greatly different from this ratio for most of the other iron-group halogenides.

Data for MnF_3 are shown in Fig. 2. The Néel point is 47°K with a possible error estimated at 2°. Above 50°K the Curie-Weiss law of the form $\chi_m = 3.10/(T-8)$ is obeyed, in fair agreement with the results of Klemm and Krose[8] for temperatures above 90°K. The increase in χ_m with decreasing temperatures below 20°K indicates that a small part of the MnF_3 may have decomposed, with the decomposition products obeying the Curie law to low temperatures. Measurement of the material not protected from contact with moisture showed that it followed the law $\chi_m = 2.97/T$ to temperatures below 4°K, with a small irregularity, within the experimental error, near 50°K. The results on both of the specimens can be explained by the presence of the order of 90% decomposition in the first specimen and 10% decomposition in the second.

The value of μ_{eff} for MnF_3 is 5.0, to be compared with the value of 4.9 observed near room temperature by Hepworth, Jack, and Nyholm.[9]

It is surprising to find θ_N so much greater than θ_p in absolute magnitude, in this compound.

[8] W. Klemm and E. Krose, Z. anorg. allgem. Chem. **253**, 226 (1947).

ACKNOWLEDGMENTS

We are indebted to Dr. R. G. Shulman for supplying the MnF_3, which was the same as that used in the experiments of Shulman and Jaccarino[10] on the nuclear resonance shift in F^{19}. We are also indebted to Dr. W. C. Koehler and Dr. E. O. Wollan of Oak Ridge National Laboratory for telling us of their neutron diffraction experiments on MnF_3 prior to publication.

Note added in proof.—Examination of a single crystal indicates that the spins are aligned along the c axis, the direction of nearest Cu-Cu neighbors according to the structure determined by Geller and Bond (to be published).

[9] Hepworth, Jack, and Nyholm, Nature **179**, 211 (1957).
[10] R. G. Shulman and V. Jaccarino, Phys. Rev. **109**, 1084 (1958).

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Oscillatory Galvanomagnetic Effects in *n*-Type Indium Arsenide*

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The Hall coefficient and the resistivity of n-type InAs, measured as a function of magnetic field strength at low temperatures, reveal de Haas-van Alphen type oscillations. The period is in good agreement with theoretical predictions. An electron effective mass of the order of 0.02 m_0 is calculated from the field and temperature dependence of the amplitude.

INTRODUCTION

Quantization of the orbital motion of electrons in a magnetic field gives rise to a quasi-periodic variation of the density of states as a function of energy.[1] Consequently the magnetic susceptibility and also the transport phenomena will show an oscillatory behavior as a function of the magnetic field strength. Such oscillations were first discovered in bismuth[2,3] and later in several other metals.[4,5]

A recent study of the magnetoresistance of InSb [6] has shown that this oscillatory behavior can also be observed in degenerate semiconductors. An additional interesting feature in the case of InSb is the fact that the conduction band is isotropic as opposed to the energy bands of all metals studied so far.

We have investigated the possibilities of observing the de Haas-van Alphen oscillations in other semiconductors. The conditions for observation of quantization effects in the transport phenomena are:

$$\omega\tau > 1, \tag{1a}$$

$$\hbar\omega > kT, \tag{1b}$$

$$\zeta_0 > kT, \tag{1c}$$

where $\omega = eH/m^*c$ = cyclotron frequency, τ = collision time, and ζ_0 = Fermi energy in the absence of a magnetic field. In germanium it is difficult to fulfill all three conditions. A high mobility [condition (1a)] requires a rather pure material; in that case, however, the electrons will condense in the impurity levels at low temperatures. It is perhaps possible to obtain germanium which is still degenerate at hydrogen tempera-

