Here the term $n^3 B_I$ in Eq. (1) has been neglected, and A_I has been eliminated by using Eq. (1). Implicit in the derivation of Eq. (3) is the assumption that the distribution function follows any change in applied voltage with a time lag small compared to τ. This is expected to be a valid assumption since the distribution should change in a time on the order of several collision times, or $\sim 10^{-10}$ sec, while τ from Fig. 2 is much greater. The first term on the right-hand side of Eq. (3) arises from the coefficient of n^2 in (1). Unfortunately, from the measurements made so far, one cannot determine B_T and $B_I N_A$ separately, and thus it is not known as yet whether the term in n^2 is determined by Auger recombination or simply a bimolecular recombination process. If the coefficients in Eq. (3) vary slowly along the steep part of the current-voltage characteristic, τ reaches a maximum when

$$n^2 (B_T + B_I N_A) = A_T (N_D - N_A).$$

For this condition

$$\tau_{max} = [A_T (B_T + B_I N_A)(N_D - N_A)]^{-\frac{1}{2}}. \qquad (4)$$

The only term on the right-hand side of Eq. (4) that varies rapidly with lattice temperature, and hence would explain the large temperature variation of τ_{max} shown in Fig. 2, is the term A_T, which should vary approximately as $\exp(-\epsilon/kT)$, where ϵ is the activation energy for donors. In Fig. 3 the temperature dependence of τ_{max} is plotted. The agreement with the slope expected from the value of ϵ for Sb donors in Ge, 0.0097 ev, is quite good. The agreement can be improved (the dashed line in Fig. 3) by taking into account, roughly, the dependence of the coefficients in Eq. (4) on low powers of the lattice temperature. On incorporating a correction for this in Fig. 3, one finds the value of ϵ

FIG. 3. The variation of τ_{max} (solid curve and experimental points) and $\tau_{max}/\sqrt{T}$ (dashed curve) with temperature. The considerations pertaining to Eq. (4) show that the slope of the dashed curve should correspond to an energy equal to $\frac{1}{2}$ the activation energy for Sb donors in Ge.

obtained from these data to be in excellent agreement with the value 0.0097 ev.

The significance of these results is that they demonstrate that the phenomenological interpretation of the low-temperature static and dynamic electrical characteristics of n-type germanium, as given essentially by Eq. (1), is correct. Breakdown must be considered in the light of the earlier discussion pertaining to Eq. (2) rather than[5] as occurring at the electrical field for which the mean electron energy becomes equal to ϵ. For example, from Eq. (2) the breakdown field should *decrease* upon adding more donor impurities at constant N_A, as long as the donor density is low enough ($\lesssim 5 \times 10^{14}/cm^3$ in a typical case) so that neutral impurity scattering is not significant in determining the mobility.

I should like to acknowledge the able assistance of Mr. Walter Schillinger in designing equipment and obtaining the experimental data. In addition, I wish to thank P. J. Price for many helpful discussions and stimulating arguments.

[1] S. H. Koenig and G. R. Gunther-Mohr, J. Phys. Chem. Solids 2, 268, (1957).
[2] See also Ryder, Ross, and Kleinman, Phys. Rev. 95, 1342 (1954).
[3] Reference 1, p. 280; also S. H. Koenig, Phys. Rev. 110, 988 (1958), following Letter.
[4] The denominator does not become zero until the value of current appropriate to τ_{max} (to be discussed) is reached, as may be seen by combining Eqs. (1) and (4). This corresponds roughly to the end of the vertical part of the current-voltage characteristic, when terms in n^2 are important and the approximations of Eq. (2) are no longer valid.
[5] Burstein, Picus, and Sclar, in *Proceedings of the Conference on Photoconductivity, Atlantic City, 1954* (John Wiley and Sons, New York, 1956), p. 393; E. Burstein and P. Egli, in *Advances in Electronics and Electron Physics*, edited by L. Marton (Academic Press, Inc., New York, 1955), Vol. 7, p. 43.
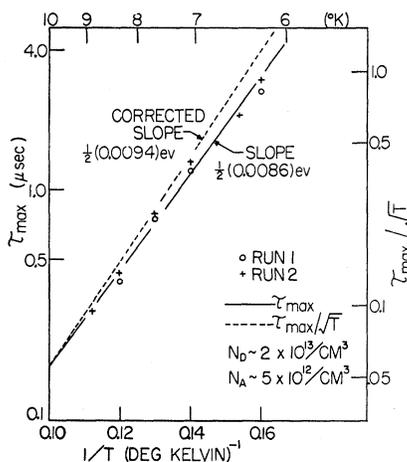
Recombination of Thermal Electrons in n-Type Germanium below 10°K

