In the limit $\epsilon\rightarrow 0$, the diagonal elements should tend to the pure-state phase shifts. In fact, if we put $\sin\epsilon=\epsilon$, and neglect terms containing ϵ^2, the foregoing form of the S matrix reduces to the form from which Eq. (15) was derived, i.e.,

$$S=\begin{pmatrix} \exp[2i\delta(T=\frac{3}{2})] & \{\exp[2i\delta(T=\frac{3}{2})]-1\}\epsilon \\ \{\exp[2i\delta(T=\frac{3}{2})]-1\}\epsilon & \exp[2i\delta(T=\frac{1}{2})] \end{pmatrix}. \tag{17}$$

From Eq. (17) and Eq. (14) we also derive $\epsilon=1/2B=-0.004$. The square of ϵ can really be neglected. Eq. (17) also verifies the relation between C_3 and g_{33} which is contained in Eq. (14).

It may be remarked that the unitarity condition of Eq. (17) is destroyed because of the presence of off-diagonal terms while the diagonal elements are still unimodular. This can be restored by using a constant multiplier in front of Eq. (17); this constant is close to 1.

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Nucleon-Nucleon Interactions and Polarization of High-Energy Protons Elastically Scattered from Carbon*

SHOROKU OHNUMA
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
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The transition matrix element in momentum space derived by Riesenfeld and Watson has been used to calculate the polarization and the triple-scattering parameter β of high-energy protons elastically scattered at small angles from carbon. As the nucleon-nucleon phase shifts which represent two-body interactions, those by Signell and Marshak, by Gammel and Thaler, and by Feshbach and Lomon have been considered. In evaluating nuclear as well as Coulomb scattering amplitudes, the first Born approximation has been employed with the assumption that, in carbon, the distribution of protons is equal to that of neutrons. Final results are then independent of the assumed distribution of nucleons. It has been found that, while one cannot discriminate between Signell-Marshak and Gammel-Thaler phase shifts, both of them being in semi-quantitative agreement with experimental data, Feshbach-Lomon phase shifts may be ruled out because of the wrong sign of the resulting β. Since only the first-order transition matrix element in momentum space has been used in the present work, the calculation does not depend on the optical model potential in the usual sense.

I. INTRODUCTION

THE polarization of high-energy protons elastically scattered from nuclei has been calculated by many authors.[1] Most of these calculations are, however, based on phenomenological potentials between incident protons and target nuclei as a whole. Therefore, it is rather difficult to relate their results to individual nucleon-nucleon interactions. Optical-model potentials directly connected to nucleon-nucleon scattering phase shifts have first been studied by Riesenfeld and Watson[2] and, more recently, by Bethe[3] in estimating the proton polarizations by carbon. Riesenfeld and Watson used the phase shifts derived by Feshbach and Lomon[4] and compared the calculated values of the polarization at $\Theta=20°$, where Θ is the scattering angle in the laboratory system, with experimental data. Bethe evaluated the scattering cross sections as well as the polarization at $E=310$ Mev, with E standing for the kinetic energy of incident protons in the laboratory system. He used five sets of phase shifts by Stapp, Ypsilantis, and Metropolis[5] for those states with isotopic spin $T=1$ together with phase shifts for those with $T=0$ that have been computed by Gammel and Thaler[6] from their potential. His conclusion is that it is difficult to discriminate between those sets for $T=1$ from his results.

In the present paper, the polarization of high-energy protons with energies E between 90 and 310 Mev scattered from carbon has been calculated as a function of scattering angles $\Theta<20°$. Also, the triple-scattering parameter β defined by Wolfenstein[7] has been estimated at $E\simeq 300$ Mev. In evaluating the transition matrix element which has been derived in RW, nucleon-nucleon phase shifts of Signell and Marshak,[8] and of Gammel and Thaler[6] have been considered. Although far from being the final answer to the problem of nuclear forces,

* This research was supported by the U. S. Atomic Energy Commission and by the Office of Ordinance Research, U. S. Army.
[1] See, for example, E. Heiberg, Phys. Rev. **106**, 1271 (1957) for detailed references.
[2] W. B. Riesenfeld and K. M. Watson, Phys. Rev. **102**, 1157 (1956). This paper will henceforth be referred to as RW.
[3] H. A. Bethe, Ann. Phys. **3**, 190 (1958).
[4] H. Feshbach and E. Lomon, Phys. Rev. **102**, 891 (1956). This work will be referred to as FL.
[5] Stapp, Ypsilantis, and Metropolis, Phys. Rev. **105**, 302 (1957).
[6] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957); **107**, 1337 (1957). This work will be referred to as GT.
[7] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954); **98**, 1870 (1955).
[8] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); **109**, 1229 (1958). This work will be referred to as SM.