* Research supported by the Office of Naval Research.
[1] L. Landau, Z. Physik **64**, 629 (1930).
[2] W. J. de Haas and P. M. van Alphen, Commun. Kamerlingh Onnes Lab. Univ. Leiden **212a** (1930) (susceptibility).
[3] J. Babiskin, Phys. Rev. **107**, 981 (1957) (galvanomagnetic effects; this paper contains a rather complete reference list).
[4] D. Shoenberg, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (Interscience Publishers, Inc., New York, 1957), Vol. 2 (review of oscillatory susceptibility).
[5] See, e.g., N. M. Nachimovich, J. Phys. U.S.S.R. **6**, 111 (1942) (magnetoresistivity of Zn).
[6] H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **108**, 1136 (1957).

ture and has at the same time a sufficiently high mobility; for an effective mass of 0.1 m_0, magnetic fields of more than 20 kilogauss are then required to satisfy condition (1b).

Even higher fields would be needed in the case of silicon because of the higher effective mass.

Another class of semiconductors which could be explored is the lead series (PbS, PbSe, PbTe). These materials have very large mobilities at low temperatures[7] and remain degenerate down to the temperature range of liquid helium. Assuming an average effective mass of[8] 0.3 m_0 and an impurity concentration of 5×10^{16}/cc, one should observe the first oscillation at about 50 kilogauss. The damping of the subsequent oscillations towards lower magnetic field strength would be very large. Such measurements could yield important information concerning the band structure of the lead compounds.

In the group of III-V compounds several members beside InSb should be considered. The effective mass of electrons in GaAs is apparently rather small and the mobility reasonably high.[9] If one can obtain sufficiently pure material, it is possible that oscillatory behavior could be observed.

TABLE I. Sample characteristics.

NOL-5	R cm³/coul	σ ohm⁻¹ cm⁻¹	μ cm²/v-sec
77°K	311	114	35 500
1.7°K	290–310	68.5	21 300[a]

[a] The mobility ($R\sigma$) is calculated on the basis of the Hall coefficient at 77°K.

An obvious material to investigate is of course indium arsenide, because of its similarity to InSb. We have chosen, therefore, to study this semiconductor and we report on the results in the present paper.[10]

EXPERIMENTAL

The samples of indium arsenide used in this investigation were obtained through the courtesy of Mr. J. R. Dickson of the Naval Ordnance Laboratory (NOL). One specimen (NOL-5) was cut from a crystal prepared in the NOL laboratories, the other came from a boule grown at the Battelle Memorial Institute. The purity and properties of these two polycrystalline samples were nearly identical; hence we will discuss only the results obtained on specimen NOL-5.

For details concerning the measuring techniques we refer to a recent paper on InSb.[6] The Hall coefficient and the resistance were measured as a function of magnetic field strength in fields up to 22 kilogauss.[11] Measurements of the Hall coefficient were made at 77°K and 1.7°K, while the magnetoresistance was determined at these two temperatures and also at 4.2 and 3.05°K. Results are shown in Figs. 1–4. Some characteristic values are given in Table I.

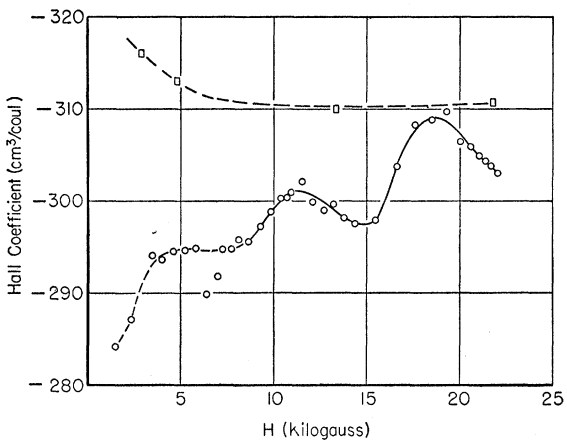

FIG. 1. Hall coefficient at 77°K (□) and 1.7°K (○).

