

## Proton-Gamma Ray Angular Correlations in the $\text{Si}^{28}(d,p\gamma)\text{Si}^{29}$ Reaction\*

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The  $\text{Si}^{28}(d,p\gamma)\text{Si}^{29}$  reaction has been studied by measuring angular distributions of the protons leaving  $\text{Si}^{29}$  in its first and second excited states and by measuring the correlation between the protons of each group and the gamma rays emitted from the corresponding level. The angular distributions of proton groups  $p_1$  and  $p_2$  exhibit large peaks in the backward direction, indicating that heavy-particle stripping may be important. The  $(p_1,\gamma)$  correlations were found to be isotropic to  $\pm 6\%$ . This is substantially different from the correlation which has been observed at a higher deuteron energy. The  $(p_2,\gamma)$  correlations are consistent with a  $\frac{5}{2}+$  level in  $\text{Si}^{29}$  at 2.03 Mev. One  $(p_2,\gamma)$  correlation shows a 20% anisotropy in the plane perpendicular to the direction taken by the recoil nucleus, while another  $(p_2,\gamma)$  correlation is essentially isotropic.

### INTRODUCTION

THE Born approximation treatment of the proton-gamma ray angular correlations in  $(d,p\gamma)$  stripping reactions neglects various interactions which tend to produce anisotropies in the correlations in the plane perpendicular to the direction taken by the recoiling nucleus.<sup>1</sup> Because of this, further study of the  $p-\gamma$  correlations was expected to throw some light upon the mechanism of the stripping process.

In the work described below, the directions of emission of the protons ( $p_1$  and  $p_2$ ) leaving  $\text{Si}^{29}$  in its first and second excited levels are correlated with the directions of emissions of the 1.28-Mev and 2.03-Mev gamma rays produced by the decay of the  $\text{Si}^{29}$  nucleus to its ground level. The  $\text{Si}^{28}(d,p\gamma)\text{Si}^{29}$  reaction was chosen for this study because previous studies elsewhere make it possible to predict the form of the angular correlations between the protons and their associated gamma rays. Some of the available information which was made use of was as follows:

(1) The ground-state spin of  $\text{Si}^{28}$  is zero, so there is no channel spin ambiguity.

(2) The momentum transfer in the  $d-p$  process is known to be  $l_n=2$  from measurements of the angular distributions of both  $p_1$  and  $p_2$  produced by bombardment with 9-Mev deuterons.<sup>2</sup>

(3) The above information indicates that the first two energy levels in  $\text{Si}^{29}$  have spins and parities of either  $\frac{3}{2}+$  or  $\frac{5}{2}+$ . The ambiguity concerning the 1.28-Mev level is removed by consideration of the  $\beta$  decay of  $\text{A}^{29}$  and of  $\text{P}^{29}$ , which indicate that this level is  $\frac{3}{2}+$ .<sup>3</sup> Similarly,  $p-\gamma$  correlations in the process  $\text{Si}^{29}(p,p'\gamma)\text{Si}^{29}$

limit the choice to  $\frac{5}{2}+$  in the case of the 2.03-Mev level in  $\text{Si}^{29}$ .<sup>4</sup>

(4) The lower excited levels of  $\text{Si}^{29}$  are spaced far enough apart so that the corresponding groups of protons can be resolved when using a NaI(Tl) detector.

(5) The theoretical angular correlation<sup>5</sup> between the protons and gamma rays associated with the 1.28-Mev level is  $W(\theta,\phi)=1+A\cos^2\theta$ , where  $-0.6\leq A\leq 1$ , and the angles  $\theta$  and  $\phi$  are defined in Fig. 1. The uncertainty in the coefficient  $A$  depends upon the possible amount of mixture of  $E2$  and  $M1$  radiation in the gamma-ray transition from the  $\frac{3}{2}+$  excited level to the  $\frac{1}{2}+$  ground level. Allen *et al.*,<sup>6</sup> using 9-Mev deuterons, found that  $A=-0.177\pm 0.04$  for  $\phi=0^\circ$ , with an anisotropy in the correlation which was a function of  $\phi$ . This value of  $A$  can be fitted by setting the ratio of  $E2$  to  $M1$  radiation widths to be 0.04.

(6) The gamma ray emitted in the transition from the  $\frac{5}{2}+$  excited level to the  $\frac{1}{2}+$  ground level will, in general, be a mixture of  $E2$  and  $M3$  radiation, but the radiation width for the  $M3$  radiation should be several orders of magnitude smaller than for the  $E2$  radiation.<sup>7</sup> If the contribution from the  $M3$  radiation is neglected, the predicted angular correlation<sup>5</sup> is  $W(\theta,\phi)=1+6\cos^2\theta-5\cos^4\theta$ .

