Polarization of Nucleons from the D(γ,p)n Reaction at Medium Energies

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The polarization of nucleons from the D(γ,p)n reaction is investigated under the assumption of tensor coupling in the n-p interaction potential. Electric dipole, electric quadrupole, magnetic dipole, and magnetic quadrupole transitions are taken into consideration. The polarization is estimated for $\hbar\omega=65$ Mev with the help of recent nucleon-nucleon scattering phase shifts. These estimates indicate the dominant role of the electric dipole and the magnetic dipole transitions from $^3S_1+{}^3D_1$ ground state to the $^3P_0$, $^3P_1$, $^3P_2$, $^3F_2$, and $^1S_0$ final states. The advantages of the measurement of the polarization are indicated.

I. INTRODUCTION

RECENTLY the theory of the polarization of nucleons from the D(γ,p)n reaction was presented[1] (hereafter referred to as I). In the derivation of the polarization formulas only $E1$ (to the ^3P_J states), $M1$ (to the $^1S_0$ state), and $E2$ (to final free D states) transitions were assumed effective and taken into consideration. The polarization was shown by means of several examples to be sensitive to the n-p interaction potential assumed. Even this oversimplified analysis was satisfactory to demonstrate that the polarization is much more sensitive to the system of the n-p phase shifts than is the angular distribution of the outgoing nucleons (compare, e.g., papers by Hsieh[2] and Austern[3]). No experiment on the polarization has been performed as yet, so that the motivation for the present study is to indicate the usefulness of such an analysis.

Recently Austern[3] mentioned that polarization measurements would be necessary to provide the additional equations required to determine the unknown $E1$ radial integrals from the experimental data. Further, the $M1$ transitions, which do not disturb the angular distribution, could thus be more carefully investigated, since the polarization is most seriously influenced by interference with the large $E1$ transitions.

The most recent results of the analysis of the D(γ,p)n differential cross section data[3,4] provide a new critical revision of the current conventional theory. This fact and the necessity of the investigation of the corrections coming from all the hitherto neglected radiative transitions assuming multipoles up to $E2$ and $M2$ require the re-examination of the analysis carried out in I. The nucleon-nucleon potential recently derived at Los Alamos[5] should also be applied to the present problem.

It was suggested in references 3 and 4 that a certain new meson effect seems to be the only reasonable explanation of the large experimental values of the ratio of the isotropic to the $\sin^2\theta$-dependent part of the differential cross section [i.e., a/b, where $\bar{\sigma}(\theta)=a+b\sin^2\theta$]. It should be noted however, that the phase shifts as given by Clementel and Villi,[6] for example, seem to produce a reasonable value for the ratio a/b at $\hbar\omega=40$ Mev, assuming the conventional theory of the D(γ,p)n reaction (see I). Austern's[3] recent analysis provides a theory of the deuteron photodisintegration at medium energies with the assumption of violation of Siegert's theorem for the $E1$ transition to the final $^3P_0$ state, and shows the importance of the previously omitted $E1$ transition $^3D_1\rightarrow{}^3F_2$. The exceptional role of the $^3P_0$ state is consistent with the assumption of a two-step process: virtual production of an S-wave pion and disintegration by re-absorption of the meson.[3,4] The $^3P_{1,2}$ final states in $E1$ transitions are assumed to be reached *via* ordinary photon absorption according to Siegert's theorem (Austern's "rigid nucleon theory"). This assumption is believed to be valid at the energies considered here, i.e., "medium" energies (up to $\hbar\omega\approx 100$ Mev).

Consequently, all radial integrals involved are computed in the present analysis using the usual radiative interaction potentials and neutron-proton forces (phase shifts are examined for various particular theories). The only radial integral for the $E1$ transitions leading to the $^3P_0$ state is computed semiempirically from the experimental total cross section data, using theoretically calculated values for all other $E1$ radial integrals.

II. NOTATION

J, M_J, the total angular momentum of a final state and its z-axis projection.

j, m, the total angular momentum of an initial state and its z-axis projection.

$\chi_1^{m_s}(\sigma)$, the triplet state spin eigenfunction.

$\chi_0^0(\sigma)$, the singlet state spin eigenfunction.

η_J, the ^3P_J state phase shift.

η_S, the $^3S_1$ state phase shift.

η_D, the ^3D_J state phase shift (no splitting assumed).

δ_S, the $^1S_0$ state phase shift.

δ_P, the $^1P_1$ state phase shift.

δ_D, the $^1D_2$ state phase shift.

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[1] W. Czyż and J. Sawicki, Nuovo cimento 5, 45 (1957).

[2] S. H. Hsieh, Nuovo cimento 4, 138 (1956); Progr. Theoret. Phys. (Kyoto) 16, 68 (1956).