DISCUSSION

Recent theoretical and experimental investigations of InAs [12–14] indicate that the effective mass of conduction electrons at the bottom of the band is of the order of 0.02 or 0.03 m_0. Hence, if $\zeta=\zeta(H)$ is the Fermi energy in the presence of the magnetic field, the value of ζ/kT at 77°K for a sample containing 2×10^{16} carriers

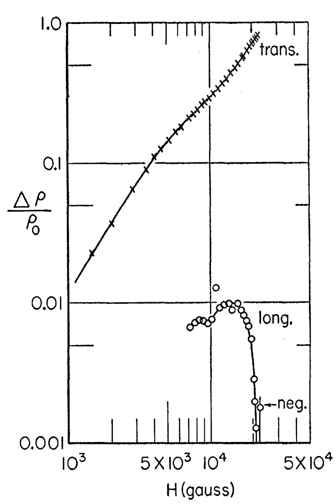

FIG. 2. Magnetoresistance at 77°K.

[7] R. S. Allgaier, Bull. Am. Phys. Soc. Ser. II, **2**, 141 (1957).
[8] T. S. Moss, Proc. Inst. Radio Engrs. **43**, 1860 (1955).
[9] L. Pincherle and J. M. Radcliffe, in *Advances in Physics* (Phil. Mag. Suppl.), edited by N. F. Mott (Taylor and Francis, Ltd., London, 1956), Vol. 5, p. 271.
[10] Just before submitting this article for publication the authors received a preprint of a manuscript by R. J. Sladek describing similar measurements concerning magnetoresistance oscillations in indium arsenide.

[11] Thanks are due the Cryogenics Physics Section of the National Bureau of Standards for making an A. D. Little magnet available for this work.
[12] W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957).
[13] R. J. Sladek, Phys. Rev. **105**, 460 (1957).
[14] F. Stern, Bull. Am. Phys. Soc. Ser. II, **2**, 347 (1957).

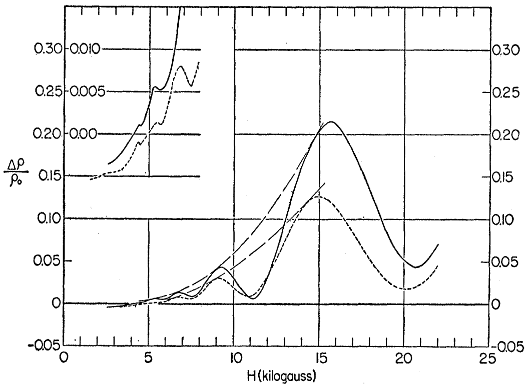

FIG. 3. Magnetoresistance at 1.7°K; ——— transverse, - - - - - longitudinal.

per cc is about 1 or 2, and such a sample is nearly completely degenerate at this temperature. The expected field dependence of the Hall coefficient R[15] is then in agreement with the results shown in Fig. 1: the saturation value at high fields is $1/nec$ and $R_{H=0}$ is somewhat higher, depending on the relative contribution of lattice and impurity scattering.

The magnetoresistance at 77°K, Fig. 2, shows a behavior very similar to that of InSb.[6] At very low fields the transverse effect approaches a quadratic dependence on H; in the region of higher field strength a linear dependence is observed, while at very high fields $\Delta\rho/\rho_0$ again tends to be proportional to H^2. The longitudinal effect is very much smaller, in agreement with the commonly accepted idea of an isotropic conduction band. The somewhat sinuous behavior above 10 kilogauss may very well be attributed to quantization effects; Eq. (1b) will be satisfied when H reaches values of 10 to 15 kilogauss. The highest maximum coincides closely with that observed at helium temperatures.

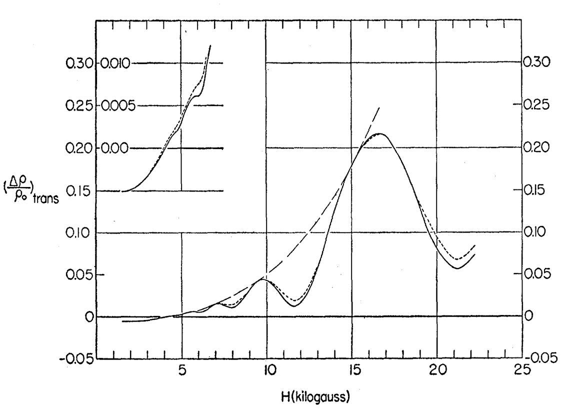

FIG. 4. Temperature dependence of the magnetoresistance; - - - - - 4.2°K, ——— 3.05°K.

