	Atomic percent silicon	Electron conc. at ~290°K (cm⁻³)	Hall mobility (~290°K) (cm²/v-sec)	Hall mobility corrected for impurity scattering (300°K) (cm²/v-sec)
674	0	0.24×10¹⁵	4250	4330
87Q	3	5.6	2710	2960
87N	4.5	5.5	2390	2590
106S	6.5	1.6	2180	2280
106R	9.5	1.3	1790	1855
106H	12.5	1.2	840	856
106I	14	1.2	495	502
106D	16	1.3	540	550
164G	25	14	438	464
164B	26	9.9	384	405

[11] D. C. Cronemeyer, Phys. Rev. 105, 522 (1957).

scattering,[12] i.e., between the two conduction bands, could explain these observations, and a fairly crude model for such a scattering process was included in an earlier paper[5] to give a satisfactory fit to the experiment. This gives the solid curve shown in Fig. 2. Since a more satisfactory treatment of this interband scattering is not yet available, the discussion of what information may be derived from the measurements will be limited to a study of the scattering in each of the bands, and alloys in which it is fairly certain the observations are limited to this case. The question of the details of the interband scattering and what can be learned about them from these data is left for a later study.

The mobility may be written as the sum of two terms, as was done in Eq. (1), over the temperature range studied. At the lowest temperatures, the corrections to the observed values for the impurity scattering become quite large. Thus at the lowest temperature, 78°K, and the point of largest magnitude of correction, the mobility of 87Q is increased by 50%, while that of 106R is increased by 26.5%. For the crystals with the conduction essentially dominated by electrons in the (111) conduction band, the germanium lattice phonon scattering (measured as $\mu_{ph}=6.56 \times 10^7 T^{-1.69}$ cm^2/v-sec) was then subtracted. In the case of the two crystals 164, in which the conduction is dominated by electrons in the (100) conduction band, the silicon lattice phonon scattering was subtracted. This was assumed to have the value $\mu_{ph}=2.81\times10^9 T^{-2.5}$ cm^2/v-sec, fitting the temperature dependence observed by Ludwig and Watters[13] together with the Hall mobility values measured by Cronemeyer.[11] Figure 3 is a plot of the resulting mobilities as a function of temperature for the six crystals which are believed to involve conduction in only one of the conduction bands in each. These mobilities can be fairly well represented by a power law dependence on the temperature, with the power between −0.7 and −0.85. Because of the magnitude of the corrections for the ionized impurity scattering and the attendant uncertainties in these, the variation in values from −0.7 to −0.85 is probably not significant.

DISCUSSION

From the measurements reported above, one might conclude that in the alloys there is an additional scattering mechanism which can be represented by a mobility having the form

$$\mu^{111}=\frac{2.8\times10^4 T^{-0.8}}{\alpha(1-\alpha)} \qquad (2)$$

[12] H. Brooks and W. Paul, Bull. Am. Phys. Soc. Ser. II, **1**, 48 (1956).
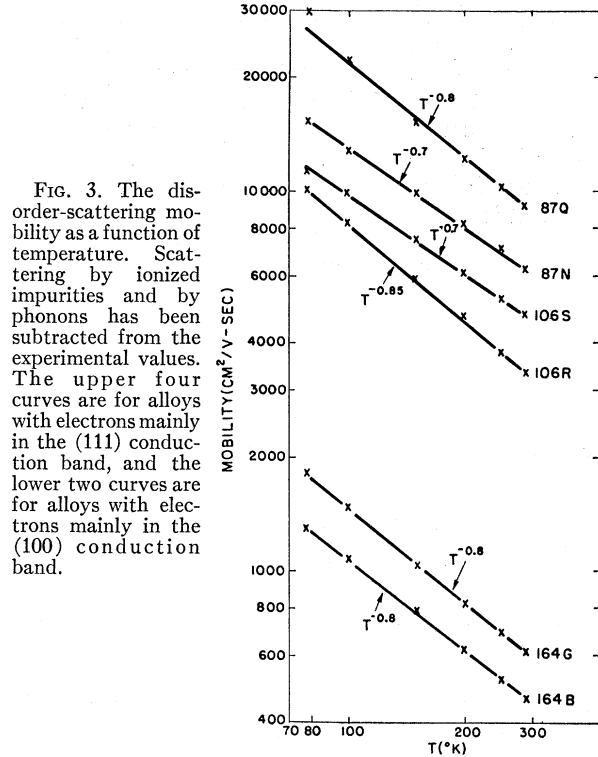[13] G. W. Ludwig and R. L. Watters, Phys. Rev. **101**, 1699 (1956).

FIG. 3. The disorder-scattering mobility as a function of temperature. Scattering by ionized impurities and by phonons has been subtracted from the experimental values. The upper four curves are for alloys with electrons mainly in the (111) conduction band, and the lower two curves are for alloys with electrons mainly in the (100) conduction band.

for electrons in the (111) conduction band, and

$$\mu^{100}=\frac{9.7\times10^3 T^{-0.8}}{\alpha(1-\alpha)} \qquad (3)$$

for electrons in the (100) conduction band. As was noted earlier, it is expected that in disordered alloys there will be a contribution to the scattering of carriers from the randomly positioned perturbations of the lattice by the alloy atoms. This can be discussed, at least in a qualitative manner, from a perturbation approach suggested by Nordheim[9] and also used by Brooks[10] with an added assumption about the magnitude of the perturbation.