[3] N. Austern, Phys. Rev. 108, 973 (1957).

[4] R. R. Wilson, Phys. Rev. 104, 218 (1956).

[5] J. L. Gammel and R. M. Thaler, Phys. Rev. 107, 291, 1337 (1957).

[6] Clementel, Villi, and Jess, Nuovo cimento 5, 907 (1957); E. Clementel and C. Villi, Nuovo cimento 5, 1166 (1957).

$\epsilon_d = 2.23$ Mev, the deuteron binding energy.

M, the nucleon mass.

$\gamma = (M\epsilon_d/\hbar^2)^{\frac{1}{2}}$, the parameter of the deuteron ground state wave function $[\lim_{r\to\infty} u = \text{const}\times\exp(-\gamma r)]$.

E, the center-of-mass system energy of the relative motion of the n-p system.

$\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, the vector from the proton to the neutron.

$\mathbf{k}$, the n-p system wave vector in the c.m. system.

$\mathbf{k}_0 = \boldsymbol{\kappa} k_0 = \boldsymbol{\kappa}(\omega/c)$, the wave vector of the incident photon.

$\mathbf{e}$, the polarization vector of the incident photon.

μ_n, μ_p, the magnetic moments of the neutron and the proton, respectively.

$\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_p$, the spin operators of the neutron and the proton, respectively.

$f\chi$, the reaction amplitude: $f\chi = g\chi_0^0 + \sum_{m_s=-1}^{1} f_{m_s}\chi_1^{m_s}$.

$\mathbf{P}$, the polarization vector.

$\{\cdots\}_{\text{Av}}$, denotes averaging over the photon polarizations and the magnetic quantum numbers m of the ground state.

u, w, the deuteron ground state radial wave functions of the S and the D states, respectively.

u_s, u_t, the $^1S_0$ and $^3S_1$ final state wave functions, respectively.

w_s, w_t, the $^1D_2$ and $^3D_1$ final state wave functions, respectively.

v_P, the $^1P_1$ final state wave function.

v_J, the ^3P_J final state wave function.

$j_l(x)$, $n_l(x)$, the spherical Bessel and Neumann functions, respectively.

$\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p)$.

III. CALCULATION

The polarization is calculated following the method of I. First one has to determine the reaction amplitude

$$f\chi = g\chi_0^0 + \sum_{m_s=-1}^{1} f_{m_s}\chi_1^{m_s},$$

and use the expression

$$\mathbf{P} = \{\langle f\chi|\boldsymbol{\sigma}_n|f\chi\rangle\}_{\text{Av}}/\{\langle f\chi|f\chi\rangle\}_{\text{Av}}. \tag{1}$$

Here the neutron polarization is determined. However, one can prove that exactly the same analytical expressions are obtained for protons, θ then having the meaning of the proton angle, provided we confine ourselves to $M1$ and $E1$ transitions only. Protons seem to be more suitable experimentally at higher energies. It should be noted that P in I was computed for neutrons, θ having the confusing meaning of the proton angle. If θ is the neutron angle, one has to substitute $\theta-\pi$ for θ and change the signs, e.g., in Tables I and II of our paper[1]; viz., $[P(-\theta') = -x] \rightarrow [P(\theta') = x]$ (to make the scattering angle always positive: $0 \leqslant \theta' \leqslant \pi$).

We shall confine ourselves in this section to the consideration of $E1$ and $M1$ transitions leading from $^3S_1 + ^3D_1$ ground state to the seemingly most important final states: $^1S_0$, ^3P_J, and $^3F_2$. Higher order transitions will be considered in Sec. 4. The $^3P_2 - ^3F_2$ mixing by the tensor force is neglected. This approximation seems reasonable for the energies under consideration.[7] Likewise η_F is assumed to be zero throughout.

Similarly to the treatment in I and in the old paper by the authors,[8] the quantization axis is chosen along the vector $\mathbf{k}\times\mathbf{k}_0$. In the present approximation, the radiative interaction operator is proportional to

$$H' = -\mathbf{e}\cdot\mathbf{r} + \tfrac{1}{2}\alpha(\boldsymbol{\kappa}\times\mathbf{e})\cdot(\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_n), \tag{2}$$

where $\alpha = (\hbar/Mc)(\mu_p - \mu_n)$. The contribution to $\mu_p - \mu_n$ of $\mu(r)$, the anomalous magnetic moment due to meson exchange current effects, was neglected. It follows from Table I of I that this correction is not essential for the polarization.