[15] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, Cambridge, 1953), p. 239.

Measurements at 4.2°K and below show clearly the oscillatory field dependence of both Hall coefficient and resistance (Figs. 1, 3, and 4). As discussed in reference 6, the minima of the magnetoresistance will occur when $\zeta=(n+\frac{1}{2})\hbar\omega$ or $\zeta_0=a\hbar\omega$ where a has the values 1.33, 2.36, 3.38, 4.40, etc. Knowing the number of carriers, we can then compare the values of $a\zeta_0/\beta^*$ (where β^* is the double effective Bohr magneton, $e\hbar/m^*c$) with the experimentally observed period $(1/H)_{n,\,n+1}$ (see Table II). The agreement is very satisfactory.

The oscillations in the Hall coefficient are also clearly distinguishable (Fig. 1). A comparison with Figs. 3 and 4 indicates a zero phase difference between the Hall effect and the magnetoresistance.

So far we have been concerned only with the period of the oscillations. In order to discuss the amplitude, we need the complete expression for the magnetoresistance. Zilberman[16] has recently derived such a formula pertaining to semimetals at low temperatures, when scattering is due to impurities. For one energy band his expression has the form:

$$\rho=\rho_0\left[1+c\left(\frac{\hbar\omega}{\zeta}\right)-d\left(\frac{\hbar\omega}{\zeta}\right)^{\frac{1}{2}}\left(\frac{2\pi^2kT}{\hbar\omega}\right)\right.$$
$$\left.\times\exp\left(-\frac{2\pi^2kT}{\hbar\omega}\right)\cos\left(\frac{2\pi\zeta}{\hbar\omega}-\frac{\pi}{4}\right)+\cdots\right]. \quad (2a)$$

This result holds under the following assumptions:

$$\exp(2\pi^2kT/\hbar\omega)\gg 1, \quad (2b)$$

$$kT\leqslant\hbar\omega\ll\zeta_0. \quad (2c)$$

The author introduces the impurity scattering of the electrons by means of a perturbation on the periodic potential of the lattice; he assumes that this perturbation can be represented by a δ function.

Lifshitz[17] has used a more sophisticated approach involving the density matrix. He simplifies the treatment considerably by assuming that the scattering

TABLE II. Period of oscillation.

n	$\frac{1}{H_n}$ (long.)	$\left(\Delta\frac{1}{H}\right)_{n,n+1}$ (long.)	$\frac{1}{H_n}$ (trans.)	$\left(\Delta\frac{1}{H}\right)_{n,n+1}$ (trans.)	$a\frac{\beta^*}{\zeta_0}$ (calc.)	$\left(\Delta\frac{1}{H}\right)_{n,n+1}$ (calc.)
	(10^{-5} gauss^{-1}) long.		(10^{-5} gauss^{-1}) trans.		(10^{-5} gauss^{-1}) calc.	
1	5.00		4.83		5.65	
		4.2⁵		4.2		4.51
2	9.26		9.02		10.16	
		4.1⁵		4.2		4.41
3	13.42		13.25		14.57	
		4.3		4.6		4.39
4	17.70		17.87		18.96	
		4.5		4.3		4.32
5	22.20		22.20		23.28	

[16] G. E. Zilberman, J. Exptl. Theoret. Phys. U.S.S.R. **29**, 762 (1955) [translation: Soviet Phys. JETP **2**, 650 (1956)].
[17] I. M. Lifshitz, J. Exptl. Theoret. Phys. U.S.S.R. **30**, 814 (1956) [translation: Soviet Phys. JETP **3**, 774 (1956)].

