TABLE I. Effects of the Coulomb interaction on the polarization.

Polarization[a]	GT, 156 Mev $\Theta=5°$	GT, 310 Mev $\Theta=5°$	SM, 90 Mev $\Theta=10°$	SM, 150 Mev $\Theta=10°$
P_{I}	0.310	0.474	0.381	0.665
P_{II}	0.270	0.467	0.370	0.657
P_{III}	0.170	0.427	0.304	0.585

[a] P_{I} = polarization that includes all effects of the Coulomb interaction. P_{II} = polarization neglecting relativistic effects arising through the Coulomb field [$\nu=0$ in Eq. (6)]. P_{III} = polarization with no Coulomb effects [$\nu=0$, $\alpha=0$ in Eq. (6)].

TABLE II. Effects of the Coulomb interaction on the triple-scattering parameter β. The symbols I, II, and III have the same meaning as in Table I.

	GT, 310 Mev $\Theta=5°$	GT, 310 Mev $\Theta=10°$	SM, 300 Mev $\Theta=3°$	SM, 300 Mev $\Theta=7°$
β_{I}	8.7°	−10.7°	15.6°	5.2°
β_{II}	9.3°	−10.1°	13.6°	4.6°
β_{III}	−3.5°	−14.3°	−2.0°	−5.6°

In Tables I and II, effects of the Coulomb interaction on the polarization and β are shown for some cases. The importance of these effects is particularly clear at 90 Mev as is seen in Fig. 2. Also, as a result of the Coulomb-nuclear interference, β changes its sign at small angles.

Since the original formalism of Watson,[30] on which the transition matrix element (1) is based, contains the relative error of the order $1/A$, it is not clear whether a certain amount of disagreement of the calculated values of the polarization with experiments is truly significant. Besides, the target nucleus has been treated as though it were infinitely heavy, which would also introduce an additional error of the same order.

[30] K. M. Watson, Phys. Rev. **89**, 575 (1953).

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Proton-Proton Scattering in the Bev Region

G. E. BROWN*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark
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Proton-proton scattering in the Bev region is analyzed in terms of an interaction which, at 1 Bev, is taken to be a hard core of radius 0.45×10^{-13} cm together with an external absorption of Gaussian form. The hard core is assumed to disappear with increasing energy and to be replaced by absorption. General features of the data are well reproduced by this simple model.

1. INTRODUCTION

In the past few years, proton-proton scattering experiments have been carried out up to an energy of 6 Bev. Whereas at energies in the range 0–300 Mev the differential cross section has been analyzed quite generally in terms of phase shifts, at energies in the Bev region, this can be done only with simplifying assumptions[1] since the number of experimental data are insufficient to determine the necessarily large number of phase shifts. One way of seeing just what such assumptions imply is to choose a simple model, such as an interaction of some definite radial dependence. One of the simplest assumptions is that the bombarded proton is equivalent to an absorbing sphere, with inverse mean free path for absorption, K, constant throughout the sphere. This means that only two parameters, K and the radius of the sphere R, have to be determined. The experimental cross sections in the range 0.8–2.75 Bev have been fitted in this way.[2] It is easy to see how K must behave. At 1 Bev the ratio of elastic to inelastic scattering is almost unity, and so K must be large enough so that the sphere is essentially black. In order to describe the decrease in the elastic cross section with energy (see Fig. 1) and the increase in the ratio of inelastic to elastic scattering, K must decrease with energy.

The absorption described here results from meson production. In terms of a picture in which the proton-meson interaction is strong, it is hard to see why the absorption should decrease as more energy becomes available for meson production. Further, the analysis of Rarita[1] at 1 Bev indicated that such a simple optical model description was inadequate for describing the angular distribution in detail, because it predicted more absorption in the s wave than in the d wave, whereas his phase-shift analysis required the opposite condition.

A reasonable way to supplement the above picture

* On leave from the Department of Mathematical Physics, University of Birmingham, Birmingham, England.
[1] W. Rarita, Phys. Rev. **104**, 221 (1956).

[2] W. B. Fowler *et al.*, Phys. Rev. **103**, 1489 (1956).

is to add to the interaction a repulsive core, of the type known to be present at low energies. We shall show that such a hard core, together with a smoothly-varying, external absorption of longer range can account quite adequately for the data at 1 Bev.