On using the same system of reference as previously,[8] we can write

$$H' = [-z + \tfrac{1}{2}\alpha(\sigma_{px} - \sigma_{nx})]\cos\phi + [-x - \tfrac{1}{2}\alpha(\sigma_{pz} - \sigma_{nz})]\sin\phi, \tag{3}$$

where $\phi = \angle(\mathbf{e}, \mathbf{k}\times\mathbf{k}_0)$. We may express f_{m_s} and g in terms of $\cos\phi$ and $\sin\phi$: $f_{m_s} = f_{m_s}{}^z\cos\phi + f_{m_s}{}^x\sin\phi$, $g = g^z\cos\phi + g^x\sin\phi$. In our frame of reference, the components P_x and P_y vanish. Finally, as in I, we get

$$P = P_z = \frac{\sum_m\{2\,\text{Re}(g^{x*}f_0{}^x) + 2\,\text{Re}(g^{z*}f_0{}^z) + |f_1{}^x|^2 - |f_{-1}{}^x|^2\}}{\sum_m\{|g^x|^2 + |g^z|^2 + \sum_{m_s}(|f_{m_s}{}^x|^2 + |f_{m_s}{}^z|^2)\}}. \tag{4}$$

After performing angular integrals, the summations over the magnetic quantum numbers of the final states, and the averaging over the magnetic quantum numbers of the initial states, we can express P in terms of the radial integrals and phase shifts:

$$P = (\sigma)^{-1}(c\sin 2\theta + \alpha d\sin\theta), \tag{5}$$

where

$$c = \tfrac{1}{6}L_0L_2\sin(\eta_0 - \eta_2) + \tfrac{1}{4}L_1L_2\sin(\eta_1 - \eta_2) - \tfrac{1}{6}L_0L_f\sin\eta_0 + \tfrac{3}{8}L_1L_f\sin\eta_1 - (5/24)L_2L_f\sin\eta_2,$$

$$d = M_SL_1\sin(\delta_s - \eta_1),$$

and[9]

$$\tilde{\sigma} = a + b\sin^2\theta,$$

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[7] This seems to be somewhat inconsistent for the "Los Alamos" potential (reference 5). However, the most important features of the interference terms are preserved. The effect of the $^3P_2 - ^3F_2$ mixing is the subject of the forthcoming publication.

[8] W. Czyż and J. Sawicki, Nuovo cimento **3**, 864 (1956).

[9] The expression for $\tilde{\sigma}$ leads to an angular distribution identical with that of W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941), provided $L_f = 0$. Equation (5) is valid for $\eta_F = 0$. If $\eta_F \neq 0$, all η_J in the arguments of all the sines and cosines should be replaced by $\eta_J - \eta_F$.

$$a=\alpha^2 M_S^2+(1/9)L_0^2-(2/9)L_0L_2\cos(\eta_0-\eta_2)$$
$$-\tfrac{1}{3}L_0L_f\cos\eta_0+\tfrac{1}{4}L_1^2+(13/36)L_2^2-\tfrac{1}{2}L_1L_2\cos(\eta_1-\eta_2)$$
$$+\tfrac{1}{2}L_f^2+\tfrac{1}{2}L_1L_f\cos\eta_1-\tfrac{1}{6}L_2L_f\cos\eta_2,$$
$$b=\tfrac{1}{3}L_0L_2\cos(\eta_0-\eta_2)+\tfrac{1}{2}L_0L_f\cos\eta_0+\tfrac{1}{8}L_1^2+(7/24)L_2^2$$
$$+\tfrac{3}{4}L_1L_2\cos(\eta_1-\eta_2)+\tfrac{1}{2}L_f^2$$
$$-\tfrac{3}{4}L_1L_f\cos\eta_1+\tfrac{1}{4}L_2L_f\cos\eta_2,$$

where

$$L_0=\int_0^\infty(\gamma r)v_0(u-2^{\frac{1}{2}}w)dr,$$

$$L_1=\int_0^\infty(\gamma r)v_1(u+2^{-\frac{1}{2}}w)dr,$$

$$L_2=\int_0^\infty(\gamma r)v_2(u-\tfrac{1}{5}2^{-\frac{1}{2}}w)dr,$$

$$L_f=\tfrac{3}{5}\sqrt{2}\int_0^\infty(\gamma r)w[krj_3(kr)]dr,$$

$$M_S=\gamma\int_0^\infty uu_s dr.$$

The normalizations of u [$\lim_{r\to\infty}u\sim\exp(-\gamma r)$] and of v_J [$\lim_{r\to\infty}v_J\sim kr(\cos\eta_J j_1(kr)-\sin\eta_J n_1(kr))$] are chosen following Austern[3] so as to make all the $E1$ radial integrals L_J and L_f have the dimensions of lengths (they are lengths of the order of those which are actually effective in this problem). The wave functions u and w were used here as given in reference 3, and the integrals $L_{1,2}$ and L_f were computed from the graphs and tables given by Austern.[3] According to Austern's analysis[3] (see also Hsieh[2] and Example 1 of I), the integrals L_J ($J=1, 2$) can be approximated by

