Scattering of Gamma Rays by a Static Electric Field*†

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In an effort to observe Delbrück scattering (the scattering of photons by a static electric field), the absolute differential cross sections for the elastic scattering of 1.33-Mev gamma rays by lead, tin, and uranium and of 2.62-Mev gamma rays by lead and tin have been measured for angles between 15 and 105 degrees. The observed scattering is the coherent sum of Delbrück, Rayleigh (bound electron), and nuclear Thomson scattering. The amplitude for the latter process is well known; recent calculations of Rayleigh scattering, which are in good agreement with data obtained previously in this laboratory for gamma-ray energies below 1 Mev, provide exact values of the amplitudes at 1.33 Mev and approximate values at 2.62 Mev. At 1.33 Mev, the difference between the observed scattering and that due to the Rayleigh and Thomson processes is not sufficiently large compared with experimental error to permit a definite identification of Delbrück scattering to be made. At 2.62 Mev, for lead, the experimental cross sections at intermediate angles (30 to 75 degrees) are substantially larger than those calculated by extrapolation of the exact calculations, even when reasonable allowance for error in the extrapolation is made. The most probable explanation for this difference is an appreciable contribution from Delbrück scattering.

I. INTRODUCTION

An interesting effect predicted by quantum electrodynamics is the scattering of light by light. Classical theory makes no such prediction since it assumes the principle of superposition which gives rise to the linearity of Maxwell's equations and expressly prohibits nonlinear effects[1] such as the scattering of light by light.

The scattering of light by light was originally discussed qualitatively by Halpern,[2] and shortly thereafter calculated in the limits of very low[3] and very high[4] photon energy. The cross section in the low-energy limit decreases rapidly with energy, is extremely small in the optical region, and accounts for the principle of superposition of classical optics not being violated experimentally. Following the recent reformulation of quantum electrodynamics, a complete calculation of the cross section was made,[5] the results of which reduce to those of the earlier calculations in the corresponding limits.

There is another scattering process which is formally closely related to the scattering of light by light and which has a similar history. This effect, the scattering of light by a static electric field, was first discussed qualitatively by Delbrück,[6] and calculations for forward scattering corresponding to those for the scattering of light by light were made in the low[7] and high[8] energy limits. More recently, a calculation of Delbrück scattering has been performed for forward scattering[9] using techniques similar to those of the photon-photon scattering calculation of Karplus and Neuman. Unfortunately, the integrals do not give known functions at arbitrary angles of scattering.

There has been an approach to Delbrück scattering through dispersion relations and the optical theorem which relate the Delbrück scattering cross section to the associated absorptive process, pair production. This method has been used by Rohrlich and Gluckstern[9] and by Toll[10]; the result is in agreement with the perturbation calculation of the former authors. These calculations were made using the Born approximation pair-production cross section, but Rohrlich[11] has shown that the effect of Coulomb corrections on Delbrück scattering at zero degrees is small. Using the method of impact parameters and the dispersion relations, Bethe and Rohrlich computed the cross section for high energies and very small angles.[12] Their results agree with those of Achieser and Pomerantschuk.

As yet no calculations have been made for arbitrary angles of scattering. Toll has made estimates for small angles at energies of a few Mev, but his results are approximate and do not go to angles larger than about 20 degrees.[10] Dispersion relations for nonzero angles of scattering are a current subject of investigation in electrodynamics[13] and recent advances toward a calculation of Delbrück scattering along these lines have been reported.[14]

In the Feynman scheme, phenomena such as the scattering of light by light or by a static electric field, which take place through intermediate states involving

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electron-positron pairs, are represented by closed electron loop diagrams. In lowest order, there are four nonvanishing closed loop diagrams leading to observable events. These are the diagrams for the scattering of light by light, the scattering of light by a static electric field, photon splitting by a static electric field,[15] and vacuum polarization. The diagram for vacuum polarization, unlike the other three, does not correspond to a specific observable event. The effect of vacuum polarization is to change the effective charge distribution of charged particles at distances of the order of the electron Compton wavelength and is observable as a small deviation from Coulomb's law at these distances. Hence vacuum polarization may be expected to play a role in many physical phenomena.

Vacuum polarization contributes -27 of the total 1057 Mc/sec to the Lamb shift. There are predicted effects of vacuum polarization on the energy levels of μ-mesonic atoms[16] and on low-energy proton-proton scattering,[17] but these have not been verified experimentally. The evidence from the Lamb shift is thus the best available. In consideration of the excellent agreement between theory and experiment, it seems that vacuum polarization is well established. However, a direct verification of a closed loop diagram is still to be desired. Photon-photon scattering is too small to observe with present techniques. The cross section for photon splitting is also quite small ($\simeq 10^{-33}$ cm^2), and this effect has not been observed. The cross section for Delbrück scattering is a factor of $\alpha^2 Z^4$ (2400 for lead) greater than that of photon-photon scattering and appears to be the most amenable to direct experimental observation.

II. TOTAL ELASTIC SCATTERING

One approach to an experimental determination of Delbrück scattering is through measurement of the cross section for elastic scattering of gamma rays of 1 to 3 Mev by medium- and high-Z atoms. In this energy region the predicted cross sections, though small, are within the limits imposed by the present techniques of measurement. There are, however, three processes in addition to Delbrück scattering that also give rise to elastic scattering by atoms in the gamma-ray region. These are scattering from the nuclear charge as a whole (nuclear Thomson scattering), scattering from the bound electrons (Rayleigh scattering), and nuclear resonance scattering. The scattering from these processes is coherent since it originates from the same charge distribution and has the same energy. Interference among the processes may lead to appreciable modifications of the angular distribution of any one of them and indicates the necessity of measuring absolute differential cross sections and of knowing with considerable accuracy the scattering amplitudes for the other processes if an identification of Delbrück scattering is to be made.