Now, such a hard core in the interaction is usually assumed to describe, in a phenomenological way, effects from the exchange of several pions, or of heavier mesons. If this is its origin, then it is unreasonable that it should exist at energies of several Bev where there is sufficient energy to produce several pions or a pair of heavy mesons. We therefore assume that the core disappears with increasing energy and is replaced by either partial or complete absorption. We shall show that this is easily able to account for the decrease in the elastic scattering.

2. DEVELOPMENT

The momentum of the incident proton in the center-of-mass system before interaction can easily be calculated from the relation

$$2(p_0^2+M^2)^{\frac{1}{2}}=E_{\text{c.m.}}+2M, \tag{1}$$

where $E_{\text{c.m.}}$ is the kinetic energy in the center-of-mass system. In the region of interaction, the momentum will

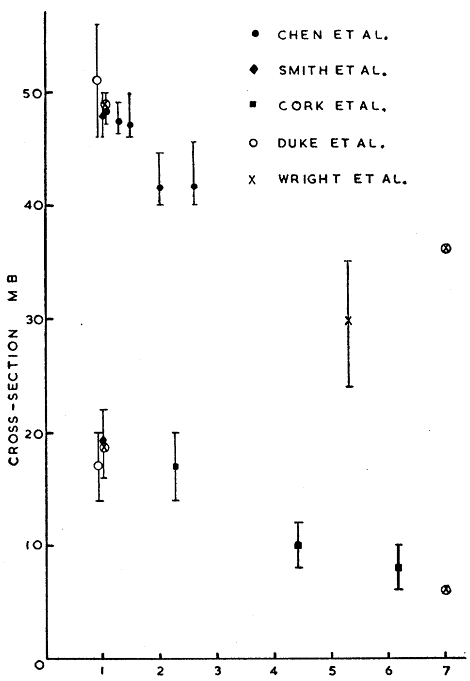

FIG. 1. Experimental data on total and elastic cross sections. The points ⊗ at 1 Bev are our points for that energy calculated with a hard core plus external absorptive interaction. The points ⊗ on the right are calculated for the case in which the hard core has been replaced by absorption. The experimental points are: ●—Chen, Leavitt, and Shapiro, Phys. Rev. **103**, 211 (1956); ◆—Smith, McReynolds, and Snow, Phys. Rev. **97**, 1186 (1955); ■—Cork, Wenzel, and Causey, Phys. Rev. **107**, 859 (1957); ○—P.J. Duke *et al.*, Phil. Mag. **2**, 204 (1957); ×—R. W. Wright *et al.*, Phys. Rev. **100**, 1802(A) (1955).

be changed. In the absence of a well-founded relativistic equation for the interaction of two nucleons, we take the equation

$$p^2\psi=[p_0+\delta p(r)]^2\psi \tag{2}$$

to describe the scattering of one of the nucleons. Here $p=\nabla/i$ is[3] the momentum operator for this nucleon. In using Eq. (2) we are assuming that the result of interaction with the target nucleon is simply to shift the local momentum of the incident nucleon.

If we now expand ψ in spherical harmonics

$$\psi(\mathbf{r})=\sum_l a_l p_l(\cos\theta) f_l(r)/r, \tag{3}$$

then f_l must satisfy the equation

$$\frac{d^2 f_l}{dr^2}+\left\{[p_0+\delta p(r)]^2-\frac{l(l+1)}{r^2}\right\}f_l=0. \tag{4}$$

If, in the region outside the hard core, $\delta p(r)$ varies smoothly, we can use the WKB method to integrate this equation. In case the classical turning point does not occur at a distance larger than the core radius a, we must start f_l from zero at a and obtain for the phase shift

$$\delta_l\cong\int_a^R\left\{\left([p_0+\delta p(r)]^2-\frac{l(l+1)}{r^2}\right)^{\frac{1}{2}}\right\}dr -(p_0R-\tfrac{1}{2}l\pi), \tag{5}$$

where a is the radius of the hard core and R is some distance larger than the range of interaction. For the case we consider, δp will be small compared to p_0, and we can expand, keeping only terms linear in δp; we obtain

$$\begin{aligned}\delta_l&\cong\int_a^R\left[p_0^2-\frac{l(l+1)}{r^2}\right]^{\frac{1}{2}}dr+\int_{(a^2-b^2)^{\frac{1}{2}}}^{\infty}\delta p dz-(p_0R-\tfrac{1}{2}l\pi)\\&=\{l-p_0b\}\pi/2-p_0(a^2-b^2)^{\frac{1}{2}}+p_0b\cos^{-1}(b/a)\\&\qquad+\int_{(a^2-b^2)^{\frac{1}{2}}}^{\infty}\delta p dz,\end{aligned} \tag{6}$$

with

$$p_0b=[l(l+1)]^{\frac{1}{2}}. \tag{6'}$$

We have been careful not to set $[l(l+1)]^{\frac{1}{2}}$ equal to $l+\frac{1}{2}$ here, as is often done in the WKB method, because the above formula will be used for small values of l where the difference is appreciable. Here b can be interpreted as the impact parameter. The first terms in δ_l give the semiclassical value of the phase shift for a hard core; the final term is just the phase shift from the external region as obtained by Fernbach, Serber, and Taylor. In case the classical turning point is outside the hard core, only the final term survives. Our above develop-