$$L_J(>r_c)\equiv\int_{r_c}^\infty(\gamma r)uv_J dr, \quad (6)$$

where $r_c=1\times10^{-13}$ cm. The integral M_s was computed by the method of Hsieh[2] and Example 1 of I, i.e., u_s was assumed to have the form $u_s=\sin(kr+\delta_S)\{1-\exp[-\rho(r-D_0)]\}$ for $r\geqslant D_0$ and $u_s\equiv0$ for $r<D_0$. The parameters ρ and D_0 were chosen such that u_s corresponds to the repulsive core of the radius D_0. (D_0 as suggested for $^1S_0$ state from the p-p scattering analysis is 0.4×10^{-13} cm according to Gammel and Thaler[5] and 0.35×10^{-13} cm according to Clementel and Villi.[6])

The integrals L_0 were calculated by using the experimental values of the total cross section. According to the equation used by Austern,[3] one has

$$9a+6b\approx L_0^2+3L_1^2+5L_2^2+(15/2)L_f^2, \quad (7)$$

provided Austern's neglect of the $\alpha^2 M_S^2$ term is justified.

The neutron (proton) polarization was calculated for $\hbar\omega=65$ Mev using the sets of phase shifts of Gammel and Thaler[5] and of Clementel and Villi.[6,10] The polarizations obtained in these cases are

$$P(\theta)=\frac{-0.215\sin2\theta+0.250\sin\theta}{0.877+\sin^2\theta} \quad (8a)$$

for the Gammel and Thaler case, and

$$P(\theta)=\frac{-0.232\sin2\theta+0.252\sin\theta}{0.877+\sin^2\theta} \quad (8b)$$

for the Clementel and Villi case. The denominators of expressions (8a) and (8b) were taken from the experimental data of Whalin *et al.*[11] This procedure seems to be justified to give an estimate for the polarization.

It should be noted that phase shifts from the potential recently proposed by Signell and Marshak[12] differ only slightly from those employed in (8a)–(8b) and so do not produce a significantly different result for $P(\theta)$.

IV. HIGHER ORDER CORRECTIONS

The surprisingly large influence upon the polarization of $M1$ transitions, which are unimportant for $\tilde{\sigma}(\theta)$, suggests the need to estimate all the interference terms including the less important transitions for the multipoles up to $E2$ and $M2$. These transitions might confuse the situation mostly *via* the interference terms with strong $E1$ transitions. The analytical expressions for the polarization were derived by use of the following additional transitions: (1) $M1$ from $^3S_1+^3D_1$ to $^3S_1$ and $^3D_1$, neglecting the tensor coupling of the final states; (2) $M1$ from $^3D_1$ to $^1D_2$; (3) $E2$ from $^3S_1$ to ^3D_J, with no splitting of the ^3D_J states; (4) $E2$ from $^3D_1$ to $^3S_1$; (5) $M2$ from $^3S_1$ to ^3P_J, taking into account only the most important interference terms of these transitions with $E1$ and $E2$ transitions; (6) $M2$ from $^3S_1$ to $^1P_1$. The most interesting point is, how do these corrections modify the general features of the angular dependence of $P(\theta)$, i.e., how large are the corrections to the coefficients of $\sin\theta$ and $\sin2\theta$ as given in Eq. (5) and what new angular functions are introduced in the modified $P(\theta)$. The transitions (1) and (3)–(6) arise from the residual interaction $\Delta H'=H_{\text{corr}}'-H'$, where H' is given by Eq. (2), while the transition (2) is already implied by H'. The residual terms of the radiative interaction are

$$\Delta H'=iq(\mathbf{e}\cdot\mathbf{r})(\boldsymbol{\kappa}\cdot\mathbf{r})+\zeta(\boldsymbol{\kappa}\times\mathbf{e})\cdot\mathbf{S}-i\xi(\boldsymbol{\kappa}\cdot\mathbf{r})(\boldsymbol{\kappa}\times\mathbf{e})\cdot\mathbf{S}$$
$$-i\eta(\boldsymbol{\kappa}\cdot\mathbf{r})(\boldsymbol{\kappa}\times\mathbf{e})(\boldsymbol{\sigma}_p-\boldsymbol{\sigma}_n), \quad (9)$$

[10] It should be noted that in a recent paper, J. Bernstein [Phys. Rev. **106**, 791 (1957)] analyzed the $D(\gamma,p)n$ total cross section using the phase shifts of Feshbach and Lomon. However, recent results in the analysis of the p-p and n-p interaction (reference 5) indicate the failure of Feshbach and Lomon's phase shifts. For those phase shifts one obtains: $(0.877+\sin^2\theta)P(\theta)=-0.147\sin2\theta+0.074\sin\theta$, as compared with the numbers given below.