In the energy region of a few Mev the photon wavelength is long compared to nuclear dimensions and the scattering from the nuclear charge is given by the classical Thomson formula, $\sigma(\theta)=(Z^2e^2/Mc^2)^2(1+\cos^2\theta)/2$, where M and Z are the nuclear mass and charge. This formula is also obtained as the low-energy limit of nuclear Compton scattering and is believed to be exact for the gamma-ray energies under consideration.

Rayleigh scattering is well known in the x-ray region where it accounts for Bragg scattering and is used extensively to measure wavelength and to study crystal structure.[18] In this energy region the differential cross section is accurately given by the cross section for Thomson scattering from an electron multiplied by the square of a form factor to take into account the atomic charge distribution. The form factor, $F(q)$, is given by

$$\int e^{i\mathbf{q}\cdot\mathbf{r}}|\psi(r)|^2 d^3r,$$

where $q=2(\hbar\omega/c)\sin(\theta/2)$ is the momentum change of the x-ray. $F(q)$ introduces a strong preference for forward scattering in the angular distribution. The form-factor approximation was originally derived using the methods of classical optics. It was obtained later by semiclassical methods and again by nonrelativistic quantum theory.[18] The first treatment for $\hbar\omega$ comparable to mc^2 was given by Franz,[19] who used the Dirac theory of the electron and second-order perturbation theory. Franz made several nonrelativistic approximations, the most important of which was $q \ll mc$. Since he was primarily interested in the total cross section, he evaluated the form factor for the Fermi-Thomas model of the atom. The form-factor approximation was also derived using the methods of Feynman by Bethe,[20] who calculated the first Born approximation and showed that for the same nonrelativistic assumptions as Franz the form-factor result was again obtained. Bethe evaluated the form factor using Dirac K-shell wave functions. Brown and Woodward[21] have calculated the second Born approximation and shown that for $q \gtrsim mc$, this term is of the same magnitude as the leading term.

Brown and Woodward point out two objectionable features in this method of calculation. First, it is an expansion in $Z\alpha$ in which only two terms have been evaluated. Second, it involves the same nonrelativistic approximations made previously. To overcome the above difficulties, Brown, Peierls, and Woodward have developed a method of summing over intermediate states, taking the effects of binding into account by

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[18] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1943).
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[21] G. E. Brown and J. B. Woodward, Proc. Phys. Soc. (London) **A65**, 977 (1952).

using relativistic Coulomb wave functions, and without making any nonrelativistic approximations.[22] They have done this for only the K-shell because it contributes about 80% of the Rayleigh cross section for $q>\alpha Zmc$. Their differential cross section is expressed as a sum of Legendre polynomials, the coefficients of which are calculated by machine. The rapid falling off with angle of the cross section comes about from the near cancellation of these polynomials. The series can be terminated when the coefficient of the last polynomial becomes sufficiently small. In principle, this method should give exact results for the K-shell. It has been carried out by Brown and his collaborators for mercury at gamma-ray energies of 0.16,[23] 0.32,[24] and[25] 0.66 and 1.33 Mev.

Nuclear resonance scattering in the gamma-ray region is not easily produced because nuclear level widths are generally quite small compared to the recoil energy of the emitting nucleus. It is possible to observe this effect if the photon energy is increased sufficiently by Doppler broadening arising, say, from a previous emission of another particle.[26] If it should appear, it would be distinguished by its large cross section and relatively isotropic angular distribution as compared to Rayleigh and Delbrück scattering. We defer a more detailed discussion until Sec. V.

III. APPARATUS AND PROCEDURE

In order to extend the earlier work in this laboratory[27] which utilized collimated photon beams from sources of 1 to 3 curies in strength, a scattering apparatus was built that was capable of yielding high count rates with smaller sources than had been used previously. This was required because sources in the 1 to 3 curie range are not readily available in the 2–3 Mev region and, in addition, the elastic scattering cross section is expected to decrease with increasing energy in this energy region. The apparatus is shown in Fig. 1. Gamma rays from a source of 100 to 200 mC are prevented from reaching the detector directly by a heavy metal cone which attenuates them by a factor between 10^5 and 10^6. The scatterers are close approximations to surfaces of constant scattering angle. The cone, scatterers, and detector have a common axis, which for experimental convenience is taken as the plumb line. The angle of scattering is changed by changing the source-detector distance along the axis and the radius of the scatterer.

The detection system consisted of a NaI scintillator and photomultiplier, the pulses from which were amplified and analyzed with either a single or multi-channel pulse-height analyzer. To reduce pileup from incoherent scattering, the scintillator was shielded with lead of thickness equal to a half-thickness for the source gamma rays.

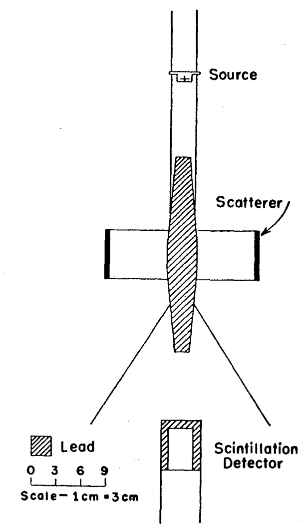

FIG. 1. Sketch of the experimental arrangement.