their potentials reproduce many essential features of nucleon-nucleon scattering data for $E \lesssim 150$ Mev (SM) and for $E \lesssim 300$ Mev (GT). Feshbach-Lomon[4] phase shifts, which do not give correct values for p-p polarization,[9] have been found to be ruled out because of the wrong sign of the resulting β.

II. LIST OF NOTATION

$\mathbf{p}(\equiv\hbar\mathbf{k})$, $\mathbf{p}'(\equiv\hbar\mathbf{k}')$ = the incident and the scattered proton momenta, respectively, in the laboratory system.

E = kinetic energy of the proton in the laboratory system.

Θ = scattering angle of the proton in the laboratory system.

m_π = π-meson mass.

$\rho_N(r)$, $\rho_C(r)$ = the nucleon density and the charge density, respectively, of the target nucleus.

$\boldsymbol{\sigma}$ = Pauli spin vector of the proton.

M = proton mass.

$k=|\mathbf{k}|=|\mathbf{k}'|$.

γ = total energy of the proton in the laboratory system divided by Mc^2.

R = radius of the target nucleus.

A, Z = mass number and atomic number, respectively, of the target nucleus.

v = velocity of the proton in the laboratory system.

$\mathbf{n}=(\mathbf{k}\times\mathbf{k}')/|\mathbf{k}\times\mathbf{k}'|$.

$q=2k\sin(\Theta/2)$.

$\eta=(e^2/\hbar v)$.

K_L = phase shift for orbital angular momentum $\hbar L$.

$\sigma_L=\arg\Gamma(1+L+i\eta)$.

$\mathbf{s}=\sin(\Theta/2)$.

III. EXPRESSIONS OF THE POLARIZATION AND THE TRIPLE-SCATTERING PARAMETER β

Since the main object of this work is to obtain information on nucleon-nucleon interactions, it is desirable to make final results independent of the detailed structure of the target nucleus. One can accomplish this by calculating the polarization and β in the first Born approximation which has been found to be valid for small scatterlng angles,[10] *i.e.*, for $\Theta<\Theta_0$, the position of the first diffraction maximum of the polarization.[11] On the other hand, as has already been pointed out,[12] the experimental angular distribution of the polarization clearly indicates significant contributions of the Coulomb-nuclear interference at such small angles. Also, relativistic effects arising through the Coulomb interaction become noticeable in this case.[13] These effects of the Coulomb forces have been taken into consideration in this work on the assumption that the distribution of neutrons is the same as that of protons and that the first Born approximation is reasonably accurate in estimating the Coulomb scattering amplitude except, of course, its phase factor. Under this assumption, the final results are entirely independent of the assumed distribution of nucleons.

The treatment of the Coulomb scattering employed below is open to criticism because the Coulomb amplitude in first Born approximation does not contain the phase factor $\exp[-i\eta\ln\sin^2(\Theta/2)]$ which is present in the exact treatment of the Coulomb wave in the case of pure Coulomb scattering. If one simply includes this factor in the Coulomb amplitude and employs the nuclear amplitude in the form used by Riesenfeld and Watson, appreciable changes result at the smaller scattering angles. Thus, for example, the values of the polarization at 90 Mev (GT) taken, respectively, with and without the phase factor are 0.40 and 0.28 for $\Theta=5°$, 0.263 and 0.270 for $\Theta=7°$, 0.284 and 0.292 for $\Theta=10°$. The employment of the phase factor in this simple manner is unjustifiable, however, as may be seen from the following argument. In the case of scattering of spinless particles the scattered wave is

$$\psi_{Sc}=\rho^{-1}\exp[i(\rho-\eta\ln 2\rho+2\sigma_0)]\{-\tfrac{1}{2}\mathbf{s}^{-2}\eta\exp(-i\eta\ln\mathbf{s}^2)+\sum_{L=0}^{\infty}(2L+1)P_L(\cos\Theta)Q_L(K_L)\exp[2i(\sigma_L-\sigma_0)]\},$$

where

$$Q_L(K_L)=(e^{2iK_L}-1)/2i,$$

with K_L denoting the phase shift for orbital angular momentum $\hbar L$. The σ_L are the Coulomb phase shifts such that

$$\sigma_L-\sigma_0=\tan^{-1}(\eta/L)+\tan^{-1}[\eta/(L-1)]+\cdots+\tan^{-1}\eta.$$

