Magnetoresistance Oscillations in Single-Crystal and Polycrystalline Indium Arsenide

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Measurements of the electrical resistivity of single-crystal and polycrystalline samples of n-type InAs have been made as a function of magnetic field strength up to 29 000 gauss at temperatures between 1.25°K and 20.2°K.

In both samples oscillations in the resistivity are observed as the magnetic field strength is varied, both when the field is parallel to and when it is perpendicular to the current direction. The oscillations are damped periodic functions of reciprocal field strength. For each sample the period observed in longitudinal and in transverse fields is the same and agrees with that calculated from the electron concentration obtained from the Hall coefficient. The observed phases, at least in the single-crystal sample, agree fairly well with that predicted by theory which is based on a quasi-free electron model.

An exponential field dependence of the amplitude of the oscillations is found, indicating the presence of energy-level broadening that can be characterized by a collision-broadening time which is related to the scattering relaxation time obtained from the mobility in a manner consistent with theory. Collision broadening also accounts for the similarity of oscillations observed in transverse and in longitudinal fields and for the absence of higher harmonics in the magnetoresistance oscillations.

The temperature dependence of the amplitude of the oscillations has been studied in a range of $kT/\hbar\omega$ not usually obtained. Our results for the temperature dependence can be fitted by the theoretically determined form, $x/\sinh x$, where $x=2\pi^2kT/\hbar\omega$, and thereby allow a value for the effective mass of conduction electrons to be determined.

I. INTRODUCTION

A STRONG magnetic field quantizes electron motion in planes perpendicular to the direction of the field. The accompanying change in the electronic energy levels in a solid may give rise to oscillations in the magnetic susceptibility and in the electrical transport properties at low temperatures, where the obscuring effect of thermal vibrations of the lattice is minimized. The oscillations occur as the magnetic field strength is varied and are quasi-periodic in reciprocal field strength.

Oscillatory effects have hitherto been studied almost exclusively in metals, both experimentally and theoretically. In metals only a small fraction of the ordinary conduction electrons, namely those associated with pathological regions of the Fermi (energy) surface can give rise to observable oscillatory effects.[1] In these pathological regions the Fermi surface has a large curvature resulting in a small effective mass for electrons.

The importance of a small mass lies in the fact that only when the mass is almost two orders of magnitude less than the free-electron mass are experimentally available magnetic fields adequate to allow quantization effects to be observed. Specifically, the magnetic quantum $\hbar\omega$ given by

$$\hbar\omega=heB/m^*c, \tag{1}$$

where h is Planck's constant over 2π, ω is the cyclotron angular frequency, e is the electronic charge, B is the magnetic induction, m^* is the effective mass of the electron, and c is the velocity of light, must be large compared to both thermal energy and to energy-level broadening.[2]

Theoretical expressions for the period and phase of oscillatory effects, occuring in strong magnetic fields, can be checked directly by comparison with experimental values only when the energy surfaces of the substance in question are simple enough so that the effective mass approximation is valid, i.e., the electrons are quasi-free. This condition is not satisfied in metals exhibiting oscillatory effects, so that no quantitative check of the theoretical period is possible without independent knowledge of the nature of the Fermi surface. When a value of the phase is deduced from the theory by assuming ellipsoidal energy surfaces, a lack of agreement with experimentally determined values in metals is usually found.[1]

There are substances, however, in which the free-electron model should be a better approximation than it is in metals and which exhibit oscillatory magnetoresistance.[3] These substances are n-type InAs and InSb. They are semiconductors with an energy gap between the valence band and the conduction band large enough so that intrinsic carriers are absent at low temperatures. However, in contrast to most semiconductors the conduction band remains populated with electrons from donor atoms down to the lowest temperatures.[4] In both materials the conduction band seems to be parabolic, at low energies at any rate, and comprised of spherical energy surfaces[5–7] centered about $\mathbf{k}=0$. In addition, the curvature of these energy

[1] See, for example, D. Shoenberg, in *Progress in Low-Temperature Physics* edited by J. C. Gorter (Interscience Publishers, Inc., New York, 1957), Vol. 2, Chap. 8.

[2] R. B. Dingle and D. Shoenberg, Nature **166**, 652 (1950).

[3] H. P. R. Frederikse and W. R. Hosler, Bull. Am. Phys. Soc. Ser. II, **2**, 347 (1957); Phys. Rev. (to be published).

[4] R. J. Sladek, Phys. Rev. **105**, 460 (1957).

[5] H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **108**, 1136 (1957).

[6] E. Kane, J. Phys. Chem. Solids **1**, 249 (1957).

[7] F. Stern, Bull. Am. Phys. Soc. Ser. II, **2**, 347 (1957).

surfaces is very large so that the conduction band electrons have a very small effective mass which is independent of energy, at least up to electron concentrations[7] at which observable oscillatory effects occur. Such electron concentrations ($\sim 10^{17}$ cc^{-1} in InAs) are many orders of magnitude less than the total conduction electron concentration in a metal, and even an order of magnitude less than the concentration of electrons associated with oscillatory effects in metals.

If the conduction band in InAs, or InSb, is parabolic, then according to theory, oscillatory transport effects should exhibit a single period, or at most exhibit harmonics of a single period, as a function of reciprocal field strength, and the value of the period should be determined by the electron concentration. Neither the effective mass of the electrons nor the crystal orientation should affect the period.

In order to test the above ideas, we have made magnetoresistance measurements on n-type indium arsenide between 1.25°K and 20.2°K, employing field strengths up to 29 000 gauss. Two samples, each having a different electron concentration n, were used. One sample was polycrystalline, allowing a novel check of the independence of magnetoresistance oscillations of crystalline direction in n-InAs. Some Hall effect measurements were also made at liquid helium temperatures in order to determine the electron concentration and to see if oscillations occur in the Hall coefficient.

A brief outline of the presentation we shall use is as follows. In Sec. II we shall present (1) pertinent magnetoresistance formulas given by transport theory and (2) some theoretical predictions of the effect of energy level broadening on the magnetic susceptibility, since available transport theory does not take this into account. Some comments on experimental details will be made in Sec. III. Then our experimental results will be presented, discussed, and analyzed in Sec. IV. Our conclusions will be presented in Sec. V.