It has been shown[27] that at small angles, to first approximation, Compton scattering may be eliminated by using scatterers of high and low Z, which have the same number of electrons, and by then taking the difference in their count rates. Scatterers of lead, tin, and aluminum were chosen for experimental convenience. (For simplicity, the following discussion is limited to lead; most remarks are equally applicable to tin.) The difference count rate is expected to exhibit a line of the same spectral shape as the unscattered gamma ray plus a continuous spectrum due to Compton scattering from the K-shell in lead[28] and bremsstrahlung from electrons produced in the scatterer. An example of this is given in Fig. 2 for the scattering from lead at 30° at 1.33 Mev. There is evidence for a peak in the lead curve at the position of the elastic peak. The aluminum curve is smooth, since the elastic scattering from aluminum is too small to observe. The difference spectrum is compared to the unscattered spectrum with no changes along the energy axis. It is seen that for the forward half of the peak the two spectra agree, while for the backward half the scattered spectrum is larger than the unscattered spectrum. In general, a line cannot be superimposed upon a continuous spectrum without distortion of the line shape. Hence, within the statistical accuracy of the difference spectrum, the amount of incoherent scattering in the forward half of the peak is small. If the unscattered spectrum is subtracted from the scattered spectrum, a quantitative estimate of the incoherent scattering can be obtained. This indicates that if the elastic-scattering count rate is taken from the high-energy side of the peak, there will be a contribution of less than 10%

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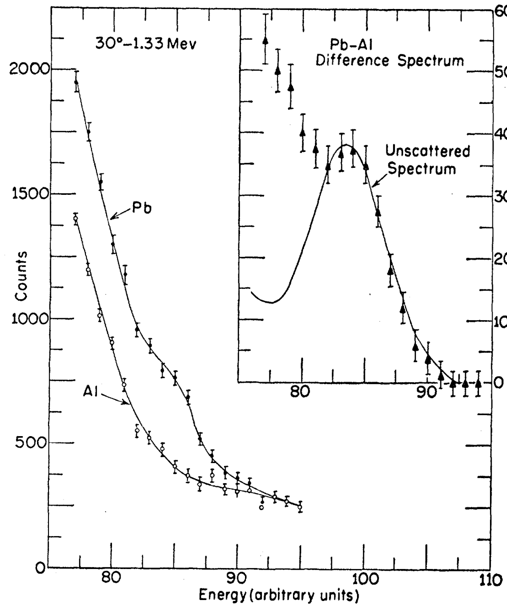

FIG. 2. Illustration of the method of eliminating incoherent scattering.

from incoherent scattering, which is true of all cases in which statistically significant spectra were obtained.

Another example, 75° at 2.62 Mev, is given in Fig. 3. For this angle, the aluminum count rate is equal to the background rate within the statistical error. The peak in the aluminum curve is due to photons coming through the cone and shows clearly that the elastically scattered peak from lead is not shifted by the scattering process. It is seen that the difference spectrum has the required shape, which again gives an upper limit of 10% incoherent scattering in the forward half peak.

These spectra have been repeated often enough to ensure that the method will work whenever the lead minus aluminum difference is large enough to be measured with adequate statistical certainty in a reasonable time. In general, if (Pb−Al)/Pb is greater than 0.10, detailed spectra can be obtained.

The stability of the detection system was checked before and after each set of scattering data which comprised a lead and aluminum run. The stability check consisted of placing a weak source of the same radiation as that used in the scattering runs in a "standard position" and measuring the spectrum. If the peak shifted by more than half a channel during a run (approximately one-tenth of the half-width of the peak), that set of data was discarded. This procedure was important for the success of the experiment, particularly at smaller angles where the elastic line rode on a steep Compton slope.

From the difference spectra, one must obtain the number of elastically scattered gamma rays per unit time. This was done by two methods. The first of these is to take the count rate directly from the total area under an unscattered spectral shape fitted to the difference spectrum as in Fig. 3. This method should eliminate any effects of incoherent scattering but is limited to cases in which good difference spectra have been obtained. The second method is to take the differences of the lead and aluminum readings for a given number of channels on the high-energy side of the elastic peak; this also should largely eliminate incoherent radiation which is small in this region. The total elastic scattering can be computed from the ratio of this fractional area to the total area of the unscattered line. For cases in which good spectra have been obtained, the two methods always agree within the statistical error. Much of the data obtained with the single-channel analyzer, where prohibitive labor would have been required to obtain a spectrum at each angle, was analyzed by the second method.

For angles smaller than 60°, where the aluminum count rate is higher than background because of the contribution from Compton scattering, one must correct the observed count rate for the differential absorption of lead and aluminum. Although the lead and aluminum targets have the same number of electrons, and hence almost the same Compton scattering, the photoelectric and pair-production effects are not the same for each scatterer. This correction can be easily made, and results in increasing the lead minus aluminum difference. These corrections are less than 15% except for lead at 2.62 Mev at 30° and 1.33 Mev at 45° where the ratios of corrected count rate to uncorrected count rate are 1.6 and 2.0, respectively. The correction is so

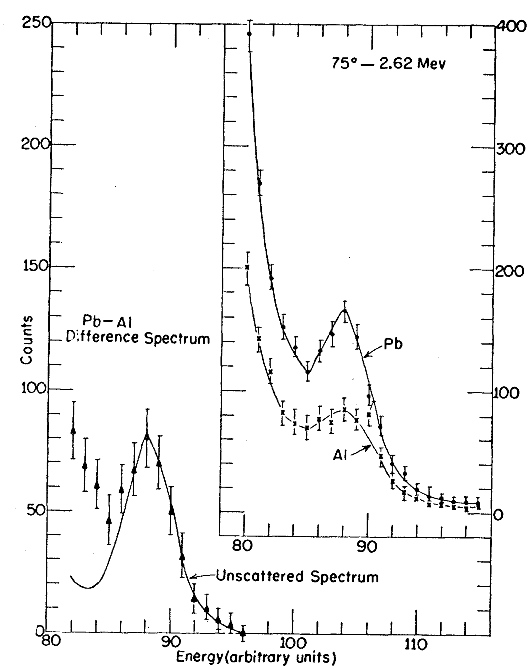

FIG. 3. Illustration of the method of eliminating incoherent scattering. The peak in the aluminum scattering is due to photons passing through the cone.

large for these cases because the ratio (Pb−Al)/Pb is approximately 0.1, while the absorption correction is about 5% of the aluminum count rate.