[11] Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956).

[12] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957).

where

$$q=\tfrac{1}{4}\frac{\omega}{c},\qquad \zeta=\frac{\hbar}{Mc}(\mu_p+\mu_n-\tfrac{1}{2}),$$

$$\xi=\frac{\hbar\omega}{2Mc^2}(\mu_p-\mu_n),\quad \eta=\frac{\hbar\omega}{4Mc^2}(\mu_p+\mu_n),$$

are the parameters characteristic for $E2$, $M1$ (to triplet final states), $M2$ (to triplet final states), and $M2$ (to singlet final states) transitions, respectively. In our system of reference, $\Delta H'$ can be written as

$$\Delta H'=[iqzy+\zeta S_x-i\xi yS_x-i\eta y(\sigma_{px}-\sigma_{nx})]\cos\phi+[iqxy-\zeta S_z+i\xi yS_z+i\eta y(\sigma_{pz}-\sigma_{nz})]\sin\phi. \quad (10)$$

For the sake of simplicity we shall neglect the $E1$ transitions ${}^3D_1\rightarrow{}^3F_2$ considered previously. In this approximation, we find

$$P_{\text{corr}}=(\tilde{\sigma}_{\text{corr}})^{-1}\{c\sin2\theta+\alpha d\sin\theta+qA_1\cos\theta\sin2\theta+\alpha qA_2\sin2\theta+qA_3\sin\theta+\zeta A_4\sin\theta+(q\zeta A_5+\zeta^2A_6)\sin2\theta+\alpha A_7\sin\theta+\alpha qA_8\sin2\theta+\xi A_9\sin2\theta+\xi qA_{10}\cos\theta\sin2\theta+\xi qA_{11}\sin\theta+\eta A_{12}\sin2\theta+\eta qA_{13}\cos\theta\sin2\theta\}, \quad (11)$$

where $\tilde{\sigma}_{\text{corr}}$ is the correspondingly modified expression which replaces $\tilde{\sigma}$ [as in Eq. (5)], given in Appendix I. The coefficients A_i are as follows:

$$A_1=\tfrac{1}{12}Q_D[5L_2\sin(\eta_2-\eta_D)-2L_0\sin(\eta_0-\eta_D)-3L_1\sin(\eta_1-\eta_D)],$$

$$A_2=-\tfrac{1}{2}M_SQ_D\sin(\delta_S-\eta_D),$$

$$A_3=\frac{1}{10\sqrt{2}}Q_S[L_1\sin(\eta_S-\eta_1)-\tfrac{4}{3}L_0\sin(\eta_S-\eta_0)+\tfrac{1}{3}L_2\sin(\eta_S-\eta_2)],$$

$$A_4=\tfrac{1}{2}\mathfrak{M}_S[\tfrac{4}{3}L_0\sin(\eta_S-\eta_0)+L_1\sin(\eta_S-\eta_1)+(5/3)L_2\sin(\eta_S-\eta_2)]-\frac{1}{8\sqrt{2}}\mathfrak{M}_D[\tfrac{4}{3}L_0\sin(\eta_0-\eta_D)-2L_1\sin(\eta_1-\eta_D)+(14/3)L_2\sin(\eta_2-\eta_D)],$$

$$A_5=[\mathfrak{M}_SQ_D+(3/40)\mathfrak{M}_DQ_S]\sin(\eta_S-\eta_D),$$

$$A_6=\frac{3}{4\sqrt{2}}\mathfrak{M}_S\mathfrak{M}_D\sin(\eta_S-\eta_D), \quad (12)$$

$$A_7=-\frac{1}{2\sqrt{2}}M_D[L_1\sin(\eta_1-\delta_D)-3L_2\sin(\eta_2-\delta_D)],$$

$$A_8=-\frac{1}{2\sqrt{2}}M_DQ_D\sin(\eta_D-\delta_D),$$

$$A_9=\tfrac{1}{6}L_0L_2'\sin(\eta_0-\eta_2),$$

$$A_{10}=\tfrac{1}{12}Q_D[2L_0'\sin(\eta_0-\eta_D)+3L_1'\sin(\eta_1-\eta_D)+7L_2'\sin(\eta_2-\eta_D)],$$