II. THEORY

A. Transport Theories

The theory of oscillatory transport phenomena has been worked out in detail only recently.[8–11] The cases considered by this theory are those in which scattering of the electrons is due to acoustic lattice vibrations[8,9] or to imperfections having delta-function potentials.[10,11] In neither case is the effect of energy-level broadening considered explicitly.

Comparison of the results of the theory for the two cases indicates that the general nature and the temperature dependence of magnetoresistance oscillations are insensitive to the particular type of scattering involved. Thus the theoretical results should be useful for analyzing our experimental data even though in our samples scattering is due to ionized impurities rather than to the types of scattering centers considered by the theory.

In this work we shall use explicit expressions for the magnetoresistance given by the transport theory of Argyres[8,9] to aid in analyzing our results. His theory applies to the case of acoustic lattice scattering. While this is not the case which occurs in our samples, where ionized impurity scattering predominates, the other case treated by theory,[10] which is for scattering by imperfections, is less applicable to our results because delta function potentials are used for the imperfections, while the potential of an ionized impurity has a long range.

For the conductivity in the presence of a longitudinal magnetic field, Argyres[8] gives

$$\frac{\sigma_H}{\sigma_0}=\left[\int_{\frac{1}{2}\hbar\omega}^{\infty}\frac{\sum_N[\epsilon-(N+\frac{1}{2})\hbar\omega]^{\frac{1}{2}}}{\sum_N[\epsilon-(N+\frac{1}{2})\hbar\omega]^{-\frac{1}{2}}}\frac{\partial f_0}{\partial\epsilon}d\epsilon\right]\Big/\int_0^{\infty}\frac{1}{3}\epsilon\frac{\partial f_0}{\partial\epsilon}d\epsilon, \tag{2}$$

where ϵ is electron energy and $f_0=1/[e^{(\epsilon-\zeta)/kT}+1]$ with $\zeta=\zeta_H$ in the presence of a magnetic field and $\zeta=\zeta_0$ in the absence of a magnetic field. The sums are over the quantized levels produced by the magnetic field which are below the Fermi energy, ζ_H. For arbitrary $\hbar\omega/kT$ but for $\zeta_H\simeq\zeta_0\gg\hbar\omega$, Eq. (2) yields

$$\frac{\sigma_H}{\sigma_0}\simeq 1-\pi^2\sqrt{2}\frac{kT}{\hbar\omega}\left(\frac{\hbar\omega}{\zeta_0}\right)^{\frac{1}{2}}\sum_{r=1}^{\infty}(-1)^r r^{\frac{1}{2}}\frac{\cos[(2\pi r\zeta_0/\hbar\omega)-\frac{1}{4}\pi]}{\sinh(2\pi^2 rkT/\hbar\omega)}, \tag{3}$$

where T is the absolute temperature. In the case of a transverse field Argyres[9] gives for the current density, in the plane perpendicular to the magnetic field,

$$J_x+iJ_y=\left(-\frac{e^2}{m}\right)(E_x+iE_y)\int_{\hbar\omega/2}^{\infty}\frac{\alpha(\epsilon)}{\tau^{-1}(\epsilon)-i\omega}\left(\frac{f_0(\epsilon+\hbar\omega)-f_0(\epsilon)}{\hbar\omega}\right)d\epsilon, \tag{4}$$

[8] P. N. Argyres, J. Phys. Chem. Solids 4, 19 (1958).
[9] P. N. Argyres, Phys. Rev. **109**, 1115 (1958). This paper also discusses previous theoretical work on the effect of quantization on transport properties in metals.
[10] G. E. Zilberman, Zhur. Eksptl. i Teoret. Fiz. S.S.S.R. **29**, 762 (1955) [translation: Soviet Phys. JETP **2**, 650 (1956)].
[11] I. M. Lifshitz and A. M. Kosevich, J. Phys. Chem. Solids 4, 1 (1958).

with

$$\alpha(\epsilon)=\frac{(2m^*/\hbar^2)^{\frac{3}{2}}}{(2\pi)^2}(\hbar\omega)^2\sum_N(N+1)[\epsilon-(N+\tfrac{1}{2})\hbar\omega]^{-\frac{1}{2}}, \tag{5}$$

$$\tau^{-1}(\epsilon)=\tfrac{1}{2}\tau_0^{-1}\epsilon^{-\frac{1}{2}}\hbar\omega\{\tfrac{1}{2}(\epsilon+\tfrac{1}{2}\hbar\omega)^{-\frac{1}{2}}+\textstyle\sum_N[\epsilon-(N+\frac{1}{2})\hbar\omega]^{-\frac{1}{2}}\}, \tag{6}$$

where τ_0 is the momentum relaxation time in zero magnetic field. In Eq. (4), E_x and E_y are electric field strengths in the x and y directions, respectively. Because of the "well function" $[f_0(\epsilon+\hbar\omega)-f_0(\epsilon)]/\hbar\omega$ in Eq. (4) rather than the sharper $df_0/d\epsilon$, which usually occurs in transport integrals, an analytical expression for the resistivity can be obtained only for the case $\hbar\omega\gg kT$. In this case the oscillatory part of the resistivity becomes, provided also that $\zeta_H\approx\zeta_0\gg\hbar\omega$,

$$(\rho_t)_{\text{oscill}}\approx\rho_0\left\{\frac{7\sqrt{2}}{16\pi}\left(\frac{\hbar\omega}{\zeta_0}\right)^{\frac{3}{2}}\sum_{r=1}^{\infty}\frac{(-1)^r}{r^{\frac{3}{2}}}\sin\left(\frac{2\pi\zeta_0 r}{\hbar\omega}-\frac{\pi}{4}\right)\right\}. \tag{7}$$

According to Eqs. (3) and (7), the magnetoresistance oscillations due to either longitudinal or transverse magnetic fields consist of a number of harmonics, each of which is periodic in reciprocal field strength, and the periods of corresponding harmonics for both field orientations are the same. When only the first harmonic is large, as seems to be true in the samples we measured, the period is given by

$$\Delta\left(\frac{1}{B}\right)=\frac{e\hbar}{\zeta_0 m^* c}=\frac{3.18\times10^6}{n^{\frac{2}{3}}}\text{ gauss}^{-1}, \tag{8}$$

where n is the electron concentration in cm^{-3}. The last equality in (8) holds when the electrons occupy a single parabolic band.