The calculated values of the absorption correction were checked experimentally by measuring the ratio of the lead and aluminum count rates at the Compton-scattered peak. The measured ratios were in good agreement with the calculated ratios. We estimate that uncertainties in the absorption corrections introduce a corresponding uncertainty in the measured elastic-scattering cross sections of not more than 5% at any angle.

There are a very large number of Compton-scattered photons producing pulses in the detector which may pile up in the detection system to give a pulse whose size is comparable with that for elastic scattering. This pileup can be largely eliminated by placing a lead shield over the detector. Further, since Compton scattering is approximately the same in both the lead and aluminum scatterers, then to first approximation, the remaining pileup will be the same for both scatterers. At large angles the aluminum and background count rates were always equal within the statistical errors, indicating the absence of pileup.

The differential cross section is given by

$$C(\theta)=(S/4\pi)\Omega_{sc}\sigma(\theta)tT(\eta\Omega)_{det},$$

where $C(\theta)$ is the count rate for elastic scattering, corrected for differential target absorption where necessary; S is the source strength; Ω_{sc} is the solid angle subtended by the scatterer; $\sigma(\theta)$ is the differential cross section in cm²/sterad; t is the target thickness in atoms/cm²; T is a correction for absorption of the elastically scattered radiation in the target; and $(\eta\Omega)_{det}$ is the product of detection efficiency and solid angle of the detector.

The target thickness is found from its weight and area. The absorption correction, T, may be calculated directly; for the target thicknesses used here, T had values between 0.70 and 0.95. Ω_{sc} may also be calculated with good accuracy since the source dimensions are quite small compared to the source-scatterer separation. S and $(\eta\Omega)_{det}$ are measured, and will now be discussed.

In elastic-scattering experiments with collimated beams of gamma rays, the product of source strength and detection efficiency may be obtained directly from the photopeak count rate produced in the detector by the source placed at a large distance from, and on the axis of, the detector. This product enters the cross-section expression at all angles since at any scattering angle the gamma rays enter the detector approximately normal to its face. In the cone scattering arrangement, gamma rays enter the detector obliquely and through its sides. The effective path length of the gamma rays in the detector, and therefore the detection efficiency, is a function of scattering angle. Consequently, the efficiency of detection and solid angle subtended by the detector must be measured at each angle. This measurement is facilitated by the use of a weak source, S_0, of the same radiation as the strong source, S, used in the scattering. It is possible to obtain the absolute value of the differential scattering cross section without knowing individually the values of S_0, S, or the efficiency of detection.

The strength of two sources can be compared, independently of the efficiency of detection, by comparing the associated count rates with the sources placed in the same position. In practice, as S was much larger than S_0, it was more convenient to compare their count rates when S was placed at a much larger distance from the detector than S_0. If the source-detector distance is large compared to the detector dimensions, the inverse square law will hold, and the source strengths can be compared accurately. Further, for a given angle of scattering, S_0 may be placed in the position of the scatterer with the detector in the proper position for that angle, and the count rate obtained is then equal to $(\eta\Omega)_{det}(S_0/4\pi)$. Thus, if one knows S in terms of S_0, the product $S(\eta\Omega)_{det}$ may be determined at every angle, and is independent of S_0. From these experimentally determined and calculated quantities, the relationship between count rate and absolute cross section was obtained at each angle with a probable error estimated to be 10%. This error was combined with the statistical error in the elastic-scattering count rate to give the total error in the final cross sections.

IV. RESULTS

The cross sections for the scattering of 1.33-Mev gamma rays from lead, shown in Fig. 4, are the averages of two sets of data obtained a year apart; the two sets were in agreement within experimental error at all angles except 45° where the difference was slightly outside the error. There is good agreement with the results of Mann,[27] Wilson,[29] and Storruste and Messelt.[30] These results are about a factor of two higher than those of Eberhard and Goldzahl[31] at smaller angles. The results of Davey[32] (not shown) are much larger than all of the above measurements.

The cross sections for tin at 1.33 Mev were also measured at the same time that the measurements on lead were made. Relative to the later values, the earlier measurements were about a factor of three higher at 60° and about 50% higher at other angles, except 15° where the two results agreed within the errors. The reason for this discrepancy is not known, but probably arose from some small systematic change in the tin count rates, since the aluminum rates were the same in both measurements. The ratio (Sn−Al)/Sn is in the range 0.05 to 0.1, except at 15° where it is about 0.2. Hence any small systematic error would

[29] R. R. Wilson, Phys. Rev. **90**, 720 (1953).
[30] A. Storruste and S. Messelt, Proc. Phys. Soc. (London) **A69**, 381 (1956).
[31] L. Goldzahl and P. Eberhard, J. phys. radium **8**, 33 (1957).
[32] W. G. Davey, Proc. Phys. Soc. (London) **A66**, 1059 (1953).