From a comparison of our observations with these predictions, we hoped to obtain other information about the need for modifications in the simple deuteron stripping theory.

### APPARATUS

The silicon target, proton detector, and gamma-ray detector were contained in a nearly spherical, evacuated chamber, 46 cm in diameter. The deuteron beam from the Minnesota electrostatic generator, after being deflected by a  $90^\circ$  magnet and refocused by a pair of strong-focusing electrostatic lenses, passed through the

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<sup>1</sup> A good discussion of the present situation in correlation studies is given by S. A. Cox and R. M. Williamson, *Phys. Rev.* **105**, 1799 (1957).

<sup>2</sup> J. R. Holt and T. N. Marsham, *Proc. Phys. Soc. (London)* **A66**, 467 (1953).

<sup>3</sup> P. M. Endt and J. C. Kluyver, *Revs. Modern Phys.* **26**, 123 (1954).

<sup>4</sup> Bromley, Gove, Litherland, Paul, and Almqvist, *Bull. Am. Phys. Soc. Ser. II*, **1**, 30 (1956).

<sup>5</sup> L. C. Biedenharn and M. E. Rose, *Revs. Modern Phys.* **25**, 729 (1953).

<sup>6</sup> Allen, Collinge, Hird, Maglič, and Orman, *Proc. Phys. Soc. (London)* **A69**, 705 (1956).

<sup>7</sup> J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

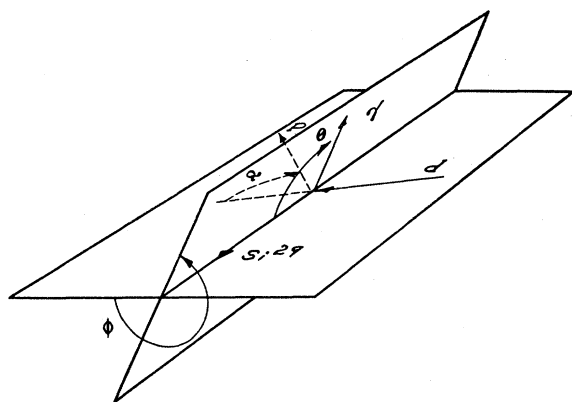


FIG. 1. Diagram defining the angles  $\alpha$ ,  $\theta$ , and  $\phi$  used in angular correlations.

sphere along a diameter and was caught for current measurement in a tantalum cup 52 cm from the center of the sphere. The proton detector [a NaI(Tl) crystal less than one millimeter thick] was mounted on the end of an RCA 6199 photomultiplier tube. The solid angle subtended at the target by this crystal varied from 0.018 steradian to 0.060 steradian during different parts of the work. This detector could be rotated about a vertical axis passing through the target at the center of the sphere. The gamma-ray detector was a 1-inch  $\times$  1-inch cylindrical NaI(Tl) crystal having one end against an RCA 6199 photomultiplier tube with the opposite end 6.25 cm from the center of the chamber. It could be rotated about the target with two degrees of freedom. The proton detection crystal was covered by an aluminum foil just thick enough to stop deuterons scattered from the target. The gamma-ray crystal was covered with lead 3 mm thick, which reduced the positron annihilation radiation due to nuclei produced in ( $d, n$ ) reactions. This arrangement of placing the detectors inside the vacuum chamber with the target eliminated any variation of absorber thickness with detector angle. The thin walls (3 mm thickness) of the sphere and the use of aluminum for most of the structure reduced the mass of material near the target and detectors which might have scattered gamma rays into the detector.

During the measurement of the angular distribution of the protons, the pulses from the proton detector were amplified and counted with a 10-channel pulse-height analyzer. For the  $p\text{-}\gamma$  angular correlation work, the pulses from the two detectors, after amplification, were fed into two single-channel pulse-height selectors having adjustable channel widths. The outputs of these circuits went into a coincidence circuit having a resolving time of 0.1 microsecond. Simultaneously the number of accidental coincidences was determined by another coincidence circuit having a one microsecond delay in one channel.