$$A_{11}=\frac{1}{10\sqrt{2}}Q_S[\tfrac{4}{3}L_0'\sin(\eta_S-\eta_0)-L_1'\sin(\eta_S-\eta_1)+(5/3)L_2'\sin(\eta_S-\eta_2)],$$

$$A_{12}=M_PL_1\sin(\eta_1-\delta_P),$$

$$A_{13}=M_PQ_D\sin(\delta_P-\eta_D),$$

where

$$M_D=\gamma\int_0^\infty ww_sdr,\quad \mathfrak{M}_S=\gamma\int_0^\infty uu_tdr,\quad \mathfrak{M}_D=\gamma\int_0^\infty ww_tdr,$$

$$Q_S=\gamma\int_0^\infty r^2wu_tdr,\quad Q_D=\gamma\int_0^\infty r^2uw_tdr,$$

$$L_J'=L_J(w=0),\qquad M_P=\gamma\int_0^\infty ruv_Pdr.$$

It is readily seen from Eqs. (11) and (12) that the numerator of P_{corr} is a simple function of θ, viz.,

$$\sigma_{\text{corr}}P_{\text{corr}}=\mathfrak{A}\sin2\theta+\mathfrak{B}\sin\theta+\mathfrak{C}\cos\theta\sin2\theta, \quad (13)$$

where

$$\mathfrak{A}=c+\alpha qA_2+q\zeta A_5+\zeta^2A_6+\alpha qA_8+\xi A_9+\eta A_{12},$$
$$\mathfrak{B}=\alpha d+qA_3+\zeta A_4+\alpha A_7+\xi qA_{11}, \quad (14)$$
$$\mathfrak{C}=qA_1+\xi qA_{10}+\eta qA_{13}.$$

It should be noted that Eq. (11) is valid for neutrons, with θ having the meaning of the neutron angle. The expression for protons, with θ now having the meaning of the proton angle, is readily obtained by changing signs of A_1, A_2, A_4, A_5, A_8, A_{10}, A_{12} (for the modification of $\tilde{\sigma}_{\text{corr}}$ see Appendix I).

We shall first estimate the corrections given by A_i for the "Los Alamos" set of phase shifts. For the numerical computation of the integrals, the ground state wave functions u and w were taken from Fig. 4(b) of Austern.[3] The final S states were assumed to have the hard core radius $D_0=0.4\times10^{-13}$ cm, as in Sec. III. Keeping the same normalization as in Eq. (8a) and (8b), the following results were obtained: (a) The most important correction to c in $\mathfrak{A}$ is $q\zeta A_5+\zeta^2A_6\approx0.043$, amounting to 20% of c; the remaining terms are negligible (e.g., αqA_2 is about 9.5%, ξA_9 is about 2.5% of c, αqA_8 is about 1.9% of c, and ηA_{12} is still smaller). (b) In the factor $\mathfrak{B}$, the most important correction to αd is $\alpha A_7\approx0.048$, amounting to 19% of αd; the remaining terms are negligible (e.g., ζA_4 is about 0.8% of αd and qA_3 and ξqA_{11} are still smaller). (c) The only important term in the factor $\mathfrak{C}$ is $qA_1\approx0.059$; ξqA_{10} and ηqA_{13} are entirely negligible. A more detailed analysis of qA_1 is given in Sec. V.

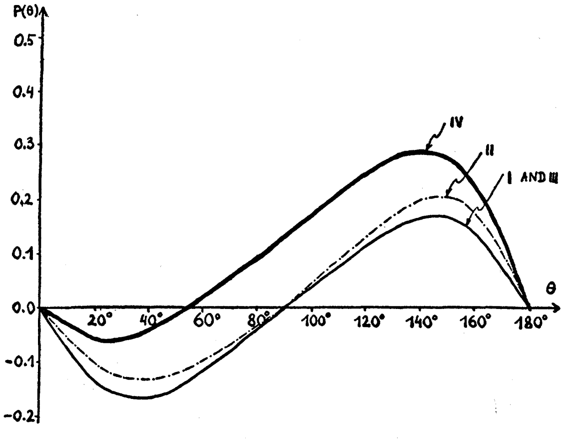

FIG. 1. Polarization of the outgoing neutrons from the $D(\gamma,p)n$ reaction. Curve I represents polarization resulting from $E1$ transitions to $^3P_0$, $^3P_1$, $^3P_2$, and $^3F_2$ final states. Curve II represents the modification introduced by the $qA_1 \cos\theta \sin 2\theta$ term in the numerator of $P(\theta)$. Curve III represents the influence of the $1-2\beta\cos\theta$ factor in the denominator of $P(\theta)$; this factor practically cancels the modification introduced by the term $qA_1 \cos\theta \sin 2\theta$ in the numerator of $P(\theta)$. Curve IV represents $P(\theta)$ as given by Eq. (8a). It illustrates the strong influence of the $M1-E1$ interference term.