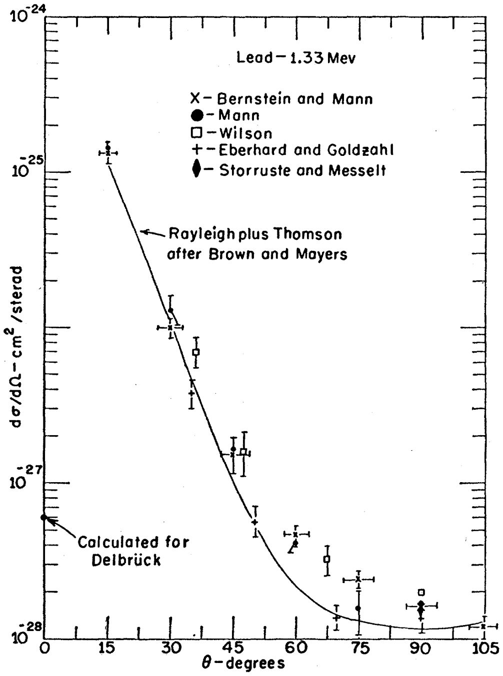

FIG. 4. Differential cross section for the elastic scattering of 1.33-Mev gamma rays by lead *versus* scattering angle. Horizontal lines through some of the points indicate the angular resolution.

change the tin results drastically. This is not so for lead at 1.33 Mev where the ratio (Pb—Al)/Pb varies from 0.25 to 0.45 at all angles except 45°.

The later data, given in Fig. 5, are more accurate for the following reasons. They were obtained with a multichannel analyzer which provided complete spectral information quite rapidly, and were taken with particular care to eliminate small sources of error; procedures such as checking the scatterer alignment and source height calibration were made much more frequently. It is possible that these additional safeguards, which had very little effect on the lead data, might have had a large influence on the tin data. Data obtained at small scattering angles with the collimator apparatus[27] are also presented and are in agreement with the cone results. The tin cross sections at large angles were too small to measure with the collimator arrangement and the point at 45° is an upper limit. The results of Hara, Benaigs, and Mey[33] which are a factor of two lower at the smaller angles are also plotted.

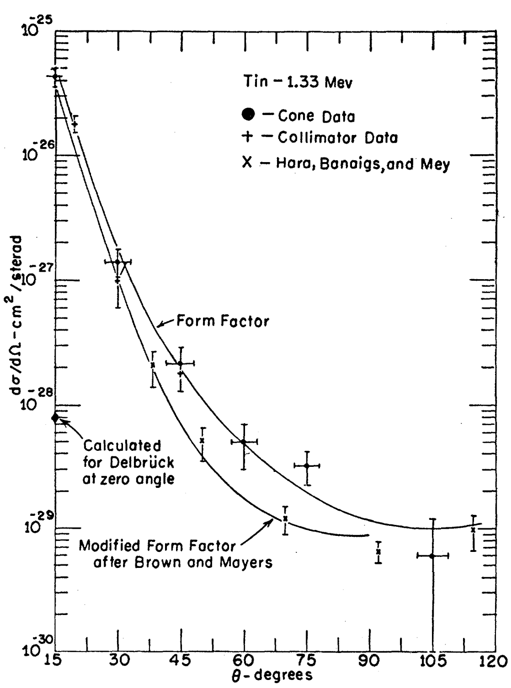

FIG. 5. Differential cross section for the elastic scattering of 1.33-Mev gamma rays by tin *versus* scattering angle.

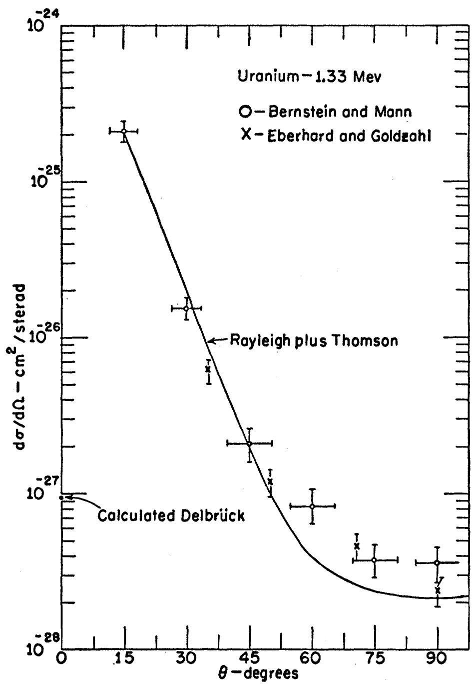

FIG. 6. Differential cross section for the elastic scattering of 1.33-Mev gamma rays by uranium *versus* scattering angle.

The scattering from uranium at 1.33 Mev was measured using the collimator apparatus. The results are shown in Fig. 6, and are in agreement with those of Eberhard and Goldzahl.[31]

Figure 7 presents the cross sections for the scattering of 2.62-Mev gamma rays by lead and also the data obtained by Goldzahl *et al.*[34] The two measurements are in good agreement and are appreciably smaller than the results of Davey[32] (not shown).

An attempt was made to measure the scattering from tin at 2.62 Mev. Spectral shapes were obtained at

[33] Hara, Benaigs, and Mey, Compt. rend. **244**, 2155 (1957).

[34] Goldzahl, Eberhard, Hara, and Mey, J. phys. radium **17**, 573 (1957); Goldzahl, Eberhard, Hara, and Alexandre, Compt. rend. **243**, 1862 (1956).

15° and 105°. For other angles the Sn–Al differences were too small to allow statistically useful spectra to be obtained and the cross sections were determined directly from the tin minus aluminum differences. For these angles only the limiting values for the cross sections were determined. The results are presented in Fig. 8.