[3] We choose $\hbar=c=1$ throughout.

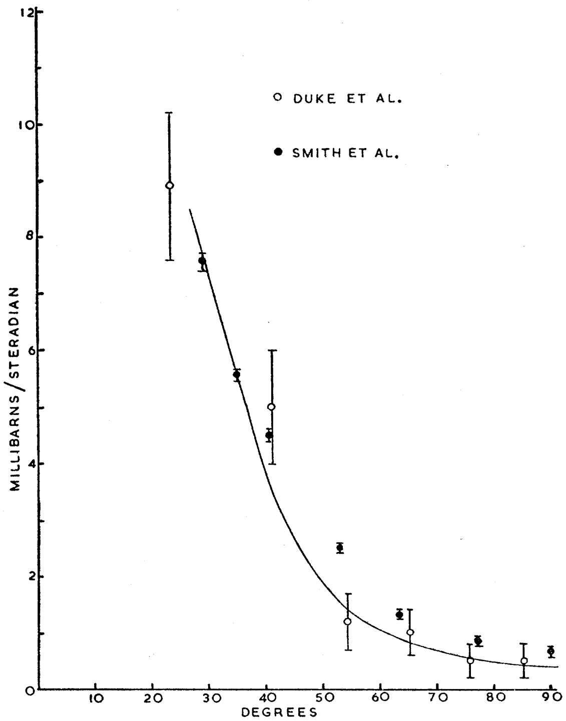

FIG. 2. Scattering at 1 Bev. Theoretical points for a hard core plus external absorption compared with experimental data. References to the experiments are given in the caption to Fig. 1.

ment shows that the effects of the core and of the external region are simply additive, as effects from different regions should be in a semiclassical treatment.

Within the linear approximation, the term in Eq. (2) responsible for the scattering is $2p_0\delta p$. We see from Eq. (6) that this particular choice of the factor multiplying δp ensures that if the interaction is absorptive, i.e., δp is imaginary, the mean free path, will be independent of energy if δp is not a function of energy. Thus, our choice of the equation (2) seems a reasonable one. For a particle in a smoothly-varying potential well, solution of Eq. (2) by semiclassical techniques gives the usual optical-model formulas.

In the case of scattering at 1 Bev and a core radius of 0.45 fermi (1 fermi$=10^{-13}$ cm), $p_0a=1.6$ and, consequently, only s and p waves are affected by the hard core. For the case of only a hard core and no external interaction, our formula gives -1.6 for δ_0 and -0.713 for δ_1. Whereas the s phase is just that from the exact solution, the exact p phase here is -0.586. Thus, our approximate formula overestimates the repulsion in p states somewhat; however, it gives zero d-state phase, whereas the exact phase shift is -0.120 here, and the exact f-state phase shift is -0.011. Whereas the method will not be expected to give accurate results, it should be sufficiently good for describing general features of the scattering.

3. CALCULATION

We have assumed the interaction to be composed of a hard core of radius 0.45 fermi and an external absorption

$$\delta p(r)=\frac{i}{R\sqrt{\pi}}\exp(-r^2/R^2), \qquad (7)$$

where the factor $i/\sqrt{\pi}$ was chosen for convenience.

Calculations of total and inelastic cross sections and of differential cross sections were carried out using standard formulas (e.g., see reference 1). The parameter R was adjusted so as to give the experimental total cross section at 1 Bev; for $R=0.86$ fermi, a σ_t of 49.1 mb was obtained. The value of the inelastic cross section was then found to be 30.0 mb, giving an elastic cross section of 19.1 mb. In Fig. 2 we show the differential cross section compared with the experimental data.