The number of deuterons passing through the target

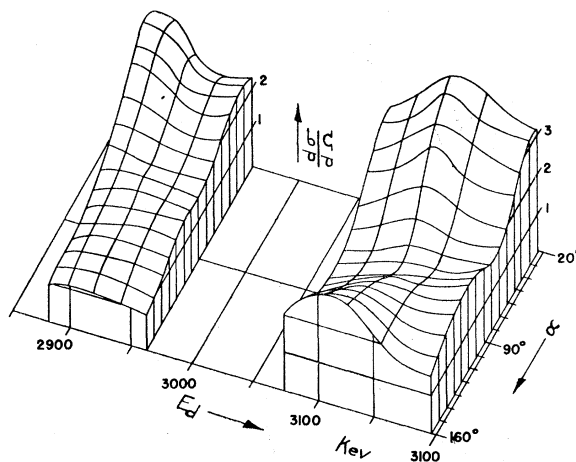


FIG. 2. Angular distribution of protons from Si<sup>28</sup>( $d, p$ )Si<sup>29\*</sup>, leaving Si<sup>29</sup> in its first excited state, as a function of deuteron energy. Cross sections in millibarns per steradian.

and entering the insulated current collection cup was measured by a current integration circuit with an error of less than one percent. The energy of the deuterons was known to  $\pm 0.1\%$  from the calibration of the flux meter used with the 90° beam deflection magnet.

#### TARGETS

The targets were prepared by evaporating normal silicon onto nickel foils  $5 \times 10^{-6}$  inch thick.<sup>8</sup> The silicon used had been purified for making transistors and contained impurities no more than one part in  $10^5$ .

Deuteron bombardment of clean nickel foils showed that no radiation was produced which would be mistaken for the radiation from silicon. During the use of the targets a thin carbon deposit developed on their surfaces, but the protons produced by ( $d, p$ ) reactions in carbon were of lower energy than the ones being studied from silicon, so no difficulty was experienced in separating their pulses with the pulse-height selector circuits.

Target thicknesses were measured by observing the difference in accelerator potential needed to reach the Li<sup>7</sup>( $p, n$ ) threshold with and without the target in the beam. The layer of silicon on the target which was used to obtain the proton angular distribution data produced an energy loss of  $42 \pm 15$  keV in a beam of 3-MeV deuterons. The target used for the angular correlation measurements was  $39 \pm 9$  keV thick for 3-MeV deuterons.

#### RESULTS

The measurements of the proton angular distributions are tabulated in Tables I and II. Figures 2 and 3 present the same data in graphical form.

The relative errors in these differential cross sections are estimated to be  $\pm 5\%$ . In addition, it is estimated that there may be an absolute error of as much as

<sup>8</sup> H. A. Hill, Rev. Sci. Instr. 27, 1086 (1956).

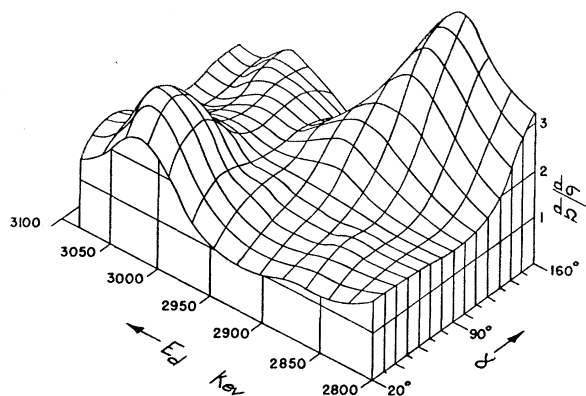


FIG. 3. Angular distribution of protons from  $\text{Si}^{28}(d,p)\text{Si}^{29*}$ , leaving  $\text{Si}^{29}$  in its second excited state, as a function of deuteron energy. Cross sections in millibarns per steradian.

$\pm 30\%$ , largely due to the uncertainty in the number of  $\text{Si}^{28}$  atoms in the target.

The uncertainties in the energy of deuterons incident upon the target are 4 or 5 keV, as given in the column headings in Tables I and II, with the energies at the center of the silicon layer reduced from the tabulated values by  $23 \pm 8$  keV.