Compton Cross Sections

Measurements of the differential Compton cross section using an aluminum scatterer were made at 15° for incident gamma-ray energies of 1.33 and 2.62 Mev. The difficulty in making precision measurements of the Compton cross section arises from the fact that the

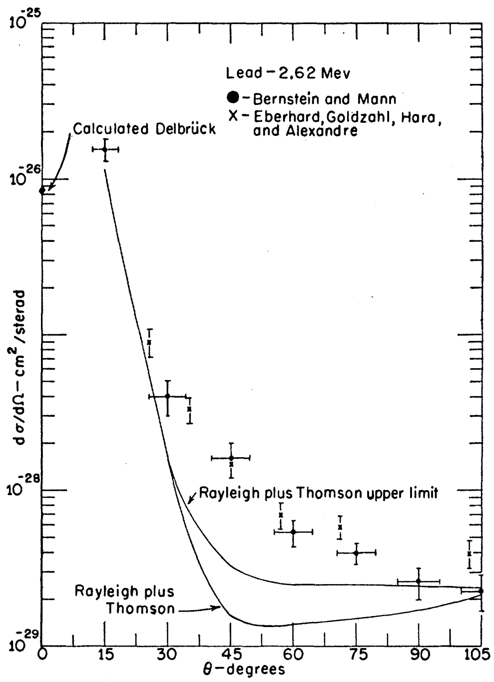

FIG. 7. Differential cross section for the elastic scattering of 2.62-Mev gamma rays by lead *versus* scattering angle.

gamma-ray energy, and hence the efficiency of detection, varies as a function of angle. Scintillation counter detection efficiencies as a function of energy are not well known. However, for small angles of scattering where the change of gamma-ray energy and the corresponding change in detection efficiency are small, the calculated efficiencies[35] as a function of energy can be utilized to make approximate corrections. These corrections decrease the observed cross sections by 10% at 1.33 Mev and by 13% at 2.62 Mev. The remainder of the measurement is similar to that of elastic scattering. The results are given in Table I. We look upon these measurements as indicating the accuracy with which known absolute differential cross sections may

TABLE I. Differential cross sections (in 10^{-26} cm²/sterad electron) for Compton scattering.

θ	$\hbar\omega$ (Mev)	Experiment	Klein-Nishina formula
15°	1.33	6.37±12%	6.50
15°	2.62	5.65±12%	5.65

be measured with the apparatus and procedure described above.

V. CONCLUSIONS

The zero-angle calculations of Delbrück scattering indicate that for gamma-ray energies below about 1 Mev the contribution of Delbrück scattering to the total elastic scattering is negligible compared with that of Rayleigh scattering. Nuclear Thomson scattering is small and may be accounted for without difficulty. This circumstance permits a direct test of the recent Rayleigh scattering calculations.[23–25]

There are, however, three minor difficulties involved in a comparison of theory and experiment. First, the calculations were made for incident energies of 160, 320, and 660 kev while the experimental data were obtained at gamma-ray energies of 411 and 662 kev. Second, the calculations were made for mercury while the experiments were done with lead (and tin) scatterers. Finally, the predicted contribution of the *L* shell of the scattering atom to the Rayleigh scattering must be included for comparison with experiment. We emphasize these corrections and their application to

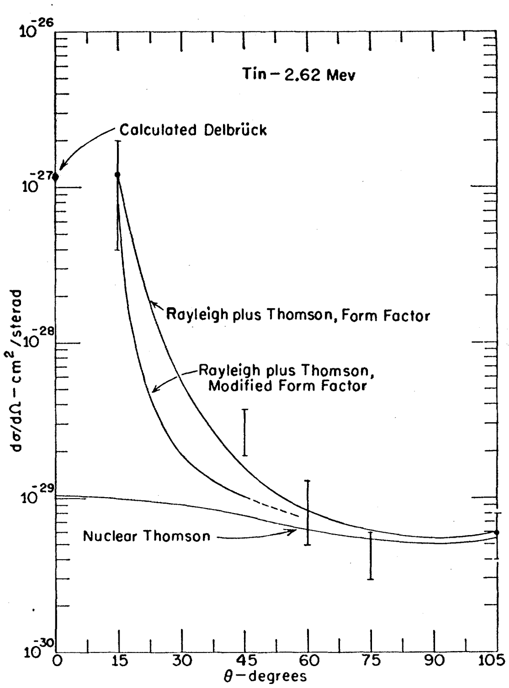

FIG. 8. Differential cross section for the elastic scattering of 2.62-Mev gamma rays by tin *versus* scattering angle. At 45°, 60°, and 75°, only limiting values are available.

[35] M. J. Berger and J. Doggett, Rev. Sci. Instr. **27**, 269 (1956).

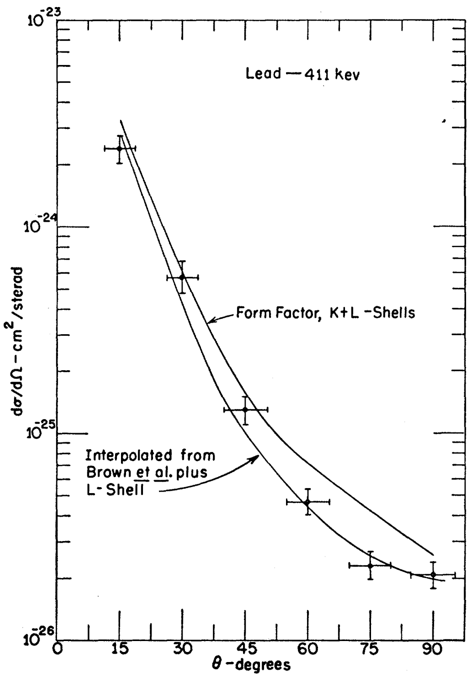

FIG. 9. Differential cross section for the elastic scattering of 0.411-Mev gamma rays by lead *versus* scattering angle. The data are from reference 27.

the low-energy results because a detailed understanding of Rayleigh scattering is necessary to any interpretation of data in the region above 1 Mev; in particular, the approximations made in estimating the L-shell contribution may be tested at low energies, i.e., small momentum transfers, where that contribution is quite large.