When ζ_H is not $\gg\hbar\omega$, the period becomes slightly field-dependent at field strengths large enough so that $\zeta_H\sim\hbar\omega$, although, even in the latter event, the field dependence is small. To show this we note that ζ_H varies somewhat with field strength according to the relation[8]

$$\zeta_0^{\frac{3}{2}}=\tfrac{3}{2}(\hbar\omega)^{\frac{3}{2}}\sum_{N=0}^{\bar{N}}\left[\frac{\zeta_H}{\hbar\omega}-(N+\tfrac{1}{2})\right]^{\frac{1}{2}}, \tag{9}$$

where $\bar{N}$ is the largest integer designating the oscillator level for which $\zeta_H>(\bar{N}+\frac{1}{2})\hbar\omega$. Next, we use Eq. (9) to calculate the value of $\zeta_0/\hbar\omega$ for field strengths at which ζ_H coincides with an oscillator level. The differences between successive values of $\zeta_0/\hbar\omega$ and hence the period is found to increase with decreasing $\bar{N}$, the period reaching a value about 5% larger than that given by Eq. (8) for the last oscillation, i.e., for ζ_H going from the $N=2$ to $N=1$ oscillator level.

There is another effect which might, in general, cause the period of oscillatory effects to be field-dependent. This effect is a nonuniform spacing between oscillator levels, i.e., ω is energy dependent. However, it arises only when the band is not parabolic,[12] up to the Fermi energy at least, or in narrow bands at energies far from the band edge.[13]

B. Effect of Energy-Level Broadening

There is no rigorous theory available for the effect of energy-level broadening on oscillatory transport phenomena. However, the ultimate cause of both steady-state and equilibrium oscillatory phenomena is the same, namely, the quasi-periodic character of the density of electronic energy states due to the quantization of electronic motion by the magnetic field. Thus theoretical predictions about an equilibrium oscillatory phenomenon should also be reflected in steady-state oscillatory phenomena. Indeed a close correspondence has been found experimentally between the oscillations which occur in the magnetic susceptibility and in the magnetoresistance in the case of metals,[14] and some theoretical results[11] for oscillatory electrical transport phenomena have been expressed in terms of the oscillatory magnetic susceptibility, thereby presumably incorporating the effect of energy level broadening via its effect on the magnetic susceptibility.

Thus, in view of their possible applicability to oscillatory magnetoresistance, we shall present here some results of a theory by Dingle[15] for the effect of collision broadening on the magnetic susceptibility. This type of energy-level broadening arises because electrons make transitions from one quantized orbit to another due to collisions with scattering centers, which are most likely impurities at the low temperatures required for observation of oscillatory phenomena. Dingle expresses the effect of collision broadening in terms of the mean time for interorbit transitions which he calls the collision broadening time, τ. Specifically, using a semiclassical derivation of the probability of an electron possessing a certain energy in the presence of scattering, he finds that a factor of

$$e^{-2\pi r/\omega\tau}, \tag{10}$$

[12] Keyes, Zwerdling, Foner, Kolm, and Lax, Phys. Rev. **104**, 1804 (1956).

[13] P. G. Harper, Proc. Phys. Soc. (London) **A68**, 874, 879 (1955); A. D. Brailsford, Proc. Phys. Soc. (London) **A70**, 275 (1957).

[14] For reference up to 1955, see M. C. Steele, Phys. Rev. **99**, 1751 (1955).

[15] R. B. Dingle, Proc. Roy. Soc. (London) **A211**, 500 (1952).

TABLE I. Characteristics of our n-type indium arsenide specimens.

Sample	Crystalline form	Orientation	Electron[a] concentration (cc^{-1})	Zero-field Hall mobility at 1.25°K ($cm^2\ v^{-1}\ sec^{-1}$)
S-1	Single crystal	Cut from (111) slice	7.6×10^{16}	16 150
P-1	Polycrystalline		2.8×10^{16}	25 800

[a] Obtained from the Hall coefficient, R_H, at lowest temperatures by using the relation $n=1/R_H ec$.

where $\omega=eB/m^*c$, τ is the collision broadening time, and r is the number of harmonic in question, occurs in the oscillatory part of the susceptibility. A quantum-mechanical treatment of energy-level broadening, using somewhat different assumptions than in the semiclassical treatment, requires that τ in (10) be replaced by 2τ.[15] The assumption of a small continuous perturbation in the quantum-mechanical treatment would seem to be more applicable to screened Coulomb scattering than the assumption of a sudden large perturbation which is used in the semiclassical derivation.

For the case of metals, Dingle states that the collision broadening time is not to be identified with the scattering relaxation time which determines the electron mobility, since in metals there are current carriers of larger mass in addition to the low-mass electrons which give rise to observable oscillatory phenomena. However, even if all carriers have the same mass, Argyres[16] has found that these two times are in general different when the scattering probability is anisotropic as it is in the case of ionized impurity scattering, for example.

In the case of degenerate semiconductors having a single parabolic band, we can hope to derive some information about the relation of the collision broadening and momentum relaxation times from experimental magnetoresistance results, since both our results and those of Frederikse and Hosler[3] indicate the presence of a single collision-broadening exponential with $r=1$.

Recently the effect of another type of energy-level broadening on the magnetic susceptibility has been considered theoretically.[13] According to the theory, this type of broadening arises because of the presence of the periodic potential of the lattice and might also cause an exponential damping of susceptibility oscillations. However, according to the theory, this lattice broadening is appreciable only when the conduction band is filled up to energies where it is no longer parabolic. Hence we do not expect to see any effect due to such broadening in our experimental results.

III. EXPERIMENTAL DETAILS

A. Specimens

N-type indium arsenide for the specimens was provided by Dr. H. Welker of the Siemens-Schuckertwerke Research Laboratory and Mr. O. Lindberg of the Materials Engineering Department, Westinghouse Electric Corporation. The specimens were about 10×3×1 mm in size and were lapped and etched. Two current leads and four potential leads of No. 36 or No. 40 copper wire were attached to the specimens with spectroscopically pure tin as a solder. The potential leads were located so as to avoid the end effects found by Weiss.[17]

Table I identifies the samples and gives some of their pertinent properties. As a rough estimate of the homogeneity of the donor distribution, we note that excellent agreement was obtained between the Hall coefficients obtained from two different sets of leads on a given sample.