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THE magnitude of the termal recombination coefficient[1] B_T has been determined for thermal electrons in n-type germanium as a function of lattice temperature in the range 4–10°K. Preliminary results are shown in Fig. 1. Also shown are the smoothed data expressed in terms of a cross section obtained by dividing B_T by a velocity $v = (3m^* kT)^{\frac{1}{2}}$, with m^*, the effective mass, taken as $\frac{1}{4}$ of the free electron mass. The magnitude, $\sim 10^{-12}$ cm^2 at 4.2°K, of the effective cross section so obtained is several times the geometrical cross section for a donor ground state in germanium.

The experimental details are discussed below to indicate the manner in which measurements of B_T for
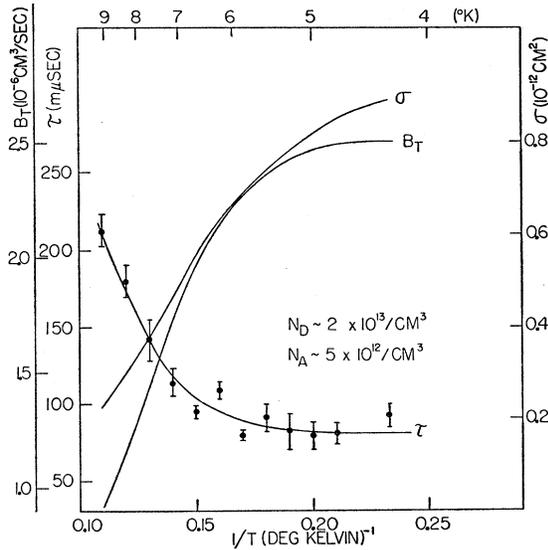
FIG. 1. Thermal recombination time τ, recombination coefficient B_T, and cross section σ, vs $1/T$ for a high-purity n-type germanium sample.

thermal electrons have been made, and to show how these techniques can be extended to obtain the dependence of B_T on electron "temperature." Measurements designed to separate the dependence of B_T on lattice temperature from that on the electron distribution are in progress.

In the present work, the sample is biased well into "breakdown"[2] at some fixed point A, and a fast-falling voltage step ($\sim 10^{-9}$ sec fall time) is applied to reduce the electric field to a value E_B, which is varied from one measurement to another. If in a time $\lesssim 10^{-9}$ sec the distribution equilibrates to that appropriate to point B, and if n, the carrier density, decays subsequently to its steady-state value for point B, one would expect to see an initial sharp drop in current determined by the relative voltages and mobilities at points A and B, followed by a slower decay as the carriers recombined. This behavior is shown in Fig. 2(a), which is an oscilloscope photograph of the applied step and the subsequent current variation.