The phase factor $\exp(-i\eta\ln\mathbf{s}^2)$ may not be included therefore without bringing in the $\exp[2i(\sigma_L-\sigma_0)]$ as well. The latter are not included however in the equations of Riesenfeld and Watson. Their inclusion would involve obtaining numerical values of the phase shifts and summing the series. This procedure is questionable because of uncertainties in the values of K_L caused by neglect of plural scattering in Watson's method. There is some compensation of the two factors however. For example, a semiclassical estimate for 100-Mev protons scattered from carbon indicates that for a nuclear radius of 3×10^{-13} cm the K_L decrease rapidly with L beginning with $L\cong6$. In this case $2(\sigma_L-\sigma_0)\cong0.4$, while for $\Theta=10°$ one has $-\eta\ln\mathbf{s}^2\cong0.5$. The two phase factors are thus approximately equal, resulting in their approximate compensation. At higher energies both factors are less important. The degree of compensation varies somewhat with Θ because of the variation of $\ln\mathbf{s}^2$ and because of the change in the relative importance of different L. The tacitly made assumption of equality of the K_L with

[9] A. M. Saperstein and L. Durand, III, Phys. Rev. **104**, 1102 (1956).

[10] For references on this point, see Bethe's article (reference 3); also E. M. Hafner, Phys. Rev. **111**, 297 (1958).

[11] In any case, the formulas in RW cannot be applied beyond this angular region.

[12] A. E. Taylor, in *Reports on Progress in Physics* (The Physical Society, London, 1957), Vol. **20**, p. 86.

[13] W. Heckrotte, Phys. Rev. **101**, 1406 (1956).

and without Coulomb field is partially justifiable because for $L=6$ the centrifugal barrier is approximately 30 times greater than the Coulomb barrier.

It may be noted that, since the present calculations use only the first-order transition matrix element in momentum space, they do not depend on the optical model potential in the usual sense but rather depend on the existence of nucleon-nucleon phase shifts. A first-order treatment in momentum space which strictly speaking corresponds to neglecting rescattering is equivalent to a first-order treatment in coordinate space and the latter leads to the use of the partial wave expansion with the approximation $Q_L(K)\simeq K$. The validity of this approximation is not essential, however, to most of the discussion regarding the Coulomb field phase factors.

According to RW, the matrix element for the transition, in which the incident and the scattered proton momenta in the laboratory system are $\mathbf{p}$ and $\mathbf{p}'$, respectively and the target nucleus is in its ground state,[14] may be written in the form

$$(\mathbf{p}'|\mathcal{V}_C|\mathbf{p})=\Big\{-(V_{CR}+iV_{CI})+(1/m_\pi c)^2 i\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{p}\times(V_{SR}+iV_{SI})\Big\}(2\pi\hbar)^{-3}\int\rho_N(r)\times\exp[-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}/\hbar]d\mathbf{r}\Big\}. \quad (1)$$

Four quantities V_{CR}, V_{CI}, V_{SR}, and V_{SI}, which are all real and functions of the incident proton energy, are connected to the nucleon-nucleon scattering amplitude as follows:

$$V_{CR}+iV_{CI}=3(A/R^3)(\hbar^2/M\gamma)[(\gamma+1)/2]^{\frac{1}{2}}\bar{M}_0,$$
$$V_{SR}+iV_{SI}=-3(A/R^3)(1/M\gamma)\times(m_\pi c/k)^2(\gamma+1)\bar{M}_1, \quad (1.1)$$

where $\bar{M}_0$ and $\bar{M}_1$ are given by Eq. (6) of RW and other symbols are as in the list of notation. These quantities may also be expressed in terms of nucleon-nucleon scattering phase shifts as given by Eq. (13) of RW.[15] It is important to note that, as one can see from Eq. (1.1), the quantities V are proportional to R^{-3}, R being the radius of the target nucleus. The nucleon density $\rho_N(r)$ in (1) is normalized to the nuclear volume,[16]

