The magnetic moment μ_{SL} arising from spin-orbit forces will contribute to the hfs anomaly,[5] namely that part of the hyperfine separation beyond the amount calculated on the basis of a point nucleus, using the experimental magnetic moment of the deuteron. Since μ_{SL} arises from proton motion in the presence of a magnetic field, the moment is "orbital" in nature[5,6]; that is, the electron moving rapidly inside a radius R will be able to follow the proton motion with a consequent relative change in the hfs,

$$\Delta_{SL} = -\frac{\mu_{SL}}{\mu_d}\left(\frac{2R}{a_0}\right), \tag{1}$$

where μ_d is the magnetic moment of the deuteron, and a_0 is the Bohr radius of hydrogen. An approximate formula for R is[7]

$$R = k\frac{mc^2}{|W_0|}\alpha a_0, \tag{2}$$

where mc^2 is the rest energy of the electron, W_0 is the binding energy of the deuteron, and α is the fine structure constant. For magnetic moments which are distributed over a distance of the size of the deuteron, $k=1.9$,[7] but for the short-range spin-orbit force, explicit evaluation indicates that k is close to unity. Thus one obtains with $k=1$, for the deuteron,

$$\Delta_{SL}/\mu_{SL} = -0.0039 \text{ (nm)}^{-1}. \tag{3}$$

The comparison between theory and experiment for the hfs of the deuteron has recently been reviewed.[8,9] When nucleon size effects are included,[9] one finds

$$\frac{\Delta\nu_{\text{D}}}{\Delta\nu_{\text{H}}} = \frac{3}{4}\frac{M_{\text{D}}}{M_{\text{H}}}\frac{\mu_d}{\mu_p}(1-\Delta), \tag{4}$$

where $\Delta_{\text{exp}} = 170.3 \pm 0.5$ ppm, and $\Delta_{\text{theor}} = 210 \pm 50$ ppm. The theoretical value for Δ [Eq. (4)] does not include relativistic and mesonic effects; these have been studied most recently by Sugawara,[10] who estimates on the basis of field theory that the effects are of the order of one to two percent of the deuteron magnetic moment. The uncertainty in Δ does not include this possibility, but simply refers to computational uncertainties in the terms included.[9] The noncovariant result of Greifinger[8] may also not contain all the important terms of a fully covariant treatment.

As a typical example, the Gammel-Thaler potential[2,4] yields $\mu_{SL} = -0.036$ nm and $\Delta_{SL} = 140$ ppm. While one cannot exclude the possibility of interaction moments which would compensate this large term, this appears unlikely. It should be noted that even if the interaction moments and/or the percentage of D state are adjusted to compensate μ_{SL} and give the correct deuteron magnetic moment, it is *still unlikely* that Δ_{SL} will also be compensated. This is because the spin-orbit moment makes its contribution as an "orbital" term and hence contributes more than the usual "Bohr" term.[6] The latter, which comes from distributed magnetism of moment μ and average radius d, contributes a relative correction to the hfs,

$$\Delta_B = -(\mu/\mu_d)(2d/a_0). \tag{5}$$

For most interaction moments one would expect a "Bohr" term with d rather less than the deuteron radius, so that for the same magnetic moment Δ_{SL} will be approximately 20 times as large as Δ_B. The contribution of the D state of the deuteron, although an "orbital" effect, gives an anomalously small contribution[7] so that for the same magnetic moment, Δ_{SL} will be approximately 7 times as large as the D-state contribution.

It is of course possible that there are no spin-orbit forces present in the ground state of the deuteron.[11] In any case it is clear that the hfs of deuterium is an experimental datum distinct from the magnetic moment of the deuteron, and the requirement that both of these numbers be predicted correctly will be useful in determining the nature of the spin-orbit force in the deuteron.

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[1] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).
[2] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957); R. G. Newton and T. Fulton, Phys. Rev. **107**, 1103 (1957); S. M. Vaidya, Nuclear Phys. **5**, 179 (1958).
[3] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950); C. H. Blanchard and R. Avery, Phys. Rev. **81**, 35 (1951); N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).
[4] H. Feshbach, Phys. Rev. **107**, 1626 (1957).
[5] A. Bohr, Phys. Rev. **73**, 1109 (1948); F. Low, Phys. Rev. **77**, 361 (1950).
[6] E. N. Adams, Phys. Rev. **81**, 1 (1951).
[7] A. M. Sessler and H. M. Foley, Phys. Rev. **98**, 6 (1955); **94**, 761 (1954).
[8] H. Bethe and E. E. Salpeter, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 35, p. 200.
[9] A. M. Sessler and R. L. Mills, Phys. Rev. (to be published).
[10] M. Sugawara, Arkiv Fysik **10**, 113 (1955).
[11] R. E. Marshak has recently fit scattering data without spin-orbit forces present in the deuteron (private communication).