B. Apparatus and Techniques

Conventional low-temperature techniques using liquid helium and liquid hydrogen were employed.

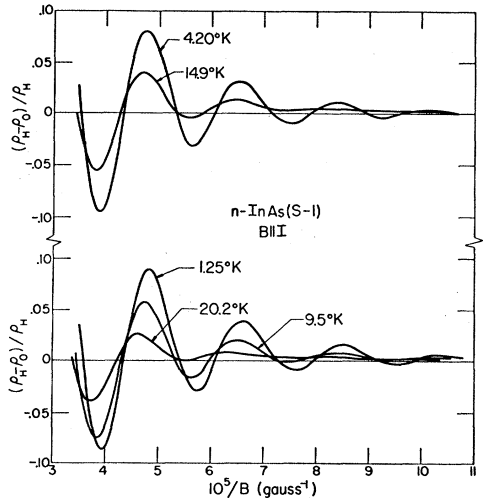

FIG. 1. Dependence of the resistivity of single-crystal n-type InAs, sample S-1, on longitudinal magnetic field strength at low temperatures. Curves of relative resistivity change $(\rho_H-\rho_0)/\rho_H$ *versus* reciprocal field strength are given for five temperatures between 1.25°K and 20.2°K. Data points were taken at field strengths 500 gauss or less apart and have been omitted to avoid clutter.

Magnetic fields up to 29 000 gauss were provided by an Arthur D. Little Electromagnet and measured with a Rawson rotating coil fluxmeter. Care was taken to measure the field strength at exactly the same time as the resistance since no automatic regulator was available to keep the field constant.

Specimen current and potentials were measured by means of a potentiometer galvanometer setup.

C. Method of Determining the Resistance

To determine the resistance the current was kept constant, at a known value of the order of 50 ma, and

[16] P. N. Argyres (private communications).

[17] H. Weiss, Z. Naturforsch. Pt. **12a**, 80 (1957).

the voltage across a pair of resistance leads was measured at intervals of usually 500 gauss, for first one direction of the magnetic field, and then for the opposite direction of the field. The values obtained for opposite directions were averaged to give the true resistance voltage. This method limits the accuracy of our resistance determinations to something like a few tenths of 1% while their precision is of course much better than this.

D. Hall Effect Errors

Hall effect measurements which we made are accurate to a few percent. The uncertainty is determined by the accuracy of the calibration the fluxmeter used. The precision of the Hall voltage *versus* field is greater than this since for any given set of measurements the relative fluxmeter readings are much more precise than accurate.

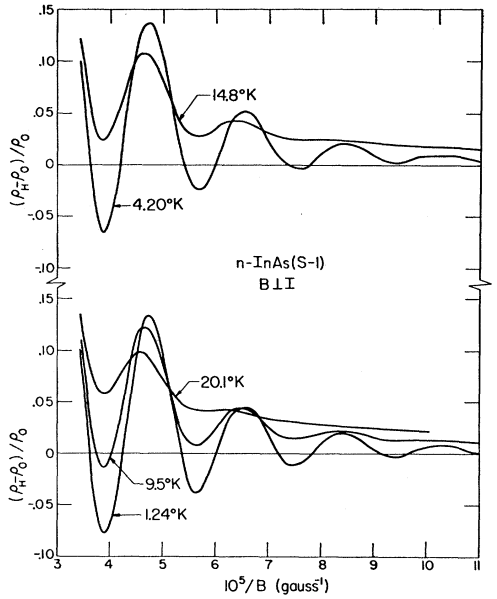

FIG. 2. Dependence of the resistivity of single-crystal *n*-type InAs, sample *S*-1 on transverse magnetic field strength at low temperatures. Curves of relative resistivity change $(\rho_H-\rho_0)/\rho_0$ *versus* reciprocal field strength are given for five temperatures between 1.25°K and 20.2°K. Data points were taken at field strengths 500 gauss or less apart and have been omitted to avoid clutter.

IV. RESULTS

A. Experimental Data

Our magnetoresistance data, for temperatures between 1.25°K and 20.2°K, are summarized in Figs. 1 to 4. Note that the abscissas are reciprocal field strength. Figures 1 and 2 pertain to the single crystal sample *S*-1, and Figs. 3 and 4 to the polycrystalline sample *P*-1. Longitudinal magnetic field data are presented in Figs. 1 and 3 and transverse field data in Figs. 2 and 4. (By longitudinal and transverse fields we mean that the magnetic field is respectively parallel to, or perpendicular to, the long sample dimension and thus current direction.)

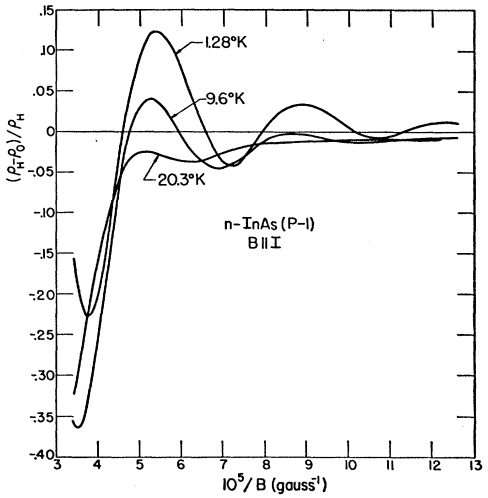

FIG. 3. Dependence of the resistivity of polycrystalline *n*-type InAs, sample *P*-1, on longitudinal magnetic field strength at low temperatures. Curves of relative resistivity change $(\rho_H-\rho_0)/\rho_H$ *versus* reciprocal field strength are given for three temperatures between 1.28°K and 20.2°K. Curves for 4.2°K and 15.2°K were also obtained but are not shown. Data points were taken at field strengths 500 gauss or less apart and have been omitted to avoid clutter.