$$\int\rho_N(r)d\mathbf{r}=(4\pi/3)R^3. \quad (1.2)$$

RW also show how to compute V_{CI} from experimental values of the total scattering cross sections σ_{np} and σ_{pp}. This value will be distinguished from the V_{CI} having its primary significance in terms of phase shifts, by the superscript σ. According to Eq. (14) of RW,

$$V_{CI}^{\sigma}=(\hbar v/2)\bar{\sigma}A(4\pi R^3/3)^{-1}. \quad (2)$$

Here v is the velocity of the incident proton, A is the mass number, and

$$\bar{\sigma}=\tfrac{1}{2}\gamma_G(\sigma_{np}+\sigma_{pp}), \quad (2.1)$$

while γ_G is the correction factor for binding effects.[17] Since γ_G is a function of the nuclear radius R, V_{CI}^{σ} is *not* proportional to R^{-3}.

In the first Born approximation, the proton-carbon scattering amplitude resulting from the matrix element (1) takes the form

$$M_N\equiv f_N+g_N\boldsymbol{\sigma}\cdot\mathbf{n}$$
$$=(2k/\hbar v)(V_{CR}+iV_{CI})(1/4\pi)F_N(q)(1+\lambda\boldsymbol{\sigma}\cdot\mathbf{n}), \quad (3)$$

where $\hbar k=|\hbar\mathbf{k}|=|\hbar\mathbf{k}'|$, $\hbar\mathbf{k}$ and $\hbar\mathbf{k}'$ being, respectively, the momentum of the incident and scattered proton, $\mathbf{n}=(\mathbf{k}\times\mathbf{k}')/|\mathbf{k}\times\mathbf{k}'|$, and $q=2k\sin(\Theta/2)$. The form factor $F_N(q)$ of the nucleon density $\rho_N(r)$ is defined as

$$F_N(q)=4\pi\int_0^\infty\rho_N(r)(\sin qr/qr)r^2dr, \quad (3.1)$$

while

$$\lambda(\Theta)=i(V_{SR}+iV_{SI})(V_{CR}+iV_{CI})^{-1}\times(\hbar/m_\pi c)^2(k^2\sin\Theta), \quad (3.2)$$

is independent of R.

On the other hand, for the Coulomb-scattering amplitudes, one obtains, in the same approximation,

$$M_C\equiv f_c+g_c\boldsymbol{\sigma}\cdot\mathbf{n}$$
$$=-(2k/\hbar v)(e/q^2)F_C(q)(1+\nu\boldsymbol{\sigma}\cdot\mathbf{n}), \quad (4)$$

where[13]

$$\nu(\Theta)=-i(E/Mc^2)(\mu-\tfrac{1}{2})\sin\Theta, \quad (4.1)$$

and $\mu(=2.793)$ is the magnetic moment of the proton in nuclear Bohr magnetons. The form factor $F_C(q)$ is defined in the same way as $F_N(q)$ [see Eq. (3.1)] using the charge (or proton) density $\rho_C(r)$ which is, in turn, normalized to the total charge of the target nucleus:

$$\int\rho_C(r)d\mathbf{r}=Ze, \quad (4.2)$$

where $Z=6$ for carbon.

If $\rho_C(r)$ is put equal to $\rho_N(r)$ except for its normalization,[18] the total scattering amplitude $M_T\equiv M_N+M_C$ takes the simple form

$$M_T\equiv(f_N+f_C)+(g_N+g_C)\boldsymbol{\sigma}\cdot\mathbf{n}$$
$$=f_N[(1+\alpha)+(\lambda+\alpha\nu)\boldsymbol{\sigma}\cdot\mathbf{n}], \quad (5)$$

[14] The target nucleus is treated as being infinitely heavy.

[15] The expression for V_{CI} in terms of phase shifts, which is not given in RW explicitly, may be obtained from that for V_{CR} by simply replacing each $\sin(2\delta)$ with $2\sin^2\delta$.

[16] If $\rho_N(r)$ were normalized to A, the mass number of the target nucleus, V_{CR}, etc., would be independent of R and A.

[17] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).

[18] The assumption $\rho_C(r)=\rho_N(r)$ is in good agreement with results of comparisons of neutron and proton distributions obtained in different ways as discussed at the Stanford Conference on Nuclear Sizes, December 1957 (unpublished).