Nordheim described the potential energy of the electron in terms of a nonperiodic function, which he separated into a periodic part, consisting of the average of the constituent potentials, and the nonperiodic residue which contributed the scattering. The result of such a calculation is the following expression for the relaxation time:

$$\frac{1}{\tau}=\frac{8\pi^3}{h^4}(UV)(2m_l m_t{}^2)^{\frac{1}{2}}E^{\frac{1}{2}}\alpha(1-\alpha). \qquad (4)$$

Here U is an integral over scattering angles of the square of the matrix element for the difference of the potentials of the germanium and silicon atoms, V is the atomic volume, m_l and m_t are the electron effective masses in the longitudinal and transverse directions of

the energy ellipsoid, respectively, and E is the electron energy. There is then one unknown, the parameter U. Brooks has suggested that this difference in potential be represented by ΔE, the difference between the germanium conduction-band energy and the silicon conduction-band energy. If the various numbers are substituted in the resulting expression for the mobility, the result is

$$\text{Nordheim:} \quad \mu^{111} = \frac{2.91 \times 10^4 T^{-\frac{1}{2}}}{\alpha(1-\alpha)\bar{U}}, \qquad (5)$$

$$\text{Brooks:} \quad \mu^{111} = \frac{7280 T^{-\frac{1}{2}}}{\alpha(1-\alpha)(\Delta E)^2}, \qquad (6)$$

for the electrons in the (111) conduction band; and

$$\text{Nordheim:} \quad \mu^{100} = \frac{8141 T^{-\frac{1}{2}}}{\alpha(1-\alpha)\bar{U}}, \qquad (7)$$

$$\text{Brooks:} \quad \mu^{100} = \frac{2035 T^{-\frac{1}{2}}}{\alpha(1-\alpha)(\Delta E)^2}, \qquad (8)$$

for the electrons in the (100) conduction band. $\bar{U}$ is appropriately averaged over the energy and both $\bar{U}$ and $(\Delta E)^2$ are in ev^2. These results should then be compared with experiment.

First, it is apparent that the calculated temperature dependence is not quite the same as that observed. The experiments yield a power law dependence as T^{-n} with n being something between +0.7 and 0.8, and the results represented in Eqs. (2) and (3) use the most likely value of 0.8. The difference between this power and the $\frac{1}{2}$ power may be in some dependence of the mean free path on the energy. Of course, under Brooks' assumption that the scattering potential is a delta-function, the mean free path is energy independent. Actually, however, the integral U may be dependent on the energy—fairly weakly, since the contribution to the mobility would indicate a dependence like $E^{+0.2}$ or $E^{0.3}$, or a scattering which is stronger for the higher energy electrons. This direction is opposite to what one would get for a Coulombic or screened Coulombic potential. This proportionality of the scattering to the energy could arise from a scattering potential which involves a hard core of opposite sign.

Of interest is the strength of the scattering in the (111) band as compared to that in the (100) band. The ratio of the calculated mobilities is

$$\mu^{111}/\mu^{100} = 3.58,$$

independent of temperature, while the experimental value of this ratio is 2.9, and also independent of tem-

perature within experimental uncertainty. This agreement (about a 20% difference) is reasonably good, considering the uncertainties in the experimental figures (about 10%) and the relative crudeness of the theories. In addition, one can derive values for $\bar{U}$ or ΔE by comparing Eqs. (2) and (3) with Eqs. (5)–(8). For $\bar{U}$, neglecting the difference in temperature dependence, one gets the value 1.04 ev^2 from the (111) expression and 0.84 ev^2 from the (100) expression. If the difference in temperature dependence is ascribed to an energy dependence in the integral U, this value for $\bar{U}$ would be affected. For ΔE, again neglecting the difference in temperature dependences, one gets 0.51 ev for the (111) scattering, and 0.46 for the (100) scattering, or about 0.5 ev for the scattering energy. On Brooks' model, this is the energy difference between the silicon and germanium atom conduction bands in the alloy. This value is not far from what one might then expect, being about equal to the difference in band gaps.

CONCLUSIONS

The observations show fairly clearly these two features: the scattering of electrons in the alloys includes a contribution from scattering by the disorder present which depends on the composition as $\alpha(1-\alpha)$, and the disorder scattering mobility depends on the temperature as something like $T^{-0.8}$. These observations are in qualitative agreement with calculations based on the work of Nordheim and Brooks. In comparing the theory with experiment, one finds that the strength of the scattering potential is about the same for the electrons in the (111) band as in the (100) band: this is what is expected since the interacting potential should in major share be determined by the type of atoms and their electronic states, and not in major share by the band index or reciprocal lattice momentum of the electron being scattered. The increased temperature dependence over the theoretical $T^{-0.5}$ may be due to an energy dependence in the mean free path. If so, this could arise from a scattering potential with a hard repulsive core (giving stronger scattering at higher energies). However, the difference in temperature dependence between the observed values and the energy-independent mean free path calculations is not large, and such an energy-dependent scattering term should probably be a small correction.