An expression for μ_B, the mobility, as a function of E_B may be readily obtained from the initial current drop on the above assumptions:

$$\mu_B = j_{AB}\mu_A E_A / E_B j_A, \qquad (1)$$

where j_{AB} is the current just after the initial drop, and the other subscripts indicate steady-state values. Thus the variation of *drift* mobility with E_B, (though not the absolute values) may be obtained. In addition, from the same considerations that led to Eq. (2) of the preceding letter, one can write for the time dependence of n

$$dn/dt = A_T(N_D - N_A) - n[B_T N_A - A_I(N_D - N_A)]. \qquad (2)$$
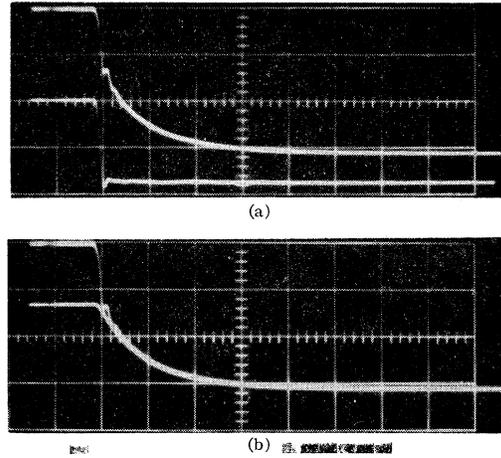
FIG. 2. (a) Time dependence of the applied voltage and the resulting current variation. (b) Comparison of the current decay with an exponential decay obtained by integrating the applied voltage step and superimposing the two traces.

According to Eq. (2), the decay in Fig. 2(a) should be exponential with a time constant

$$\tau = [B_T N_A - A_I(N_D - N_A)]^{-1}$$

if the assumptions to this point are valid. Figure 2(b) is a photograph of the applied voltage pulse of Fig. 2(a) integrated by an RC network and superimposed on the current decay, to demonstrate the exponential form of the current decay. The experimental values of τ are in fact measured by this technique, using a calibrated variable capacitor in the integrating circuit.

It is apparent from Eq. (2) that the steady-state concentration of carriers at point B is given by

$$n_B = A_T(N_D - N_A)\tau_B. \qquad (3)$$
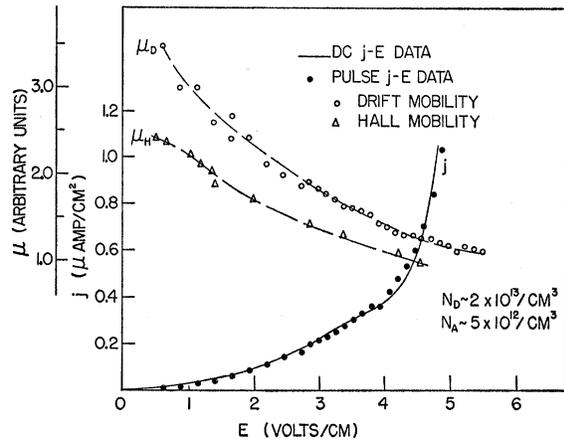
FIG. 3. Comparison of the dc j-E data with the same curve reconstructed from pulse measurements. In addition the measured Hall mobility and the drift mobility determined from pulse measurements are shown on a scale of arbitrary units. No attempt has been made either to normalize one mobility curve to the other or to have the same relative scale for both.

From Eqs. (1) and (3) it is seen that by using measurements of both the initial drop and the decay constant of Fig. 2(a), it should be possible to reconstruct the variation of conductivity, and hence current density, with electric field. In Fig. 3 the variation of current density (obtained as just described) and drift mobility determined by pulse measurements is compared with the Hall mobility and current density determined as a function of E by dc measurements. The j-E curve obtained from the pulse data was normalized to the dc data at one point. It is this excellent agreement of the pulse measurements with the dc data that justifies the fundamental assumption that, even for the large pulses involved in these experiments, the distribution function follows changes in applied electric field in a time short compared to the time constant, τ, for change in carrier density. It is the value of τ obtained from pulsing to a value of E_B *in the linear conductivity range* that was used, with Eq. (2), to obtain the values of B_T plotted in Fig. 1. The additional assumption that

$$A_I \ll B_T N_A/(N_D - N_A)$$

in the linear range was made. A nominal value for N_A of $\sim 5 \times 10^{12}/\text{cm}^3$ was obtained from comparison of the measured Hall mobility and conductivity with results from similar data on other samples.[2] This value should be correct to within 30%. The experimental values of τ, and their rms spread, are also shown in Fig. 1.