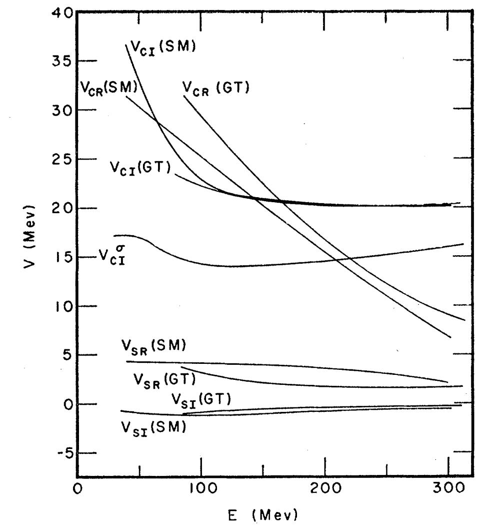

FIG. 1. Parameters in the transition matrix element, Eq. (1), calculated from nucleon-nucleon phase shifts of Signell and Marshak (SM) and of Gammel and Thaler (GT). The kinetic energy E of the incident proton is in Mev.

where

$$\alpha(\Theta)=f_C(\Theta)/f_N(\Theta)$$
$$=-(1/2k)(6e^2/\hbar v)(\sin\tfrac{1}{2}\Theta)^{-2}$$
$$\times(3\hbar v/2k)[R^3(V_{CR}+iV_{CI})]^{-1}. \quad (5.1)$$

Here, it must be emphasized that, since V_{CR} and V_{CI} are proportional to R^{-3}, α is independent of R. The polarization $P(\Theta)$ and the triple-scattering parameter $\beta(\Theta)$ may then be expressed in terms of α, λ, and ν,

$$P=\mathrm{Re}\left(\frac{2(1+\alpha^*)(\lambda+\alpha\nu)}{|1+\alpha|^2+|\lambda+\alpha\nu|^2}\right), \quad (6)$$

$$\sin\beta=-\mathrm{Im}\left(\frac{2(1+\alpha^*)(\lambda+\alpha\nu)(1-P^2)^{-\frac{1}{2}}}{|1+\alpha|^2+|\lambda+\alpha\nu|^2}\right), \quad (7)$$

where α^* is the complex conjugate of α. Equation (6) is equivalent to Eq. (38) of RW when α is put equal to zero. The quantity β is connected to the rotation parameter $R(\Theta)$ of Wolfenstein through the relation[7]

$$R(\Theta)=(1-P^2)^{\frac{1}{2}}\cos(\Theta-\beta). \quad (7.1)$$

From Eqs. (6) and (7), it is clear that in the approximation used here these quantities are independent of the assumed nucleon density distribution and the radius R of the target nucleus.[19,20]

IV. RESULTS AND DISCUSSION

Substituting nucleon-nucleon phase shifts into Eq. (13) of RW and making use of reference 15, one can obtain V_{CR}, V_{CI}, V_{SR}, and V_{SI} as a function of energy. This has been done for SM phase shifts at 40, 100, 150, and 300 Mev, and for GT phase shifts at 90, 156, and 310 Mev. Figure 1 shows these parameters together with V_{CI}^{σ} of Eq. (2) that has been computed from the experimental[21] σ_{np} and σ_{pp}. In Fig. 1, $R=3.06\times10^{-13}$ cm has been used as the radius of carbon[22] although

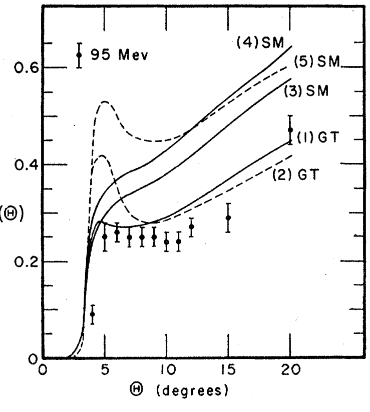

FIG. 2. Polarizations of protons elastically scattered from carbon. (1) GT at 90 Mev; (2) GT at 90 Mev with V_{CI}^{σ}; (3) SM at 90 Mev; (4) SM at 100 Mev; (5) SM at 100 Mev with V_{CI}^{σ}. Experimental points are those of Dickson and Salter[23] at 95 Mev. Errors are due to counting statistics only.