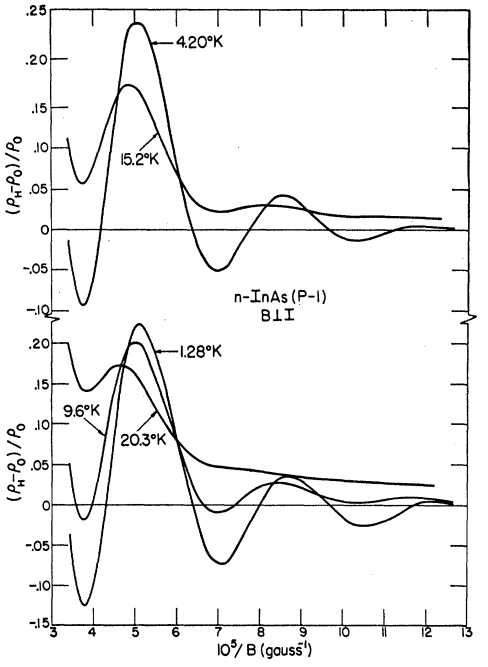

FIG. 4. Dependence of the resistivity of polycrystalline *n*-type InAs, sample *P*-1 on transverse magnetic field strength at low temperatures. Curves of relative resistivity change $(\rho_H-\rho_0)/\rho_0$ *versus* reciprocal magnetic field strength are given for five temperatures between 1.28°K and 20.2°K. Data points were taken at field strengths 500 gauss or less apart and have been omitted to avoid clutter.

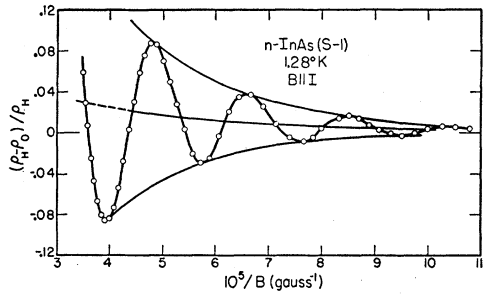

FIG. 5. Graphical analysis of the magnetoresistance of sample S-1 at 1.25°K due to longitudinal magnetic fields.

Hall effect data which we took will not be presented here since no large oscillations were observed in the Hall coefficient. Some poorly defined oscillations did occur in the Hall voltage, corresponding to a maximum amplitude of oscillation in the Hall coefficient of 1%. We note that Argyres' theory[9] predicts no oscillations in the Hall coefficient, at least in the limit of very large $\omega\tau$.

B. Discussion of the Magnetoresistance Oscillations

From Figs. 1–4 we can see that the resistivities of both samples exhibit oscillations which seem to be periodic in reciprocal field strength and whose amplitudes are dependent on temperature and magnetic field strength, with the temperature dependence becoming very small at liquid helium temperatures. There is a nonzero mean magnetoresistance component, also, which we shall not discuss in this work.

1. Period and Phase

In order to determine the period of the oscillatory part of the magnetoresistance, we need to separate out any nonoscillatory component which may be present. This was done graphically as follows, with a particular case being illustrated in Fig. 5. Smooth curves were drawn through plots of the data, expressed as relative change in resistivity, $\Delta\rho/\rho$, *versus* the reciprocal of the magnetic field strength. Next, envelope curves were drawn tangent to the curves through the data points. Finally, halfway between the envelopes a curve was drawn which presumably represents the nonoscillatory part of $\Delta\rho/\rho$, or the mean $\Delta\rho/\rho$. The nodes of the oscillatory part of $\Delta\rho/\rho$ are given by the intersection of the curves through the data points with this mean $\Delta\rho/\rho$ curve.

Then to find the period and phase, at a given temperature, we plotted the field strengths at which the nodes of the oscillations occur *versus* half integers. (See Fig. 6, for example.) If this plot is a straight line, as we expect, and as actually happens, the slope will give the period and the intercept, i, allows the phase, ϕ, to be determined from the relation

$$\phi=(2i+\tfrac{1}{2})\pi, \tag{11}$$

which is obtained from Eq. (3) by replacing $-\frac{1}{4}\pi$ by ϕ.

We have used relation (11) to determine the phase in the case of transverse fields as well as in the case of longitudinal fields even though Eq. (7), given by Argyres' theory for the transverse field case, would lead us to a different relation between the intercept and the phase.

Nevertheless, the reason why we use Eq. (11) for transverse fields is that the nodes of the magnetoresistance oscillations observed in transverse fields occur at almost exactly the same places as those observed in longitudinal fields. The identity of the nodal positions is due, we believe, to the effect of collision broadening. This may be seen qualitatively as follows. If a collision-broadening exponential having a broadening time τ which increases with energy is inserted into the transport integral for transverse fields, given by Eq. (4), it causes most of the contribution to the integral to be from energies in the neighborhood of the Fermi energy rather than from all energies between $\zeta_H-\hbar\omega$ and ζ_H as specified by the $[f(\epsilon+\hbar\omega)-f(\epsilon)]/\hbar\omega$ factor in Eq. (4). Thus inclusion of collision broadening reproduces to some extent, at least, the action of the $df_0/d\epsilon$ factor in the longitudinal-field case.

Table II gives the periods and phases obtained by the above method for both samples in both transverse and longitudinal fields at a number of temperatures. The calculated periods, obtained using Eq. (8) with the electron concentration determined from the Hall coefficient at lowest temperatures for the sample in question (see Table I), are also given.

The accuracy of all entries in Table II is not the same, those at higher temperatures being less accurate

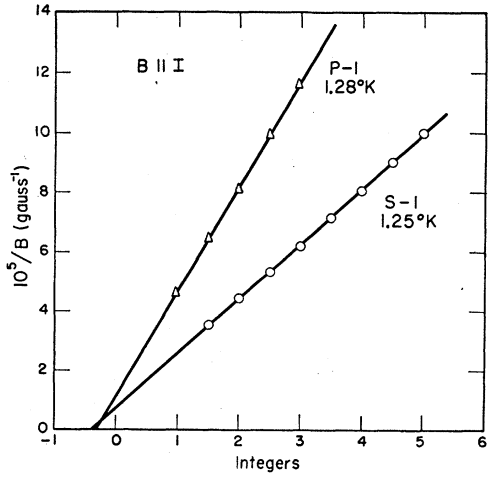

FIG. 6. Positions of the nodes of the magnetoresistance oscillations in samples S-1 and P-1 plotted *versus* integers. The magnetic field direction is longitudinal. The period and phase of the oscillations in each sample are obtained from the slope and intercept respectively of each straight line. (See text.)

TABLE II. Period and phase of magnetoresistance oscillations in n-InAs.

