Putting $R_D=90$ in., $L=30$ in., $R_s=60$ in., $A=0.786$ in.^{2}, and $l/\lambda=2$, we get

$$\left(\frac{\Delta\sigma_t}{\sigma_t}\right)_{\text{inscatter}}=0.26\times10^{-2}\left(\frac{kR+1}{8\pi}\right)^2.$$

To check this experimentally, we placed a detector just outside the neutron cone (and near the normal detector position) and compared its counting rate with and without a 2λ copper sample in the neutron beam. This geometry was slightly, but not importantly, different from the inscattering situation in our transmission experiments. The measured inscattering agreed with the theoretical (modified slightly for the changed geometry) well within the statistics, which were only ±40%.

PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

THE nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be characterized in first approximation in terms of independent particle motion. For instance, the statistical level density, at not too low excitation energies, is expected to resemble that of a Fermi gas. Still, in both systems, important correlations in the particle motion arise from the action of the forces between the particles and, in the metallic case, from the interaction with the lattice vibrations. These correlations decisively influence various specific properties of the system. We here wish to suggest a possible analogy between the correlation effects responsible for the energy gaps found in the excitation spectra of certain types of nuclei and those responsible for the observed energy gaps in superconducting metals.

We first briefly recall the evidence for an energy gap in the spectra of nuclei, and shall especially consider nuclei of spheroidal type. The single-particle level spectra for such nuclei exhibit a particularly close similarity to that of a Fermi gas, since the degeneracies characterizing the particle motion in a spherical potential are largely removed by the distortion in the shape of the nuclear field. The levels remain doubly degenerate, and their average spacing may be most directly obtained from the observed spectra of odd-A nuclei. These exhibit intrinsic states which may be associated with the different orbits of the last particle, and the observed single-particle level spacing is approximately[1]

$$\delta\approx 50A^{-1}\text{ Mev},\tag{1}$$

where A is the number of particles in the nucleus.

If the intrinsic structure could be adequately described in terms of independent particle motion, we would expect, for even-even nuclei, the first intrinsic excitation to have on the average an energy $\frac{1}{2}\delta$, when we take into account the possibility of exciting neutrons as well as protons. Empirically, however, the first intrinsic excitation in heavy nuclei of the even-even type is usually observed at an energy of about 1 Mev (see Fig. 1). The only known examples of intrinsic excitations with appreciably smaller energy are the $K=0-$ bands which occur in special regions of nuclei, and which may possibly represent collective octupole vibrations.[2]

Such an energy gap between the ground-state and first intrinsic excitation indicates an important departure from independent-particle motion, a departure arising from the residual forces between the particles. In lowest order, such forces give rise to a pairing effect, since the attractive interaction is expected to be especially strong for a pair of particles in degenerate orbits. This effect implies a shift upwards, relative to the ground state, of states involving the breaking of a pair. However, to this order, one still expects that levels corresponding to the simultaneous excitation of two particles remaining as a pair will have an average energy spacing of about δ. Such low-lying $K=0$ bands

* National Science Foundation Senior Post-Doctoral Fellow on leave of absence from Princeton University, Princeton, New Jersey, 1957–1958.

[1] B. R. Mottelson and S. G. Nilsson (to be published); F. Bakke (to be published).

[2] See, e.g., K. Alder *et al.*, Revs. Modern Phys. 28, 432 (1956).

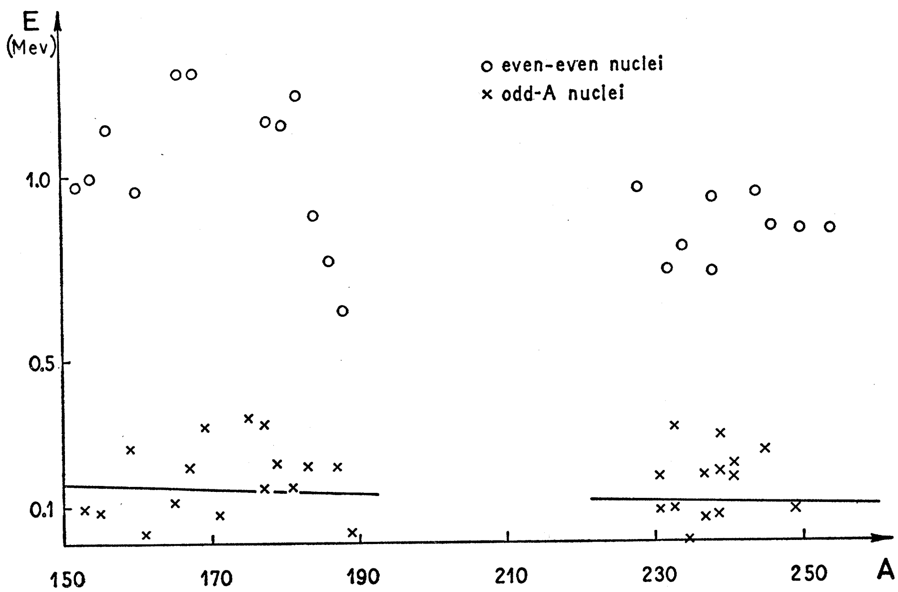

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.

are not observed, and their absence implies significant correlations in the intrinsic nucleonic motion.