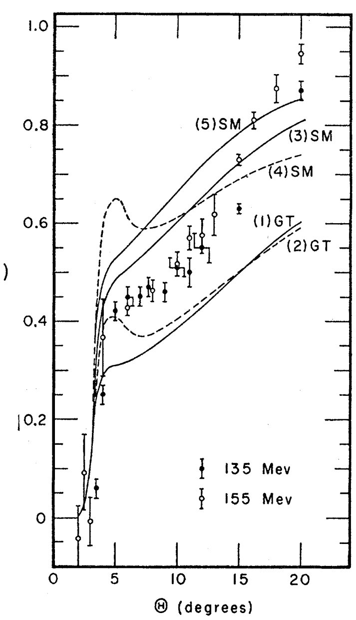

FIG. 3. Polarizations of protons elastically scattered from carbon. (1) GT at 156 Mev; (2) GT at 156 Mev with V_{CI}^{σ}; (3) SM at 135 Mev; (4) SM at 135 Mev with V_{CI}^{σ}; (5) SM at 150 Mev. Experimental points are those of Dickson and Salter[23] at 135 Mev and of Alphonse, Johansson, and Tibell[24] at 155 Mev. Errors are due to counting statistics only.

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[19] However, the fact that, in carbon, the number of neutrons is equal to that of protons and the nuclear spin is zero has been incorporated in this result.

[20] The fact that for $\rho_C(r)=\rho_N(r)$ the first-order result for the polarization is independent of $\rho(r)$ but that for $\rho_C(r)\neq\rho_N(r)$ this is not necessarily the case, was noted by G. Breit and J. S. McIntosh in connection with their article in the *Encyclopedia of Physics* [*Handbuch der Physik* (Springer-Verlag, Berlin, to be published)], Vol. 41.

[21] In Eq. (2.1), σ_{pp} should not include contributions due to the Coulomb interaction between two protons. Values for σ_{pp} used here are those given by W. N. Hess, University of California Radiation Laboratory Report UCRL-4639 (unpublished), where care has been taken not to include the Coulomb scattering. However, especially at lower energies ($E\lesssim100$ Mev), these values are rough estimates of pure "nuclear" scattering cross sections rather than well-defined quantities.

[22] R. Hofstadter, Revs. Modern Phys. 28, 214 (1956).

polarizations and β calculated from these V_{CR}, V_{CI}, V_{SR}, and V_{SI} are independent of this choice; P and β obtained from $V_{CI}{}^{\sigma}$ are, on the other hand, affected by the value of R. Polarizations calculated from them are compared with experimental data[23–26] in Figs. 2 to 5. In these measurements at 95, 135, 155, and 220 Mev,[23–25] inelastic contributions have been separated to get polarizations due to the elastic scattering only. However, no such separation has been done at 289 and 313 Mev.[26] Experimental uncertainties indicated in Figs. 2 to 5 take into consideration counting statistics only; additional uncertainties quoted in references 23 and 26 are $\pm 10\%$ at 95 Mev, $\pm 7\%$ at 135 Mev, $\pm 7.5\%$ at 289 Mev, and $\pm 4\%$ at 313 Mev. Interpolated values of V_{CR}, etc., from Fig. 1 have been used to estimate polarizations at 90 and 135 Mev for SM and at 220 and 250 Mev for GT. In Fig. 6 are plotted theoretical values

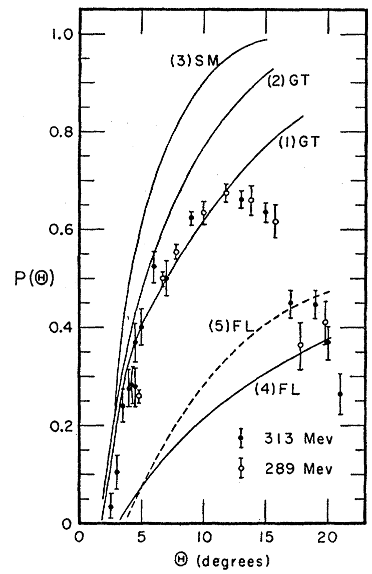

FIG. 5. Polarizations of protons elastically scattered from carbon. (1) GT at 250 Mev; (2) GT at 310 Mev; (3) SM at 300 Mev; (4) FL, set A, at 274 Mev; (5) FL, set A, at 274 Mev with $V_{CI}{}^{\sigma}$. Experimental points are those of Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis[26] at 289 and 313 Mev. Errors are due to counting statistics only.