TABLE III. Values for effective mass, "Dingle temperature," and mobility.

	transv.	long.	
m^*/m	0.018	0.017	0.030,[a] 0.020,[b] 0.024[c]
T_{Av}' (°K)	15.2	16.8	
μ(cm^2/v-sec)	$1.5^7\times10^4$	$1.5^3\times10^4$	2.13×10^4 [d]

[a] Reference 12. [b] Reference 13. [c] Reference 14. [d] See Table I.

mechanism can be described by a classical collision time independent of the magnetic field. His result indicates a different power ($\frac{3}{2}$) of ($\hbar\omega/\zeta$) in the coefficient of the oscillatory term.

Several years ago Dingle[18] discussed the energy level broadening due to collisions. He showed that this broadening influences the damping of the oscillations; the form of the damping term remains the same except that the actual temperature T in the exponential is replaced by a somewhat higher temperature $(T+T')$, where $T'=h/2\pi^2k\tau$ (τ = collision time).

We have analyzed our data in terms of Eq. (2a) in which T is replaced by $T+T'$. It is evident that the coefficient of the cosine in the third term represents the amplitude A of the oscillations. Plotting $\ln(AH^{\frac{1}{2}})$ *vs* $1/H$, one expects a straight line with a slope of $-2\pi^2k(T+T')/\beta^*$.[19] From data at different temperatures T' and β^* can be calculated. Such a plot is shown in Fig. 5 and the results are listed in Table III. The

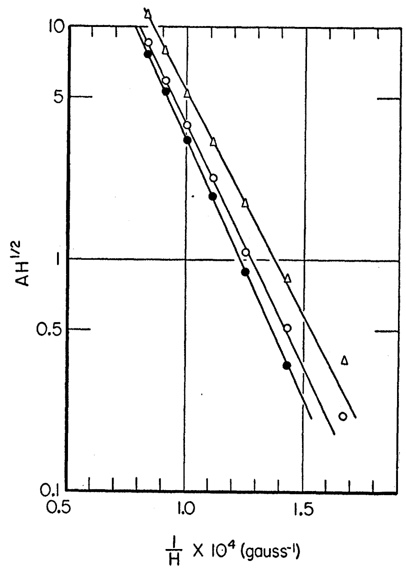

FIG. 5. Magnetic dependence of the amplitude; ● 4.2°K, ○ 3.05°K, △ 1.73°K.

accuracy of the values obtained is small due to the relatively large value of T' (causing very little difference in the slopes). The last column contains for comparison the effective masses reported in earlier experimental and theoretical work and the observed Hall mobility $R\sigma$. The mobility values agree much better than is commonly found in metals.[4]

ACKNOWLEDGMENT

The authors want to thank Dr. A. H. Kahn for helpful discussions.

[18] R. B. Dingle, Proc. Roy. Soc. (London) **A211**, 500 (1952).

[19] It was mentioned before that there is some doubt with regard to the power of H in the proportionality factor of the amplitude. It was impossible, however, to make an unambiguous decision on the basis of our data.

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Lattice Conductivity of Tin*

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The thermal conductivities of several impure tin specimens have been measured at liquid helium temperatures. Impurities of antimony, bismuth, and indium between 0.1% and 6% were used to lower the electronic conductivity. The conductivities are found to be consistent with the equation $K=\alpha T+\beta T^2$, where the first term may be ascribed to the electronic conductivity and the second to the lattice conductivity. For impurities of 3% or less the values of α, combined with measurements of the residual electrical resistivity, agree to about one percent with the prediction of the Wiedemann-Franz law. For the same samples β is approximately constant with a mean value of $(3.5\pm0.4)\times10^{-4}$ watt/cm deg^3. The comparative constancy of this value of β indicates that it is characteristic of the intrinsic conductivity of the tin lattice.

INTRODUCTION

THIS paper will describe and discuss measurements of the thermal and electrical conductivities of several impure tin specimens between 1.7°K and 4.2°K. The investigation was carried out in order to obtain a measure of the lattice contribution to the thermal conductivity of tin.

The theory of thermal conductivity has been extensively treated in the literature[1] and we merely state

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[1] A summary of the theory and an exhaustive bibliography are given by P. G. Klemens, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1956), Vol. 14, p. 198.