The angular correlations of the 1.28-Mev gamma rays with the protons leaving  $\text{Si}^{29}$  in its first excited state are presented in Figs. 4 and 5. Figures 6 and 7 represent the angular correlations between the 2.03-Mev gamma ray and the protons leaving the  $\text{Si}^{29}$  nuclei in the second excited state. The caption of each figure gives the

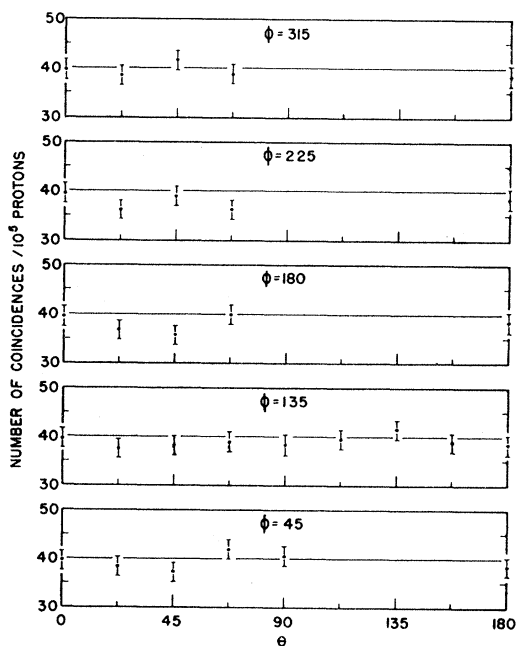


FIG. 4.  $p$ - $\gamma$  angular correlations for first excited state proton group from  $\text{Si}^{28}(d,p\gamma)\text{Si}^{29}$ ; proton angle  $\alpha = 35^\circ$ , deuteron energy  $= 2935 \pm 10$  keV.

angle with respect to the incident deuteron beam at which the proton is observed,  $\alpha$ , and the energy of the deuterons at the surface of the target. At the center of the silicon layer the deuteron energy was reduced from the tabulated values by  $22 \pm 5$  keV.

The missing points in the angular correlation data were not taken because of mechanical interference between the detectors and the beam defining system or the beam current collection system.

The errors shown for the individual points in Figs. 4, 5, 6, and 7 were obtained from consideration of the following factors: (a) Fluctuations in the intensity of the deuteron beam produced a variable loss of counts due to the "dead time" of the pulse-height selector circuits. (b) Variations in the amplifier, pulse-height analyzer, and coincidence circuit changed the efficiency with which proton-gamma ray coincidences were detected. (c) The most intense portion of the deuteron beam did not always remain in the center of the bombarded spot. This produced slight variations in the effective solid angles subtended by the detector crystals at the target. (d) The correction for accidental coincidences was uncertain due to variations in the circuits detecting true and accidental coincidences. These factors together were estimated to contribute an uncertainty of  $\pm 3\%$  to each point. The statistical fluctuations due to the limited number of coincidences counted varied from point to point and contributed the larger portion of the uncertainties shown in the figures.

The data shown in Figs. 4 and 5 have an additional uncertainty due to the fact that the proton pulses due to the reaction  $\text{Si}^{29}(d,p_2)\text{Si}^{30}$  could not be separated from the proton pulses of interest due to the  $\text{Si}^{28}(d,p_1)\text{Si}^{29}$  reaction. However, an estimate of the number of  $p$ - $\gamma$  coincidences due to the  $p_2$  group from  $\text{Si}^{29}$  could be made, since the coincident gamma ray had an energy of 3.15 Mev, whereas the gamma ray in coincidence with the  $p_1$  group from  $\text{Si}^{28}$  had an energy of 1.28 Mev. By using gamma-ray detection crystals on both photomultiplier tubes and a  $\text{Co}^{60}$  gamma-ray source at the target position, the absolute efficiency of the detectors for gamma rays of known energy was measured. With

TABLE I. Differential cross section for proton group  $p_1$  in  $\text{Si}^{28}(d,p\gamma)\text{Si}^{29}$  as a function of proton angle and deuteron energy. Deuteron energy in keV, cross sections in millibarns per steradian.