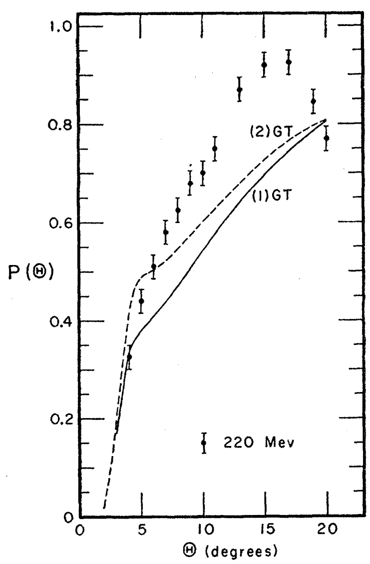

FIG. 4. Polarizations of protons elastically scattered from carbon. (1) GT at 220 Mev; (2) GT at 220 Mev with $V_{CI}{}^{\sigma}$. Experimental points are those of Chesnut, Hafner, and Roberts[25] at 220 Mev. Errors are due to counting statistics only.

of β in conjunction with experimental points[26] at $E \simeq 300$ Mev. Broken curves in Figs. 2 to 6 represent polarizations or β that have been evaluated using $V_{CI}{}^{\sigma}$; in Figs. 2 and 3 they seem to be in definite disagreement with trends of the experimental polarization.

In Fig. 1, one notices that quantities V_{CR}, etc. resulting from SM and GT phase shifts are quite similar. In particular, values of V_{CI} are almost identical for $E \gtrsim 100$ Mev. It is, therefore, difficult to discriminate between SM potentials and GT potentials on the basis of present work, both potentials being in semiquantitative agreement with the available experimental data. However, there is a marked difference between the SM and GT phase shifts and those of Feshbach and Lomon at higher energies ($E \gtrsim 200$ Mev). As one can see from Fig. 3 or Table II of RW, the phase shifts of Feshbach and Lomon give either negative (for set B) or positive but very small (for set A) V_{CR} whereas V_{CR} of SM and GT are always positive for $E \lesssim 300$ Mev and fairly large, i.e., $V_{CR} > 10$ Mev for $E < 260$ Mev.[27] Since, without Coulomb effects, P is roughly proportional to $(V_{CI}V_{SR} - V_{CR}V_{SI})/\sin\Theta$ and β to $-(V_{CR}V_{SR} + V_{CI}V_{SI})/\sin\Theta$, and since V_{CI} and V_{SR} are always positive whereas V_{SI} always negative, positive but small or negative V_{CR} generally yield small polarization and positive β in disagreement with experiments.[28] In Fig. 5, polarizations at 274 Mev computed from set A of FL are plotted for the sake of comparison.[29]

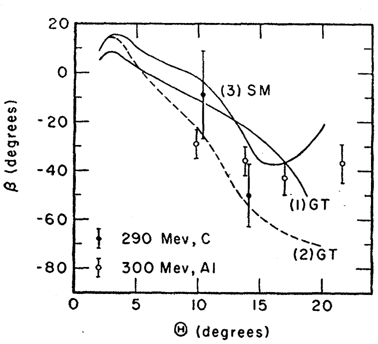

FIG. 6. Triple-scattering parameter β of protons elastically scattered from carbon. (1) GT at 310 Mev; (2) GT at 310 Mev with $V_{CI}{}^{\sigma}$; (3) SM at 300 Mev. Experimental points are those of Chamberlain *et al.*[26] at 290 Mev for carbon and at 300 Mev for aluminum.

[23] J. M. Dickson and D. C. Salter, Nuovo cimento **6**, 235 (1957).
[24] Alphonce, Johansson, and Tibell, Nuclear Phys. **3**, 185 (1957).
[25] Chesnut, Hafner, and Roberts, Phys. Rev. **104**, 449 (1956).
[26] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **102**, 1659 (1956).

[27] In RW, $R = 3.23 \times 10^{-13}$ cm has been used as the radius of carbon. Therefore, values of V_{CR} in Table II of RW must be multiplied by $(3.23/3.06)^3 = 1.18$ for this comparison. This, however, does not affect the subsequent argument.
[28] Set A of FL phase shifts gives, at 274 Mev, $\beta(5°) = 12.9°$, $\beta(10°) = 25.6°$, $\beta(15°) = 38.0°$, and $\beta(20°) = 49.8°$.
[29] It is clear from Fig. 5 that the agreement near $\Theta = 20°$ of the Feshbach-Lomon polarization with experimental points at $E \simeq 300$ Mev, which has been indicated in Fig. 4 of RW, is rather accidental and cannot be taken seriously.