The A_i for the set of phase shifts of Clementel and Villi[6] are very similar. Only a certain decrease of αA_7 and increases of ζA_4 and qA_1 should be noted.

V. CONCLUSIONS

As was mentioned before, there are no data on $P(\theta)$ available as yet, and therefore the present short discussion is based on numerical factors obtained in previous sections and plotted in Fig. 1. These results are based on rough estimates of the integrals L_J from Austern's paper and recent phase shifts. There is, however, some inconsistency among the various sets of phase shifts now in the literature. The numerical results of the present analysis are, therefore, only illustrative. These results also illustrate the importance of all transitions up to $E2$ and $M2$. The measurements of $P(\theta)$ at many angles can provide an argument for or against a given set of phase shifts and can provide new equations to determine the L_J and M integrals.

Equations (8a) and (8b) of Sec. III represent the main features of $P(\theta)$ (see Fig. 1). It appears that the "Los Alamos" set of phase shifts[5] gives about the same results for $P(\theta)$ in the entire interval $0<\theta<\pi$ as does the set of Clementel and Villi.[6] The higher order corrections do not introduce essential differences between these two cases.

Let us now consider the most important higher order corrections given by qA_1 ($E1-E2$ interference) and αA_7 ($E1-M1$ interference). The $E1-E2$ interference introduces the factor $qA_1 \cos\theta \sin 2\theta$ in the numerator and for neutrons changes the denominator from $a+b\sin^2\theta$ to $(a+b\sin^2\theta)(1-2\beta\cos\theta)$. For a photon energy of 65 Mev, $2\beta=0.25$, according to Hanson *et al.*[10] The influence of $E1-E2$ interference on $P(\theta)$ is shown in Fig. 1. It is readily seen, however, that the contributions to the numerator and denominator largely cancel each other. This effect does not depend upon the details of the radial integrals and phase shifts involved. It is sufficient that their signs and the sign of the c coefficient remain the same as in the calculations presented above for the cancellation to take place. The term αA_7 ($E1-M1$ interference) can give rise to a correction as large as 19% of αd. It has, however, no important influence on the general character of the curve $P(\theta)$. On the other hand, there is a great uncertainty in our estimates of αA_7 and αd. It seems, therefore, that any data on $P(\theta)$ could provide the following useful information on the $M1$ radial integrals: (1) the $E1-M1$ interference terms give rise to destructive interference in the forward and constructive interference in the backward direction; the deviation from the symmetry of $P(\theta)$ about the point $\theta=90°$ thus indicates the $M1$ contribution; (2) the polarization at $\theta=90°$ is due only to the $E1-M1$ interference:

$$\tilde{\sigma}_{\rm corr}(90°)P_{\rm corr}(90°) =\mathfrak{B}\approx\alpha(d+A_7) =\alpha\Big\{-M_SL_1\sin(\eta_1-\delta_S) -\frac{1}{2\sqrt{2}}M_D[L_1\sin(\eta_1-\delta_D)-3L_2\sin(\eta_2-\delta_D)]\Big\}. \quad (15)$$

Once the integrals $L_{1,2}$ are known, one can in principle determine the $M1$ matrix elements. Estimation of the $M1$ transitions is simpler in the low-energy region. In this case, throughout the interval $0<\theta<\pi$, only the $E1-M1$ interference is important, since the phase shifts of the P states vanish.

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APPENDIX I

The angular distribution factor denoted by $\tilde{\sigma}_{\rm corr}$ in Eq. (11) has the following form:

$$\begin{aligned}\tilde{\sigma}_{\rm corr}=a&+b\sin^2\theta+qB_1\sin^2\theta\cos\theta+q^2B_2\sin^2\theta\cos^2\theta\\&+qB_3\cos\theta+q^2B_4+\zeta B_5\cos\theta+(q\zeta B_6+\zeta^2B_7)\sin^2\theta\\&+q\zeta B_8+\zeta^2B_9+\eta^2B_{10}\cos^2\theta+\alpha\eta B_{11}\cos\theta\\&+\alpha^2B_{12}(3\cos^2\theta-1)+\alpha^2B_{13}(5-3\cos^2\theta)\\&+\alpha\eta B_{14}(3\cos^2\theta-1)\cos\theta+\xi B_{15}\sin^2\theta+\xi B_{16}\\&+\xi qB_{17}\sin^2\theta\cos\theta+\xi qB_{18}\cos\theta, \quad \text{(A1)}\end{aligned}$$