The exact K-shell calculations make use of states of circular polarization and give the cross section for Rayleigh scattering in the form

$$\sigma_K(\theta)/r_0^2 = |a_{1K}+ib_{1K}|^2+|a_{2K}+ib_{2K}|^2.$$

Subscript $1K$ stand for the non-spin-flip K-shell amplitude, subscript $2K$ for the spin-flip K-shell amplitude, and r_0 is the classical electron radius. The imaginary parts are obtained naturally from the calculations and are related to the lower order absorptive process which is the photoelectric effect from the K shell. Up to 1.33 Mev, the imaginary amplitudes are much smaller than the real amplitudes so that one may take

$$\sigma_K(\theta) = r_0^2(a_{1K}^2+a_{2K}^2)$$

as a good approximation for energies less than about 1 Mev.

It can be shown simply by plotting the calculated values that these amplitudes may be written in the form

$$a_{1K} = F_{1K}(q)(1+\cos\theta),$$
$$a_{2K} = F_{2K}(q)(1-\cos\theta).$$

F_{1K} is accurately a function of q only for values of q between 0.6 mc and the highest value for which an exact calculation was made. For values less than 0.6 mc, F_{1K} is a multivalued function whose value depends on the gamma-ray energy. F_{2K} is a function of q only for all values of q up to the maximum value for which an exact calculation was made and, indeed, is closely equal in magnitude to the form-factor approximation, F_K, over that range. These facts may be used to obtain F_{1K} and F_{2K} at any energy up to 1.33 Mev, and form the basis for extrapolation to higher energies.

The K-shell form-factor approximation, F_K, has a Z dependence of Z to $Z^{\frac{5}{2}}$, depending upon the magnitude of q. To convert the exact calculations from mercury to lead, the average value of the ratio $F_K(\text{Pb})/F_K(\text{Hg})$, for a given gamma-ray energy, may be used; the functions F_{1K} and F_{2K} are then multiplied by this ratio. This procedure has the advantage of leaving the angular dependence of the exact calculations unchanged. The ratios were 1.00, 1.03, and 1.06 for 411, 662, and 1330 kev, respectively.

To estimate the L-shell functions, F_{1L} and F_{2L}, we have assumed that $F_{iL}/F_L = F_{iK}/F_K$, where the F_{iK} are obtained from the exact K-shell calculations and F_L is calculated using Dirac L-shell wave functions.[36] Alternatively, as suggested by the Birmingham group,[37] F_{1L} may be calculated from a modified form factor,

$$\int e^{i\mathbf{q}\cdot\mathbf{r}}|\psi(r)|^2\frac{mc^2}{(E+V)}d^3r,$$

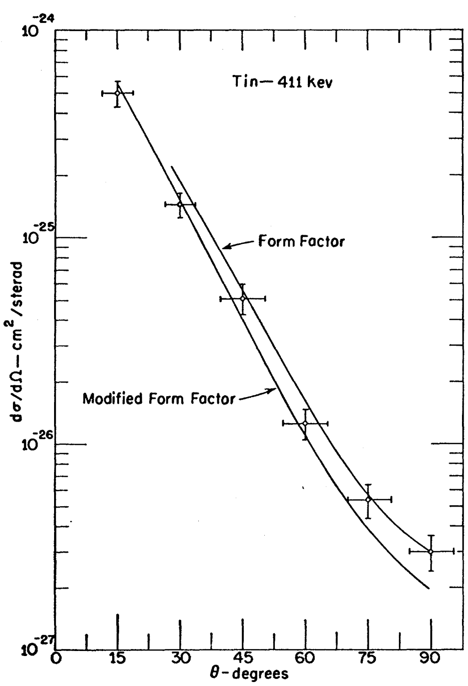

FIG. 10. Differential cross section for the elastic scattering of 0.411-Mev gamma rays by tin *versus* scattering angle. The data are from reference 27.

[36] J. B. Woodward, thesis, University of Birmingham, 1953 (unpublished).
[37] See reference 25 for a detailed discussion.

where E is the total energy of the scattering electron and $V=Ze^2/r$. This expression is expected to be a good approximation for small αZ and momentum transfers, but the conditions for its validity are not established. The F_{1L} obtained by the two methods are substantially the same. In the absence of exact numerical calculations for tin, the modified form factor is used to estimate both F_{1K} and F_{1L}.

For lead at 411 kev (Fig. 9), the L shell contributes more than the K shell at 15° and comparable amounts at 30° and 45°. The relatively good agreement between theory and experiment indicates that the L-shell estimates are reasonable. The form factor is high at large angles, suggesting the inadequacy of this approximation for increasing momentum transfer. For tin at 411 kev (Fig. 10), both form factor and modified form factor seem to fit about equally well.

For lead at 662 kev (Fig. 11), the L shell contributes an amount comparable to the K shell only at 15°. The good agreement of the Brown and Mayers calculation with experiment and the large error in the form-factor approximation indicate forcibly the necessity for the exact calculation of Rayleigh scattering from the K shell. For tin at 662 kev (Fig. 12), the modified form factor appears to be a better approximation than the form factor, but the difference is not appreciable.

For lead at 1.33 Mev (Fig. 4), the L shell contributes approximately 20% to the cross section at all angles. Brown and Mayers did not include an L-shell contribution in the non-spin-flip term so that their result is 20% lower than ours at 30°.

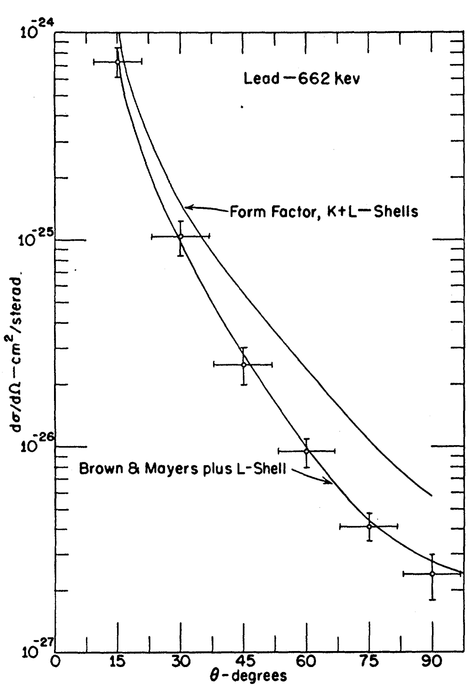

FIG. 11. Differential cross section for the elastic scattering of 0.662-Mev gamma rays by lead *versus* scattering angle. The data are from reference 27.