$\alpha \backslash E_d$	2882 $\pm 5$	2898 $\pm 4$	2919 $\pm 4$	2941 $\pm 4$	2963 $\pm 4$	3077 $\pm 4$	3100 $\pm 4$	3129 $\pm 4$	3157 $\pm 4$	3197 $\pm 4$
20°	3.25	3.23	2.48	2.12	2.34	2.65	3.14	3.91	3.76	3.20
30°	3.46	3.30	2.85	2.41	2.41	2.59	2.90	3.85	3.40	3.22
40°	3.11	3.06	2.48	2.36	2.55	2.18	2.72	3.36	3.16	2.70
50°	2.96	2.92	2.56	2.21	2.37	1.95	2.24	3.11	2.58	2.49
60°	2.44	2.41	2.13	1.92	2.08	1.40	1.88	2.40	2.13	1.94
70°	2.07	2.11	1.93	1.82	1.83	1.13	1.48	1.93	1.75	1.83
80°	1.84	1.81	1.65	1.64	1.67	0.91	1.17	1.54	1.59	1.68
90°	1.55	1.72	1.74	1.68	1.58	0.96	1.19	1.45	1.51	1.98
100°	1.36	1.51	1.45	1.63	1.60	0.89	1.17	1.46	1.63	2.01
110°	1.21	1.48	1.53	1.60	1.63	1.03	1.34	1.78	1.78	2.18
120°	1.20	1.36	1.45	1.60	1.55	0.99	1.59	1.95	1.96	2.11
130°	1.14	1.37	1.45	1.45	1.38	1.17	1.74	2.16	1.98	2.07
140°	1.10	1.31	1.28	1.38	1.24	1.21	2.16	2.38	2.00	1.93
150°	1.07	1.20	1.28	1.15	1.02	1.74	2.43	2.52	2.02	1.66
160°	0.96	1.08	1.05	0.98	0.89	2.07	2.75	2.74	1.99	1.49

TABLE II. Differential cross section for proton group  $p_2$  in Si<sup>28</sup>(d, p $\gamma$ )Si<sup>29</sup> as a function of proton angle and deuteron energy. Deuteron energies in kev, cross sections in millibarns per steradian.

$\alpha \backslash E_d$	2799 $\pm 5$	2840 $\pm 5$	2861 $\pm 5$	2882 $\pm 5$	2898 $\pm 4$	2919 $\pm 4$	2941 $\pm 4$	2963 $\pm 4$	2986 $\pm 4$	3008 $\pm 4$	3031 $\pm 4$	3054 $\pm 4$	3077 $\pm 4$
20°	1.76	1.22	1.25	1.14	0.99	0.87	1.11	1.41	2.11	2.47	2.39	1.94	1.45
30°	1.63	1.36	1.23	1.27	1.20	1.13	1.31	1.61	2.42	3.01	2.81	2.01	1.81
40°	1.70	1.52	1.65	1.29	1.27	1.25	1.53	1.92	2.75	3.05	2.96	2.11	1.85
50°	1.63	1.54	1.59	1.32	1.34	1.48	1.63	1.91	2.69	3.04	2.87	2.07	1.91
60°	1.41	1.61	1.69	1.45	1.45	1.39	1.69	1.83	2.49	2.72	2.50	1.92	1.69
70°	1.58	1.44	1.52	1.44	1.52	1.60	1.66	1.72	2.18	2.28	2.25	1.77	1.65
80°	1.49	1.45	1.59	1.61	1.66	1.62	1.63	1.66	1.95	1.92	1.86	1.58	1.50
90°	1.53	1.48	1.52	1.77	2.04	1.87	1.78	1.60	1.76	1.83	1.73	1.45	1.48
100°	1.48	1.61	2.02	2.09	2.29	2.22	1.92	1.62	1.79	1.74	1.55	1.25	1.30
110°	1.59	1.85	2.18	2.40	2.69	2.40	1.99	1.67	1.71	1.69	1.46	1.24	1.42
120°	1.63	2.21	2.83	3.07	2.87	2.60	2.08	1.67	1.81	1.74	1.57	1.16	1.21
130°	1.96	2.67	2.91	3.28	3.09	2.61	1.93	1.52	1.69	1.79	1.65	1.17	1.41
140°	2.34	3.15	3.83	3.85	3.18	2.36	1.81	1.36	1.72	1.72	1.80	1.20	1.34
150°	2.75	3.80	4.10	4.00	3.22	2.36	1.69	1.20	1.52	1.66	1.72	1.29	1.47
160°	3.20	3.77	4.36	4.14	3.17	2.37	1.65	1.24	1.30	1.57	1.72	1.29	1.32

the pulse analyzer adjusted to favor the detection of the 1.28-Mev gamma rays, the efficiency of detection of the 3.51-Mev gamma rays was less by a factor of 2.6. A comparison of the observed  $p$ - $\gamma$  coincidence rate from the silicon target (see Figs. 4 and 5) with that calculated on the basis of the measured detector efficiency showed that the ratio of the differential cross section of the Si<sup>29</sup>(d, p $\gamma$ )Si<sup>30</sup> reaction to that of the Si<sup>28</sup>(d, p $\gamma$ )Si<sup>29</sup> reaction was  $1.5 \pm 1.7$  and  $1.0 \pm 1.7$  for the conditions of observation represented by the data in Figs. 4 and 5 respectively. Considering the natural isotopic abundance ratio in silicon, one can then estimate that the Si<sup>29</sup>(d, p $\gamma$ )Si<sup>30</sup> reaction contributed  $2.8\% \pm 3.2\%$  and  $1.9\% \pm 3.2\%$  of the  $p$ - $\gamma$  coincidences in the data of Figs. 4 and 5.