We represent the disappearance of the core expected for energies of several Bev by simply extending the absorption, Eq. (7), in to the origin (in the preceding it applied, of course, only to the external region). We then find $\sigma_t=37.4$ mb and $\sigma_{el}=6.4$ mb. We have used here a p_0 corresponding to 6.4 Bev but the result is very insensitive to the particular value of p_0 as we shall soon see. The angular distribution obtained with no core is compared with the 6.4 Bev experimental points in Fig. 3. The theoretical curve gives too little large-angle scattering here. This may be because the hard core has not yet completely disappeared at this energy, in which case the theory would predict a slightly larger total cross

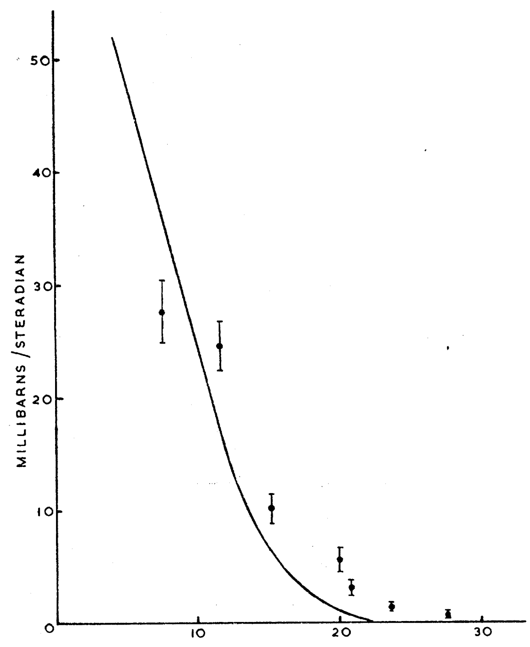

FIG. 3. Theoretical points for a purely absorptive interaction compared with the 6.4 Bev experiments of Cork, Wenzel, and Causey referred to in the caption to Fig. 1.

section and more large-angle scattering. On the other hand, better agreement could probably also be obtained by decreasing R slightly and increasing the absorption which is possible without destroying the fit at 1 Bev.

Although our description is rather rough, it is clear that the decrease in elastic scattering can be accounted for by the disappearance of the hard core in the nucleon-nucleon interaction.

Quantitative features of our description can easily be understood if we neglect the identity of the two protons and describe the scattering by optical model formulas in which the sums over partial waves have been replaced by integrals. Then

$$\sigma_t=4\pi\int_0^\infty \{1-e^{-2\delta_I(b)}\cos 2\delta_R(b)\}bdb,$$
$$\sigma_i=2\pi\int_0^\infty \{1-e^{-2\delta_I(b)}\}bdb, \tag{8}$$

where δ_R and δ_I are the real and imaginary parts of δ, the former arising from the hard core and the latter from the external absorption. For a hard core of large extent, it is clear that the total cross section will be equal to that for a black disk, since $\langle\cos 2\delta_R(b)\rangle_{\text{Av}}$ will be zero. At 1 Bev the core is, of course, not large. But for $b=0$, $2\delta_R=3.2$, and consequently, $\cos 2\delta_R\cong -1$ corresponding to resonance scattering in the s state. Thus, the total cross section can be substantially supplemented by the presence of the hard core. As the hard core disappears, $\delta_R\rightarrow 0$, and the total cross section goes down, leaving the inelastic cross section almost unchanged.

4. DISCUSSION

It is seen from the preceding that our description accounts for the main features of p-p scattering in the Bev region. It is, of course, only a rough one, and would not be expected to describe more detailed features, such as polarization. (It would predict zero polarization.) For a description of these, other elements will have to be added. However, we believe that the disappearance of the hard core may be the essential factor in the decrease in elastic scattering with increasing energy.

We have chosen to describe the hard core as a velocity-dependent interaction. It is clear that we could also choose an imaginary core of height H such that H lies between 1 and 6.4 Bev. At the lower energies, the effect of such a core would be similar to that of the hard core, since the wave function must be nearly zero at the edge. For energies above it, it will act as a completely absorbing region. Such a purely absorbing core has been used recently in nucleon-antinucleon scattering by Ball and Chew.[4]

If the external absorption is really more or less constant, as we have postulated it, then the elastic cross section should stop decreasing after the hard core disappears, and it will soon be possible to check this by experiments at higher energies.

It is, of course, to be assumed from our picture that neutron-proton scattering should exhibit the same general characteristics, although the absorption may have to be rather smaller to take account of the fact that meson production from the state of isotopic spin $T=0$ is less likely than from $T=1$.

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[4] J. S. Ball and G. F. Chew, Phys. Rev. **109**, 1385 (1958).