Sample	Calc. period (gauss^{-1})	T (°K)	Observed period $B\|I$ (gauss^{-1})	Observed period $B\perp I$ (gauss^{-1})	Observed phase $B\|I$	Observed phase $B\perp I$
S-1	$1.7_7\times10^{-5}$	1.25	1.85×10^{-5}	1.84×10^{-5}	-0.25π	-0.20π
		4.20	1.79	1.85	-0.32π	-0.12π
		9.54	1.87	1.82	-0.05π	-0.20π
		14.9	1.90	1.78	$+0.08\pi$	-0.34π
		20.2	1.82	1.81	-0.14π	-0.20π
P-1	3.4_2	1.28	3.44	3.50	-0.10π	-0.14π
		4.20	3.39	3.50	-0.18π	-0.06π
		9.65	...	3.50	...	0
		15.25	...	3.52	...	0.04π

because, at higher temperatures, the oscillations are smaller in amplitude so that fewer of them can be analyzed well enough to locate the nodes. Apparently this causes more trouble in determining the phase than in determining the period, as might be expected from the fact that the phase is related to the intercept of the nodes *versus* integer plot by an additive constant, see Eq. (10), while the period is given directly by the slope of such a plot. In sample P-1 the situation is worse, especially in longitudinal fields, than in sample S-1 and is the reason for the missing entries in Table II. The fact that we are dealing with lower oscillator levels in sample P-1 may be partly responsible. It could be connected with the fact that P-1 is polycrystalline.

From Table II we see that for each sample the observed period is definitely independent of temperature and field direction, at least in the temperature range where the period can be determined. The observed period, in each sample, agrees very well with the calculated period. This agreement is evidence for the validity of the assumptions used to calculate the period, namely a single parabolic conduction band and the electron concentration is that given by the Hall effect.

The observed values of the phase are not very constant. However, the most accurate values, i.e., those for lowest temperatures, are in fair agreement with the value of $-\frac{1}{4}\pi$ predicted by theory. This is satisfying since this theoretically predicted phase is based on the assumption of quasi-free electrons, and if this condition is ever to be realized in real substances, it should be in a material like InAs.

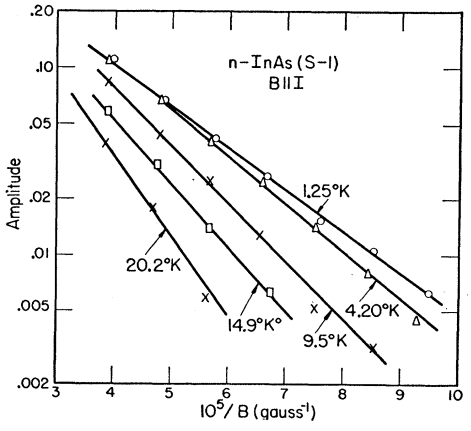

FIG. 7. Dependence of the amplitude of the resistivity oscillations in sample S-1 upon longitudinal magnetic field strength at various low temperatures. The slopes depend on the effective electron mass, the collision broadening time, and the temperature. (See text.)

2. *Field Dependence of the Amplitude*

Next we shall discuss the amplitude of the magnetoresistance oscillations. First we note that the oscillations are somewhat larger in transverse fields than in longitudinal fields. (See Figs. 1–4.) This observation disagrees with the transport theory of Argyres which predicts smaller oscillations in transverse fields than in longitudinal fields. [Compare Eqs. (3) and (7) in Sec. II.] Furthermore, we observe a stronger dependence of the amplitude of the oscillations on magnetic field strength than is predicted by transport theory.[8–10] However, these discrepancies could be reconciled, we believe, if the effect of collision broadening were included in transport theory. The evidence for this view lies in how well the observed field dependence of the oscillation amplitudes can be explained in terms of collision broadening, and it will be presented below.

To study the field dependence of the oscillation amplitudes, we first plot the logarithm of the amplitude for a given sample at a fixed temperature *versus* reciprocal field strength. (See Figs. 7 and 8 for the longitudinal-field case.) A very good straight line results for each of a number of temperatures with the slope of the line depending on temperature, especially at the higher temperatures. In order to explain the occurrence and slopes of the lines for different temperatures, we need to include an exponential damping factor, presumably due to collision broadening. Specifically, the field dependence of the amplitude, A, can be summarized by the relation[3,5]

$$A(T=\text{const})\sim\exp[-2\pi^2k(T+T')/\hbar\omega], \quad (12)$$

where k is Boltzmann's constant, $T'=\hbar/\pi k\tau$ with τ the semiclassical collision broadening time,[15] and $\omega=eB/m^*c$. We shall see later that the temperature

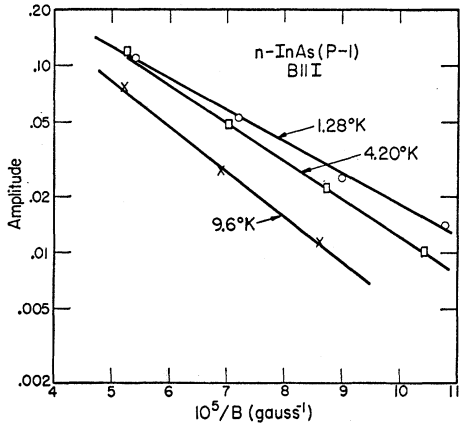

FIG. 8. Dependence of the amplitude of resistivity oscillations in sample P-1 upon longitudinal magnetic field strength at various low temperatures. The slopes depend on the effective electron mass, the collision broadening time, and the temperature. (See text.)

dependence is more properly represented by a different function which, however, at high enough temperatures yields an exponential temperature dependence like that included in Eq. (12). The reason that Eq. (12) can represent our experimental results fairly well at liquid helium temperatures is that T' turns out to be large enough to make the inclusion of T unimportant in that temperature range. This was of course kept in mind when we decided to use Eq. (12) to summarize the field dependence.

If we regard the effective mass as known, or use the value obtained in the next section, we can obtain a value of T', and hence the collision broadening time, from the slope of each log amplitude *versus* $1/B$ curve. The value of T' for a given sample would presumably be independent of temperature if the collision broadening time can be identified with the relaxation time occurring in the mobility, since the mobility is independent of temperature. The values which we find for T' from the slopes assuming an effective mass of $0.02m_0$ are given in Table II. We also include the value of a quantity called T_μ' for each sample in Table II. The values of T_μ' are calculated by assuming that the relaxation time in the mobility is the same as the semiclassical collision-broadening time. Thus

$$T_\mu' = (\hbar/\pi k)(e/m^*\mu), \tag{13}$$

where μ is the mobility.