The appearance of a gap is reminiscent of the well-known repulsion effect between coupled levels, but in order to obtain a gap of the observed magnitude, which is several times larger than δ, it appears necessary to consider the coupling between a large number of states of independent particle motion. It seems likely that the resulting excitation spectrum may show regularities of collective type, and indeed there is some evidence for the occurrence of vibrational levels among the first intrinsic excitations in the spheroidal nuclei of even-even type.[2]

The correlations giving rise to the energy gap may also affect many other nuclear properties; thus, they appear to be responsible for the observed fact that the rotational moments of inertia are appreciably smaller than the values corresponding to rigid rotation.[3] Moreover, the well-known mass difference between even-even and odd-A nuclei[4] appears intimately connected with the occurrence of the gap. While we have here considered the nuclei of spheroidal type, similar differences between the intrinsic spectra of odd and even nuclei appear also for nuclei of spherical equilibrium shape. To exhibit the gap in these spectra one must, however, subtract the relatively low-lying collective excitations of vibrational type.

In the superconducting metal we may possibly be dealing with somewhat similar correlation effects in the electronic motion. Measurements of the thermal and electromagnetic properties of superconductors[5] indicate that the low-energy electronic excitation spectrum differs essentially from that of a Fermi gas in that there exists a finite energy gap between the ground state of the metal and the states representing electronic excitation.

Recently, new insight concerning the behavior of interacting fermions has been obtained from a detailed study of the correlations arising from the part of the interaction which acts between particles with equal and opposite momenta.[6] Treating only this part of the interaction, it was found that even very weak attractive interactions lead to a major change in the low-energy spectrum of the system. Quite apart from the extent to which additional interactions may further modify the spectrum, it would seem that the results obtained are already of considerable interest in connection with the features of the nuclear spectra discussed above and indicate that a modified structure of the Fermi surface is a general feature of Fermi systems with attractive interactions. This qualitative result is perhaps not

[3] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, 1 (1955).

[4] For a review of this effect, see C. D. Coryell, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1953), Vol. 2, p. 304.

[5] For discussions of evidence for an energy gap, see Blevins, Gordy, and Fairbank, Phys. Rev. **100**, 1215 (1955); Corak, Goodman, Satterthwaite, and Wexler, Phys. Rev. **102**, 656 (1956); R. E. Glover and M. Tinkham, Phys. Rev. **104**, 844 (1956); **108**, 243 (1957).

[6] Bardeen, Cooper, and Schrieffer, Phys. Rev. **106**, 162 (1957), and Phys. Rev. **108**, 1175 (1957). This model has also been treated by N. N. Bogoliubov, J. Exptl. Theoret. Phys. (to be published) and J. Valatin (to be published).

surprising, since it may easily be shown that a perturbation treatment of the interactions leads to divergencies in the region of the Fermi surface.

Due to the simplicity of the argument, it may perhaps be useful to sketch such a perturbation calculation.[7] Consider two particles with opposite momenta and each having a kinetic energy $E_0=E_F-\epsilon$, where E_F is the Fermi energy. We evaluate the amplitude for their excitation to a level $E=E_F+\epsilon$, the other particles in the system remaining unperturbed. To first order we obtain, in obvious notation,

$$af^{(1)}=\frac{\langle 0|V|f\rangle}{2(E_0-Ef)}, \tag{2}$$

and to second order

$$af^{(2)}=\sum_i \frac{\langle 0|V|i\rangle\langle i|V|f\rangle}{4(E_0-E_i)(E_0-Ef)} \approx af^{(1)}\int_{E_F}^{E_0+\Delta}\frac{VN(0)dE}{E-E_0}, \tag{3}$$

or

$$af^{(2)}\approx af^{(1)}N(0)V\ \ln[\Delta/(E_F-E_0)]. \tag{4}$$

We have approximated the interaction matrix element by a constant negative value $-V$ over an effective energy interval Δ. The density of the states at the Fermi surface is denoted by $N(0)$.

It is seen that the perturbation expansion represents a power series in the parameter

$$x=N(0)V\ \ln[\Delta/(E_F-E_0)]. \tag{5}$$

Thus, the series diverges for $x>1$, i.e., for E_0 in an energy region around the Fermi surface of extension

$$\epsilon\approx\Delta\ \exp[-1/N(0)V]. \tag{6}$$

[7] A corresponding divergence in the two-body scattering equation for particles in a Fermi gas has been pointed out by J. Goldstone (private communication).

This estimate corresponds to the result obtained in the above-mentioned model of superconductivity[6] for the energy region in which the particle motion is essentially correlated. It was also found[6] that this model leads to a gap in the energy spectrum of order given by (6).

It is of interest to attempt to apply these considerations to the nuclear case. Estimates of the quantity $N(0)V$ for the nucleus are somewhat uncertain, but indicate values of the order of $\frac{1}{5}$. Owing to the sensitivity of (6) to this quantity, it is difficult to make quantitative estimates of ϵ, but it appears likely that $\epsilon\ll\Delta$, with Δ representing an energy of the order of E_F. Under such circumstances the correlation effect would be relatively unimportant for bulk nuclear properties like the total binding energy, but would, of course, have essential effects on the low-energy excitation spectra. A quantitative estimate of ϵ would also be important for the derivation of a criterion for the occurrence of shell structure; thus, since the energy spacing between shells tends to zero as the size of the system increases, one would expect the shell structure to be washed out for a sufficiently large system.

It thus appears that there may exist interesting similarities between the low-energy spectra of nuclei and of the electrons in the superconducting metal. However, it must be stressed that the former are significantly influenced by the finite size of the nuclear system. Thus, the energy gap is observed to decrease with A, and the present data are insufficient to indicate the limiting value for the gap in a hypothetical infinitely large nucleus. Moreover, the degrees of freedom associated with the variation in shape play an especially important role in the low-energy nuclear spectra.

ACKNOWLEDGMENT

One of us (D. P.) would like to take this opportunity to thank Professor Niels Bohr and the members of the Institute for Theoretical Physics for the warm hospitality extended to him during the summer of 1957.