DISCUSSION OF RESULTS

The angular distributions of the protons do not follow the predictions of Butler's theory as they do for

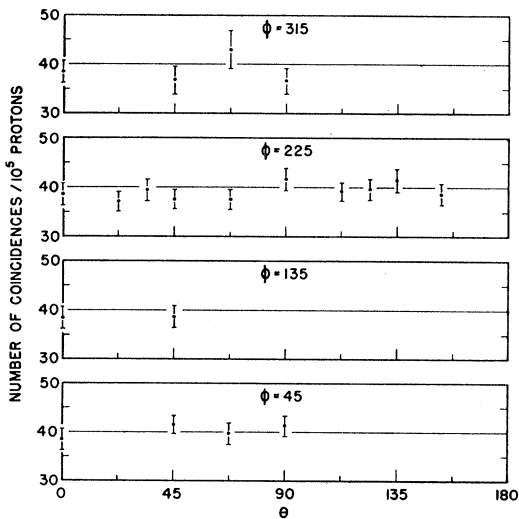


FIG. 5.  $p$ - $\gamma$  angular correlations for first excited state proton group from Si<sup>28</sup>(d, p $\gamma$ )Si<sup>29</sup>; proton angle  $\alpha = 135^\circ$ , deuteron energy =  $2935 \pm 10$  kev.

deuterons of 9 Mev.<sup>6</sup> Instead the peaks in the forward angles are very broad; so much so that the value of the angular momentum,  $l_n$ , of the captured neutron cannot be determined from the Butler theory. In addition, peaks appear at backward angles, and the amplitudes

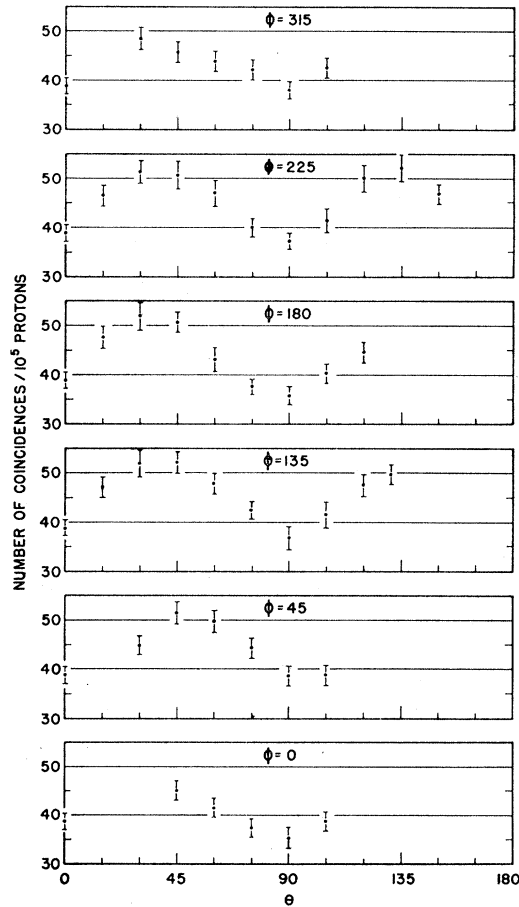


FIG. 6.  $p$ - $\gamma$  angular correlations for second excited state proton group from Si<sup>28</sup>(d, p $\gamma$ )Si<sup>29</sup>; proton angle  $135^\circ$ , deuteron energy =  $2892 \pm 10$  kev.