TABLE III. The collision broadening temperature, T_{obs}' obtained from the observed field dependence of the amplitude of the magnetoresistance oscillations, and T_μ' calculated from the zero-field Hall mobility at 1.25°K using Eq. (13).

Sample	T_μ' (°K)	Temp. (°K)	T_{obs}' (°K) $B\|\|I$	$B\perp I$
S-1	13.2	1.24	16.2	16.8
		4.20	16.9	14.7
		9.54	14.7	16.3
		14.8	11.9	14.8
		20.2	13.7	
P-1	8.28	1.28	11.9	12.6
		4.20	11.7	...
		9.65	9.2	10.1

From Table III we can see that the collision-broadening "temperatures," T_{obs}', deduced from the observed field dependence of the amplitude of our magnetoresistance oscillations, are approximately constant for a given sample and are approximately equal to that calculated from the mobility, T_μ', for the sample in question. Specifically, we note that according to the quantum-mechanical derivation of collision broadening, T_{obs}' should be equal to $\tau_\mu T_\mu'/2\tau_b$, where τ_μ is the momentum relaxation time and τ_b is the collision-broadening time. Evaluating the theoretical formulas

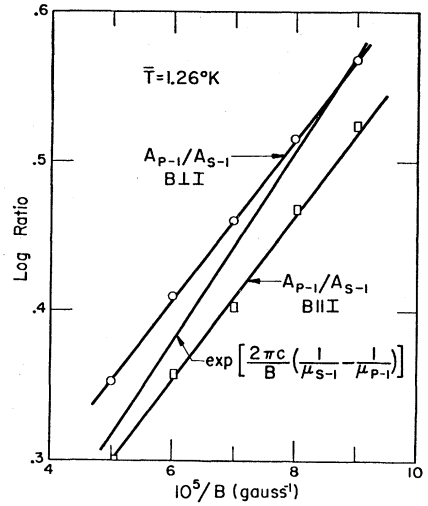

FIG. 9. Comparison of the ratio of the oscillation amplitudes in sample P-1 and S-1 in both longitudinal and transverse magnetic fields with the ratio of collision broadening exponentials for the two samples. The collision broadening times in these exponentials have been replaced by the relaxation times deduced from the mobilities, i.e., $m^*\mu/e$.

for[18] τ_μ and[16] τ_b for the case of ionized impurity scattering by using the electron concentrations present in our samples, we find $\tau_\mu/\tau_b \approx 3$, thus requiring T_{obs}' to be about $\frac{3}{2}$ times T_μ'. From Table III we note that T_{obs}'/T_μ' is about 1.2 for sample S-1 and about 1.3 for sample P-1.

Now to consider the relative sizes of the oscillation amplitudes in the two different samples. In view of the foregoing results, collision broadening should control the ratio of the amplitudes in our samples. To test this we plotted the logarithm of the amplitude ratio *versus* reciprocal field strength, for both transverse and longitudinal fields, in Fig. 9. A straight line results for each of the two field orientations which agrees quite well with the ratio of the collision-broadening exponentials for the two samples as given by the function

[18] P. N. Argyres and E. N. Adams, Phys. Rev. **104**, 900 (1956).

$\exp[-(2\pi c/B)(1/\mu_{P\text{-}1}-1/\mu_{S\text{-}1})]$, where B is the field strength, c is the velocity of light, and the μ's are the zero-field Hall mobilities of the samples at 1.25°K. Available transport theory, which neglects collision broadening, predicts no field dependence for the amplitude ratio, in sharp contrast to our results.

3. *Temperature Dependence*

Finally, we shall consider the temperature dependence of the amplitude of the magnetoresistance oscillations. The temperature range of our measurements is such that $2\pi^2kT$ is not always large compared to $\hbar\omega$. Thus the temperature dependence is not expected to be given simply by an exponential function of $-1/T$, as in metals for example, and the higher harmonics which are predicted by transport theory must be damped out by other means than by the occurrence of an exponential function of $-r/T$ in the rth harmonic. The other means

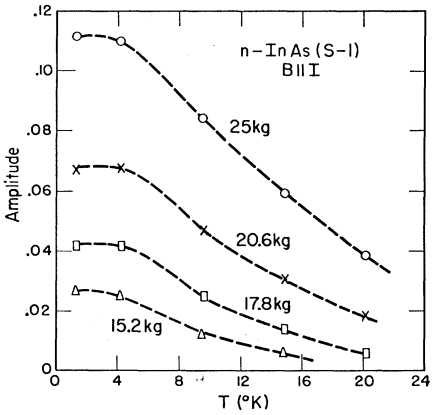

FIG. 10. Dependence of the oscillation amplitudes in single-crystal n-InAs, sample S-1, on temperature in the case of longitudinal magnetic fields. Each set of points gives the oscillation amplitude at a particular field strength.

is undoubtedly collision broadening, since Dingle's exponential collision-broadening factor does indeed provide successively stronger exponential damping of the higher harmonics in the susceptibility, and this present work and that of other authors[3] provide evidence that the first harmonic of magnetoresistance oscillations are exponentially damped.

The observed amplitudes of the magnetoresistance oscillations for various field strengths are plotted *versus* temperature for sample S-1 in Figs. 10 and 11. The results for sample P-1 are similar, but less complete, and will not be presented explicitly. They are of course contained in the "data" plots given in Figs. 2 and 3. For both samples the amplitudes are just about independent of temperature at liquid helium temperatures. In order to see if our data would fit the theoretical temperature dependence, given by Eq. (3) in Sec. II, and if so to deduce an effective mass, we plotted curves of $x/\sinh x$ *versus* x with different abscissa scales but