To compare Hall and drift mobilities requires a knowledge of the absolute value of drift mobility for at least one value of electric field. This can be obtained in the linear conduction range from a measurement of the Hall constant at large (effectively infinite) magnetic fields. Such measurements are currently in progress. It should then be possible to determine the ratio of Hall to drift mobility as a function of electric field for a wide range of lattice temperature.

It is felt, partly because of the relatively large value of the cross section τ, and partly because of additional (as yet unpublished) data on the dependence of B_T on the "electron temperature," that the mechanism of capture is similar to that discussed by Lax[3] in connection with "giant traps."[3] Recombination on this model involves capture in an excited state as the result of an appropriate, but improbable, energy-losing collision followed either by a cascade of transitions to the ground state, or by an immediate ejection of the electron (by absorption of a phonon) into the conduction band with no net contribution to the recombination rate. Because the event which brings the electron into orbit is unlikely, the calculated cross section is much smaller than the "geometrical" area of the orbits that make the major contribution to the total cross section but large compared with that of the ground state.

The experimental cross section is in qualitative agreement with the theory of Lax for the temperature range of Fig. 1.[4] The "saturation" of τ at the lower temperatures is to be associated with a cutoff for orbits larger than the spacing between recombination sites.

I should like to thank Melvin Lax for the several enlightening discussions of his theory, P. J. Price for his critical comments throughout, and Walter Schillinger for his invaluable aid in obtaining the results.

[1] S. H. Koenig, Phys. Rev. 110, 986 (1958), preceding Letter.
[2] S. H. Koenig and G. R. Gunther-Mohr, J. Phys. Chem. Solids 2, 268 (1957).
[3] M. Lax, Bull. Am. Phys. Soc. Ser. II, 1, 128 (1956).
[4] M. Lax (private communication).

Luminescence from X-Ray Colored KBr Crystals during Plastic Deformation*

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IN a previous report[1] the effect of strain, strain rate, and irradiation time on the luminescence induced by plastic deformation of various alkali halides contaiing F centers was discussed. However, it was not possible at that time to determine the spectral distribution accurately.

The spectral distribution of this luminescence from 2000 to 9000 A has now been determined in the case of potassium bromide. The emission spectrum, shown in Fig. 1, is a broad, bell-shaped curve with its maximum at 4000 A and showing a long-wavelength tail.

The experimental details are as follows: single crystals, about $5 \times 5 \times 10$ mm, were cleaved from large pieces supplied by the Harshaw Chemical Company. The crystals were heated overnight at 100°C below
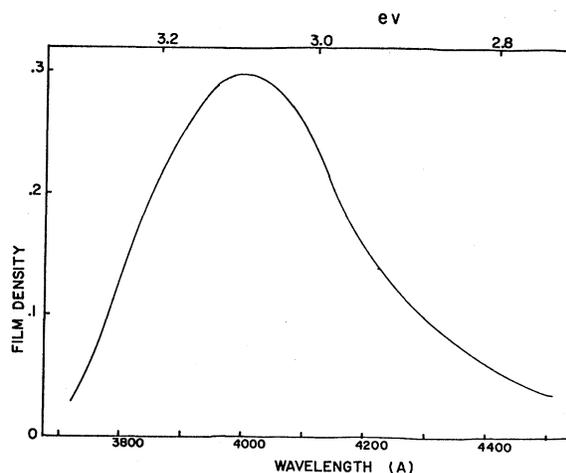
FIG. 1. Luminescence spectrum from x-ray colored KBr (50 crystals). Kodak 103-F Plate.

FIG. 2. (a) Time dependence of the applied voltage and the resulting current variation. (b) Comparison of the current decay with an exponential decay obtained by integrating the applied voltage step and superimposing the two traces.