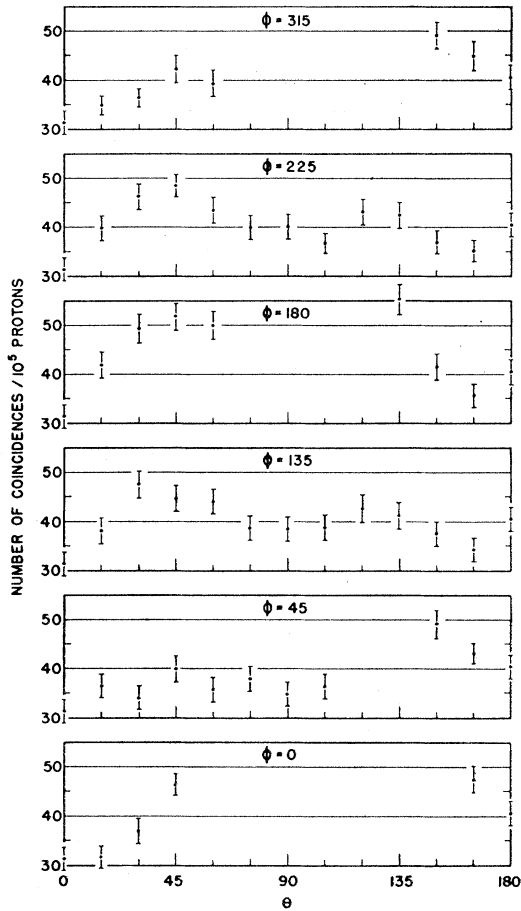


FIG. 7.  $p$ - $\gamma$  angular correlations for second excited state proton group from  $\text{Si}^{28}(d, p_1\gamma)\text{Si}^{29}$ ; proton angle  $40^\circ$ , deuteron energy  $= 3024 \pm 10$  kev.

of both the forward and backward peaks are strong functions of deuteron energy. The presence of these backward peaks may indicate the occurrence of heavy particle stripping, which appears to be a strong function of deuteron energy.

The angular correlations observed between the protons and gamma rays from the  $\text{Si}^{28}(d, p_1\gamma)\text{Si}^{29}$  reaction have an insignificant departure from isotropy. On the assumption that any such departure should be of the form  $1 + A \cos^2\theta$ , since this is the predicted form of the angular correlation, values of  $A$  were calculated from the data of Figs. 4 and 5. Table III lists these results.

As explained above, these data include a small effect due to the protons and gamma rays from  $\text{Si}^{29}(d, p_2\gamma)\text{Si}^{30}$ . Since the fraction of the observed coincidences due to this reaction was shown above to be  $(2.8 \pm 3.2)\%$  for Fig. 4 and  $(1.9 \pm 3.2)\%$  for Fig. 5, the corrections to the anisotropy in the angular correlations should be no greater than these amounts. To fit these nearly isotropic angular correlations, the ratio of the radiation widths  $E2/M1$  has to be approximately 1. These results are in marked contrast to those of Allen *et al.*,<sup>6</sup> mentioned above, in which  $A = -0.177 \pm 0.04$  and

TABLE III. Values of the coefficient  $A$  required to fit the  $\text{Si}^{28}(d, p_1\gamma)\text{Si}^{29}$  angular correlations to the equation  $1 + A \cos^2\theta$ .

$\phi$	Data from Fig. 4	Data from Fig. 5
	$\alpha = 35^\circ, E_d = 2935$ kev $A$	$\alpha = 135^\circ, E_d = 2935$ kev $A$
$45^\circ$	$-0.06 \pm 0.06$	$-0.05 \pm 0.07$
$135^\circ$	$-0.01 \pm 0.03$	
$180^\circ$	$-0.04 \pm 0.08$	
$225^\circ$	$-0.06 \pm 0.07$	$-0.05 \pm 0.05$
$315^\circ$	$-0.02 \pm 0.05$	

$E2/M1 = 0.04$ . In the account of their work the effect of the radiation from the  $\text{Si}^{29}$  in their target was not discussed, and the effect of this radiation was not necessarily as small in their work as in the present experiment because of the great difference in the deuteron energies in the two cases.

If the apparent decrease in the anisotropy of this angular correlation, due to the reduction of the deuteron energy from 9 Mev to 2935 kev, is real, it is consistent with modifications of stripping theory<sup>9-11</sup> which predict that interactions neglected in the simple theory will decrease the anisotropy in the  $\phi = 0^\circ$  plane for lower deuteron energies.

The angular correlation data shown in Fig. 6 for the  $\text{Si}^{28}(d, p_2\gamma)\text{Si}^{29}$  reaction, where the protons were observed in the backward direction,  $\alpha = 135^\circ$ , possess the symmetry about  $\theta = 90^\circ$  which would be predicted for deuteron stripping. This is in spite of the fact that the proton angular distribution data for this deuteron energy, shown in Fig. 3, exhibit a large peak in the backward direction so that one might expect that heavy particle stripping would be more important here than deuteron stripping. These data are also almost isotropic with respect to  $\phi$ .