where a and b are the same as in Eq. (5) and

$$B_1=-Q_D[\tfrac{1}{3}L_0\cos(\eta_0-\eta_D)+L_1\cos(\eta_1-\eta_D)+(5/3)L_2\cos(\eta_2-\eta_D)],$$

$$B_2=\tfrac{3}{2}Q_D^2,$$

$$B_3=-\frac{\sqrt{2}}{5}Q_S[\tfrac{1}{3}L_0\cos(\eta_0-\eta_S)-\tfrac{1}{2}L_1\cos(\eta_1-\eta_S)+\tfrac{1}{6}L_2\cos(\eta_2-\eta_S)],$$

$$B_4=(1/25)Q_S^2,$$

$$B_5=-\mathfrak{M}_S[\tfrac{2}{3}L_0\cos(\eta_0-\eta_S)+L_1\cos(\eta_1-\eta_S)+\tfrac{1}{3}L_2\cos(\eta_2-\eta_S)]+\frac{\mathfrak{M}_D}{4\sqrt{2}}[\tfrac{2}{3}L_0\cos(\eta_0-\eta_D)+L_1\cos(\eta_1-\eta_D)+\tfrac{1}{3}L_2\cos(\eta_2-\eta_D)],$$

$$B_6=-(3/20)Q_S\mathfrak{M}_D\cos(\eta_S-\eta_D),$$

$$B_7=\frac{3}{2\sqrt{2}}\mathfrak{M}_S\mathfrak{M}_D\cos(\eta_S-\eta_D)-\tfrac{3}{16}\mathfrak{M}_D^2,$$

$$B_8=(1/20)Q_S\mathfrak{M}_D\cos(\eta_S-\eta_D)+\frac{\sqrt{2}}{10}Q_S\mathfrak{M}_S, \tag{A2}$$

$$B_9=-\frac{1}{\sqrt{2}}\mathfrak{M}_S\mathfrak{M}_D\cos(\eta_S-\eta_D)+\tfrac{3}{2}\mathfrak{M}_S^2+\tfrac{9}{16}\mathfrak{M}_D^2,$$

$$B_{10}=4M_P^2,$$

$$B_{11}=-4M_SM_P\cos(\delta_S-\delta_P),$$

$$B_{12}=-\frac{1}{\cdot}M_SM_D\cos(\delta_S-\delta_D),$$

$$B_{13}=\tfrac{1}{4}M_D^2,$$

$$B_{14}=\sqrt{2}M_PM_D\cos(\delta_P-\delta_D),$$

$$B_{15}=-(7/12)L_2L_2'+\tfrac{1}{3}L_0L_0'\cos(\eta_0-\eta_2)+\tfrac{1}{4}L_1L_1',$$

$$B_{16}=-(2/9)L_0L_0'-\tfrac{1}{2}L_1L_1'+(11/24)L_2L_2'-(1/18)L_0L_0'\cos(\eta_0-\eta_2)-\tfrac{1}{6}L_0L_0'\cos(\eta_0-\eta_2),$$

$$B_{17}=-Q_D[\tfrac{5}{6}L_2'\cos(\eta_2-\eta_D)-\tfrac{1}{3}L_0'\cos(\eta_0-\eta_D)-\tfrac{1}{2}L_1'\cos(\eta_1-\eta_D)],$$

$$B_{18}=-\frac{1}{15\sqrt{2}}Q_S[-2L_0'\cos(\eta_0-\eta_S)+3L_1'\cos(\eta_1-\eta_S)+2L_2'\cos(\eta_2-\eta_S)].$$

Certain B_i are already known: B_1 and B_2 were given by Schiff,[13] Marshall and Guth[14] (for $\eta_2=\eta_D=0$), and Sasaki[15]; B_3 and B_4 were given by Sasaki[15]; and B_{12} and B_{13} were reported by Austern.[16] It is readily seen that $\tilde{\sigma}_{\text{corr}}$ is of the form:

$$\tilde{\sigma}_{\text{corr}}=\mathfrak{A}'(1+\alpha'\cos\theta)+\mathfrak{B}'\sin^2\theta(1+\beta'\cos\theta+\beta''\cos^2\theta).$$

Equation (A1) is valid for neutrons, θ having the meaning of the neutron angle. $\tilde{\sigma}_{\text{corr}}$ for protons, θ now having the meaning of the proton angle, is readily obtained by changing signs of B_1, B_3, B_5, B_{11}, B_{14}, B_{17}, and B_{18}.†

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† *Note added in proof.*—A recent paper by R. E. Marshak and J. J. de Swart (to be published) using the potentials of reference 12 and assuming Siegert's theorem for all the transitions obtains $\tilde{\sigma}$ in agreement with the data. A numerical computation of $P(\theta)$ for this case is in preparation.