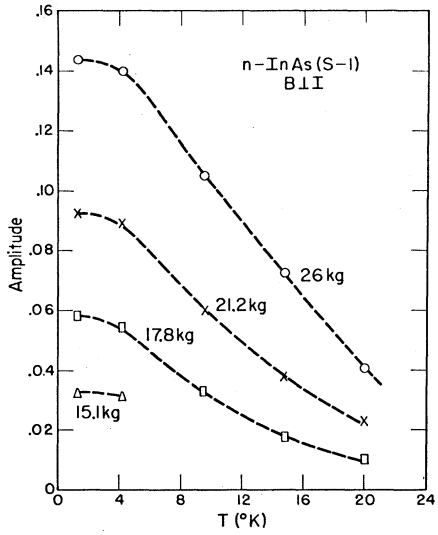

FIG. 11. Dependence of the oscillation amplitudes in single-crystal n-type InAs, sample S-1, on temperature for the case of transverse magnetic fields. Each set of points gives the oscillation amplitude at a particular field strength.

keeping the ordinate scale the same. We then superimposed these various curves on the experimental amplitude *versus* temperature plot for a given field strength. In practically all cases our data plots could at least be approximated by one of the $x/\sinh x$ curves and in some cases the fit was quite good. See Figs. 12 and 13 for examples. Note that Fig. 13 is for transverse magnetic fields. Thus, experimentally we find the same type of temperature dependence for the oscillation amplitudes in transverse fields as in longitudinal fields, whereas Argyres' transport theory gives no explicit prediction for the case of transverse fields.

In the event of a good fit between our amplitude *versus* temperature data and an $x/\sinh x$ *versus* x plot, we could identify the value of x with a temperature. Thus, since according to theory[8] $x=2\pi^2kT/\hbar\omega$, with $\omega=eB/m^*c$, we could calculate a value for the effective

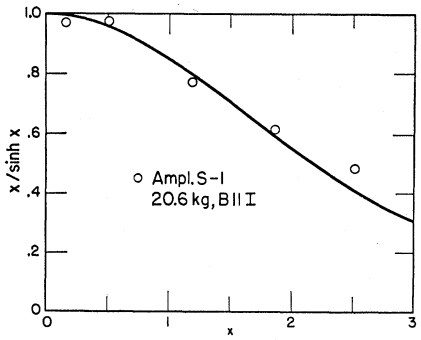

FIG. 12. The curve $x/\sinh x$ *versus* x fitted to oscillation amplitude *versus* temperature data for sample S-1 in a longitudinal field of 20.6 kilogauss. The correspondence found between x and temperature allows the effective electron mass to be deduced. (See text.)

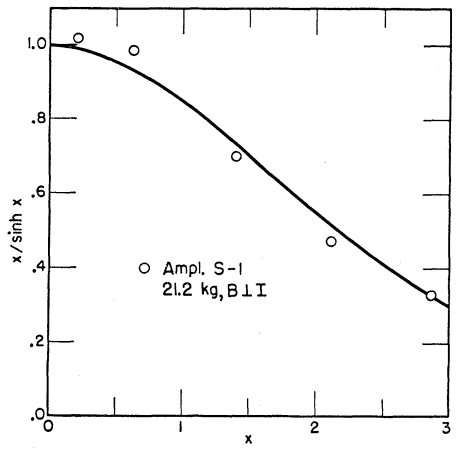

FIG. 13. The curve $x/\sinh x$ *versus* x fitted to amplitude *versus* temperature data for sample S-1 in a transverse magnetic field of 21.2 kilogauss. The correspondence found between x and temperature allows the effective electron mass to be deduced. (See text.)

mass m^*. The values we obtain for m^* by fitting amplitude *versus* temperature data at various field strengths are given in Table IV. The most reliable values are in boldface; the best reliability being attributed to values obtained from the most accurately known experimental amplitudes which at the same time have enough temperature dependence to be fitted accurately.

Only one value of m^* is given for sample P-1, since only for the field indicated were there available enough experimental amplitude values to allow fitting by an $x/\sinh x$ function. The reason for the paucity of experimental amplitude points in sample P-1 is that fewer oscillations occur in this sample than in sample S-1 because of the longer period in P-1. Thus, when oscillations at the lower field strengths disappear as the temperature is raised, we can no longer analyze the few oscillations still remaining at higher field strengths.

From Table IV we see that an effective mass of about $0.02m_0$ is obtained from the temperature dependence of our most reliable amplitude data. This value is the same as that obtained by Frederikse and Hosler[3] from magnetoresistance oscillations for n-InAs and is in good agreement with the value of m^* obtained by us from analysis of mobility *versus* temperature data,[4] $0.020m_0$; and that calculated theoretically,[7] $0.025m_0$. Most experimental determinations of m^* give larger values than these, but they usually involve greater electron concentrations so that the conduction band is filled up to energies where m^* increases with energy.

V. CONCLUSIONS

Magnetoresistance oscillations due to strong magnetic fields occur in polycrystalline as well as in single-crystal n-type indium arsenide samples. Their occurrence in the polycrystal is a novel confirmation of the sphericity of the conduction-band energy surfaces.

Collision broadening is very important in n-InAs. It seems to be responsible for both qualitative and quantitative features exhibited by our experimental results. Among the qualitative features are the absence of higher harmonics in the magnetoresistance oscillations, and the occurrence of oscillations in transverse fields which are in phase and of comparable amplitude to those in longitudinal fields. Among the quantitative features is the exponential field dependence of the amplitude of the oscillations, which involves a collision-broadening "temperature" corresponding to a collision-broadening time which is related to the momentum relaxation time derived from the mobility in a manner consistent with theory for ionized impurity scattering.

Very good agreement between the periods obtained experimentally and those calculated from theoretical considerations provide evidence that the conduction band is parabolic and that all conduction electrons participate in resistivity oscillations in InAs.

TABLE IV. Effective-mass values determined from the temperature dependence of the amplitude of magnetoresistance oscillations in n-InAs.

Sample	$B_{\parallel}$ (kilogauss)	m^*/m_0	$B_{\perp}$ (kilogauss)	m^*/m_0
S-1	25	0.028	26	0.035
	20.6	**0.018**	21.2	**0.021**
	17.8	0.012	17.8	0.015
P-1			14.3	**0.019**

The agreement between the observed phase of the magnetoresistance oscillations, in the single-crystal sample at any rate, and that predicted by the quasi-free electron model indicates that this model holds rather well in a degenerate semiconductor having a parabolic band.

The temperature dependence of the magnetoresistance oscillations which we observe fits the function given by transport theory and allows a value of the effective mass to be deduced which is in reasonable agreement with values obtained by other methods.

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