The theoretical angular correlation for the  $\text{Si}^{28}(d, p_2\gamma)\text{Si}^{29}$  reaction, if the level emitting the gamma ray is a  $\frac{5}{2}^+$  level, is  $W(\theta, 0^\circ) = 1 + 5.12 \cos^2\theta - 4.18 \cos^4\theta$ , when corrected for the finite resolution of the detectors. A least squares fit to a typical portion of the data of Fig. 6 gives  $W(\theta, 225^\circ) = 1 + (1.63 \pm 0.18) \cos^2\theta - (1.54 \pm 0.18) \cos^4\theta$ . If the spin and parity of the 2.03-Mev level in  $\text{Si}^{29}$  were  $\frac{3}{2}^+$ , one would expect a correlation of the form  $1 + A \cos^2\theta$ , where  $-0.6 \leq A \leq 1$ . The need for the  $\cos^4\theta$  term in the equation describing these data is consistent with this level having spin and parity of  $\frac{5}{2}^+$ , although the data do not reach low enough values at  $\theta = 90^\circ$  to agree with the predicted correlation.

The data presented in Fig. 7 were taken at a deuteron energy and proton angle where one might expect to find predominantly deuteron stripping, as indicated by the peak in the proton distribution at  $\alpha = 40^\circ$  shown in Fig. 3. However, in this case the axis of symmetry is shifted about  $10^\circ$  from the expected  $\theta = 90^\circ$ , and there is an isotropy of about 20% in  $\phi$ . It is not clear whether

<sup>9</sup> H. C. Newns, Proc. Phys. Soc. (London) **A66**, 477 (1953).

<sup>10</sup> J. Horowitz and A. M. L. Messiah, Phys. Rev. **92**, 1326 (1953).

<sup>11</sup> J. Horowitz and A. M. L. Messiah, J. phys. radium **15**, 142 (1954).

these effects are due to the influence of heavy particle stripping or are due to the interactions which are neglected in the Butler theory analysis.

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## Proton Potential Anomaly and Nonlocal Potentials

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A detailed study of the variations of the difference between the proton-nuclear potential and the neutron-nuclear potential is carried out using ground-state mass data. A previous analysis of mirror nuclei has delimited the problem and suggested that the well difference can be neither constant nor strictly  $Z$  dependent. It is found by using analytical techniques as well as calculations performed with the Oak Ridge Oracle that the potential anomaly varies with the symmetry parameter  $(N-Z)/A$  and that the proportionality constant does not change very greatly as one proceeds from a static well to a velocity-dependent well. This is somewhat surprising in view of many studies which suggest that the nuclear symmetry energy is considerably influenced by the velocity dependence of the nuclear potential. The relative insensitivity to velocity dependence or nonlocality is attributed to

surface effects not taken into account in the analyses of infinite nuclear matter.

A number of possible origins of the well differences are examined including (a) the failure of Koopman's theorem, (b) the breakdown of electrostatic laws, (c) the presence of Heisenberg forces, (d) the effect of the exclusion principle, and (e) the spin dependence of nuclear forces. It is concluded that the last two effects account for the bulk of the proton potential anomaly. Indeed, these last two effects suggest that the well depths used in neutron scattering and in proton scattering vary individually with the symmetry parameter, and it is suggested that experimentalists attempt to seek out these variations in careful, low-energy scattering experiments.

### I. INTRODUCTION

A LARGE number of papers<sup>1-15</sup> have appeared recently which bear upon the difference between the proton-nuclear potential and the neutron-nuclear potential, i.e., the proton potential anomaly. Yet there remains considerable confusion as to the magnitude and

origin of this effect. This work is an effort directed toward quantitatively characterizing the anomaly for the case of static and nonlocal or velocity-dependent potentials and to assign the origin of the effect. Bound-state data form the basis of these estimates. Calculations based upon approximate analytical techniques and calculations performed with the help of the Oak Ridge Oracle are utilized in this study.

This work follows upon a series of previous articles<sup>3-8</sup> and for brevity's sake the notation and previous results will be utilized extensively. This earlier work suggested that the proton potential anomaly is attractive and approximately equal to one-half the magnitude of the Coulomb potential seen by a proton and as such might be associated with a direct distortion of the Coulomb energy term "inside the nucleus." However, a detailed study of mirror nuclei<sup>8</sup> has indicated that no appreciable anomaly is needed for the last neutron and the last proton binding energy in mirror pairs, thereby suggesting that the anomaly is not associated with a direct modification of the Coulomb term. This study also indicated conclusively that the characterization of the proton well as having a fixed depth of about 55 Mev for all nuclei, as contrasted with a fixed neutron depth

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