TABLE I. Effects of the Coulomb interaction on the polarization.

Polarization[a]	GT, 156 Mev $\Theta=5°$	GT, 310 Mev $\Theta=5°$	SM, 90 Mev $\Theta=10°$	SM, 150 Mev $\Theta=10°$
$P_{\rm I}$	0.310	0.474	0.381	0.665
$P_{\rm II}$	0.270	0.467	0.370	0.657
$P_{\rm III}$	0.170	0.427	0.304	0.585

[a] $P_{\rm I}$ = polarization that includes all effects of the Coulomb interaction. $P_{\rm II}$ = polarization neglecting relativistic effects arising through the Coulomb field [$\nu=0$ in Eq. (6)]. $P_{\rm III}$ = polarization with no Coulomb effects [$\nu=0$, $\alpha=0$ in Eq. (6)].

In Tables I and II, effects of the Coulomb interaction on the polarization and β are shown for some cases. The importance of these effects is particularly clear at 90 Mev as is seen in Fig. 2. Also, as a result of the Coulomb-nuclear interference, β changes its sign at small angles.

Since the original formalism of Watson,[30] on which the transition matrix element (1) is based, contains the relative error of the order $1/A$, it is not clear whether a certain amount of disagreement of the calculated values of the polarization with experiments is truly significant. Besides, the target nucleus has been treated as though it were infinitely heavy, which would also introduce an additional error of the same order.

[30] K. M. Watson, Phys. Rev. 89, 575 (1953).

TABLE II. Effects of the Coulomb interaction on the triple-scattering parameter β. The symbols I, II, and III have the same meaning as in Table I.

	GT, 310 Mev $\Theta=5°$	GT, 310 Mev $\Theta=10°$	SM, 300 Mev $\Theta=3°$	SM, 300 Mev $\Theta=7°$
$\beta_{\rm I}$	8.7°	−10.7°	15.6°	5.2°
$\beta_{\rm II}$	9.3°	−10.1°	13.6°	4.6°
$\beta_{\rm III}$	−3.5°	−14.3°	−2.0°	−5.6°

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Proton-Proton Scattering in the Bev Region

G. E. BROWN*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

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Proton-proton scattering in the Bev region is analyzed in terms of an interaction which, at 1 Bev, is taken to be a hard core of radius 0.45×10^{-13} cm together with an external absorption of Gaussian form. The hard core is assumed to disappear with increasing energy and to be replaced by absorption. General features of the data are well reproduced by this simple model.

1. INTRODUCTION

In the past few years, proton-proton scattering experiments have been carried out up to an energy of 6 Bev. Whereas at energies in the range 0–300 Mev the differential cross section has been analyzed quite generally in terms of phase shifts, at energies in the Bev region, this can be done only with simplifying assumptions[1] since the number of experimental data are insufficient to determine the necessarily large number of phase shifts. One way of seeing just what such assumptions imply is to choose a simple model, such as an interaction of some definite radial dependence. One of the simplest assumptions is that the bombarded proton is equivalent to an absorbing sphere, with inverse mean free path for absorption, K, constant throughout the sphere. This means that only two parameters, K and the radius of the sphere R, have to be determined. The experimental cross sections in the range 0.8–2.75 Bev have been fitted in this way.[2] It is easy to see how K must behave. At 1 Bev the ratio of elastic to inelastic scattering is almost unity, and so K must be large enough so that the sphere is essentially black. In order to describe the decrease in the elastic cross section with energy (see Fig. 1) and the increase in the ratio of inelastic to elastic scattering, K must decrease with energy.

The absorption described here results from meson production. In terms of a picture in which the proton-meson interaction is strong, it is hard to see why the absorption should decrease as more energy becomes available for meson production. Further, the analysis of Rarita[1] at 1 Bev indicated that such a simple optical model description was inadequate for describing the angular distribution in detail, because it predicted more absorption in the s wave than in the d wave, whereas his phase-shift analysis required the opposite condition.

A reasonable way to supplement the above picture

* On leave from the Department of Mathematical Physics, University of Birmingham, Birmingham, England.
[1] W. Rarita, Phys. Rev. 104, 221 (1956).
[2] W. B. Fowler et al., Phys. Rev. 103, 1489 (1956).