Consequences of a Pseudovector Pion-Nucleon Coupling and the Universal β Decay

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On the assumption of the usual[1,2] (V-A) weak coupling it is well known that the $\pi \rightarrow \mu + \nu$ decay proceeds only through the axial vector current $\sqrt{G}\bar{\psi}\gamma_\mu\gamma_5\tau_+\psi$. Apart from the coupling constant factors, the divergence of this current is identical to the nucleon source current of a PV-coupled pion field; i.e.,

$$(4\pi)^{\frac{1}{2}} f\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\tau_i\psi) = (\Box^2 - \mu^2)\varphi_i + \delta\mu^2\varphi_i.$$

This observation allows the matrix element for the $\pi\rightarrow\mu+\nu$ decay,[3]

$$(\mu,\nu|H_I|\pi)=(\sqrt{G})F(-\mu^2)\bar{u}_\mu\gamma\cdot k_\pi \times[(1+\gamma_5)/2]u_\nu\delta^3(p_\mu+p_\nu-k_\pi), \quad (1)$$

to be related to the pion mass correction $\delta\mu^2$ by the expression

$$F(-\mu^2)=\frac{i}{(16\pi E_\mu E_\nu)^{\frac{1}{2}}}\left(\frac{\delta\mu^2}{\mu^2}\right)\frac{\sqrt{G}}{f}\langle 0|\varphi_+(0)|\pi\rangle. \quad (2)$$

The quantity $F(-\mu^2)$ can be obtained directly from the experimental π lifetime and the matrix element $\langle 0|\varphi_+(0)|\pi\rangle$ is $(16\pi^3E_\pi)^{-\frac{1}{2}}Z_3^{\frac{1}{2}}$, where Z_3 is the pion wave function renormalization coefficient. If the equality $Z_3(0)\equiv\mu^2\Delta_F(0)=\mu^2/\mu_0^2$ is now employed to rewrite Eq. (2) in terms of the measured, renormalized coupling constants,

$$f_r=Z_2Z_1^{-1}Z_3^{\frac{1}{2}}f$$

and

$$\sqrt{G_r}=Z_2Z_1^{-1}(0)\sqrt{G}=1.15\sqrt{G},$$

there results

$$F(-\mu^2)=\frac{i}{16\pi^2(E_\pi E_\mu E_\nu)^{\frac{1}{2}}}\left(\frac{\delta\mu^2}{\mu_0^2}\right)\left(\frac{Z_3Z_1(0)}{Z_3(0)Z_1}\right)\frac{\sqrt{G_r}}{f_r}, \quad (3)$$

which relates the bare pion mass to experimentally measured quantities and the *finite* ratio of renormalization coefficients $[Z_3Z_1(0)/Z_3(0)Z_1]$. This ratio is of the order of one and differs from this value by an amount which represents the variation in the pion-nucleon form factor over the range of nucleon four-momentum transfer, k, for which $-\mu^2\leq k^2\leq 0$. An explicit calculation of this ratio is now being undertaken by means of dispersion theory so that a value for μ_0 may be obtained. Preliminary estimates based upon perturbation theory indicate a bare pion mass in the neighborhood of $200m_e$.

It should be emphasized that the validity of these remarks depend entirely upon the pseudovector nature of the pion-nucleon coupling. In addition, the exactness of our conclusions would be marred if it is found that the above analogy [Eq. (1)] between the nucleon-lepton axial vector coupling and the nucleon-pion interaction cannot be extended to include the hyperons. If this requirement on the weak decay of the hyperons is not borne out by experiment, our determination of μ_0 would then be in error to the extent that the pion mass arises from virtual hyperons and to the extent that our required analogy is not fulfilled. At present, no positive statement can be made concerning the form of the axial-vector hyperon current, but in the case that the vector current is found not to be renormalized by virtual pions[1] we can conclude that this current is essentially the same as the isotopic-spin current. In this case, assuming the axial-vector hyperon current to be identical to the isotopic-spin current except for a factor of γ_5, there would be no contribution from the Λ field. Our required analogy would then break down if we assume that a direct π-Λ-nucleon interaction is needed to explain the large binding energies of hyperfragments in nuclei.

To avoid confusion it should perhaps be added that although the renormalization procedure applied to a theory with gradient coupling does not eliminate all the divergences, it is nevertheless meaningful to discuss the renormalization constants Z_1, Z_2, and Z_3 in the conventional manner. It is felt that this obvious need for a cutoff in the pseudovector theory does not eliminate it from interest, since in the eventual theory containing a fundamental length such a cutoff must appear.

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] R. E. Marshak and E. C. G. Sudarshan, Nuovo cimento (to be published).

[3] The normalization of the fermion spinors are chosen so that the projection operator takes the form $m-\gamma\cdot p$.