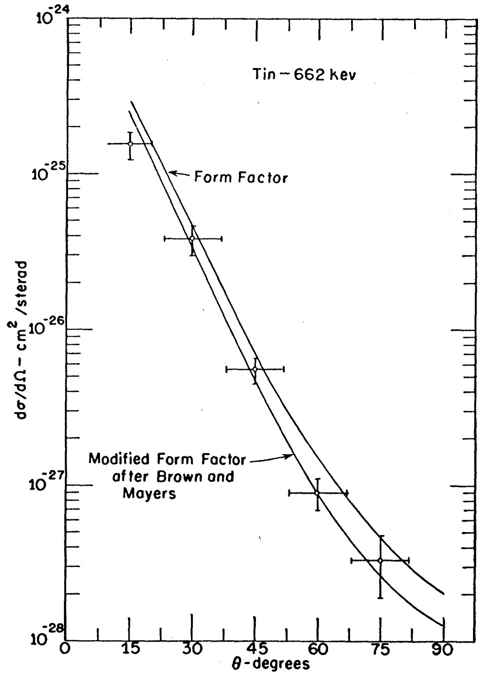

FIG. 12. Differential cross section for the elastic scattering of 0.662-Mev gamma rays by tin *versus* scattering angle. The data are from reference 27.

The Rayleigh plus Thomson theory for uranium at 1.33 Mev (Fig. 5), was obtained from the mercury calculation by assuming a Z^5 dependence. The fit of theory with experiment is qualitatively similar to that for lead at the same energy.

For tin at 1.33 Mev (Fig. 6), the difference between the modified-form-factor and form-factor calculations is larger than at lower energies, but the experiments are not in good agreement and no definitive conclusions can be reached.

The situation at 1.33 Mev with respect to Delbrück scattering is not clear. The data for lead and uranium indicate an excess of scattering in the region from 60° to 75° whose rough magnitude is not inconsistent with the zero-angle value for Delbrück scattering. This excess is, however, not large compared to the experimental accuracy and, restricted as it is to a small angular region, is not particularly convincing. It appears that at this energy the limited accuracy of the experimental data and the relative magnitudes of the Rayleigh and Delbrück amplitudes combine to prevent a definite identification of Delbrück scattering from being made with present techniques.

For lead at 2.62 Mev (Fig. 7), the real Rayleigh scattering amplitudes are obtained by extrapolation of the 1.33-Mev calculations which gives theoretical values for angles as large as 60°. For larger angles the spin-flip term dominates, and, as this is expected to be given by the form factor, it can be calculated. Here F_{1K} changes sign at about 30°, which gives rise to partial cancellation of the non-spin-flip Thomson term and makes the cross section between 45° and 75° quite small. Thus, for the first time, the imaginary amplitudes, b_1 and b_2, become important. We have estimated b_1 and b_2 at 2.62 Mev in a manner similar to that by which a_1 and a_2 were obtained, i.e., by extrapolation of the 1.33-Mev calculations. There is no adequate justification for this calculation which most likely overestimates the imaginary amplitudes. Inclusion of the imaginary amplitudes increases the theoretical cross sections by 50% at 45° and 60°, by 20% at 75°, and by less than 10% at all other angles. The experimental results are significantly higher than the theoretical values obtained in this way (plus Thomson) for all angles between 30° and 75°. If one arbitrarily neglects F_{1K} at angles larger than 30°, as suggested by Brown and Mayers,[25] the curve marked "upper limit" for the theoretical cross section is obtained. It does not seem likely that errors involved in the extrapolation of the theory to this energy could be sufficiently large to produce theoretical values lying above this curve, which is still substantially below the experimental results. The excess of scattering in the experimental results indicates that a process other than Rayleigh and nuclear Thomson is contributing to the observed scattering.

The 2.62-Mev gamma ray of ThC″ arises from a transition between the 2.62-Mev excited state and the ground state of Pb^{208} which constitutes 51.6% of the naturally occurring lead isotopes. The recoil energy of the lead nucleus on emission of the gamma ray is 18 ev, which is expected to be appreciably larger than the natural width of the 2.62-Mev state (probably less than 10^{-2} ev). The beta and gamma radiations preceding the 2.62-Mev gamma ray in the decay of RaTh (the actual source material for the scattering experiments) do not have sufficient energy to replace the 18 ev by Doppler broadening, but the decay of RaTh also involves a series of α-particle emissions which can impart recoil energies of as much as 150 kev to the emitting nuclei. This latter energy, however, appears not to contribute to Doppler broadening in solid sources at room temperature, for which resonance scattering is not observed,[26] presumably because a nucleus loses its recoil energy in a very short time, $\tau \lesssim 10^{-11}$ sec. This reasoning is further strengthened in the particular case of ThC″ since the 2.62-Mev state in Pb^{208} is reached by the beta decay of Tl^{208} which has a half-life of 3.1 min. We cannot, of course, rule out the possibility that one of the other three stable lead isotopes has a level at 2.62 Mev.

The case of nuclear resonance scattering far from resonance, i.e., the nuclear equivalent of atomic Rayleigh scattering, has been treated by Levinger.[38] For lead at 2.62 Mev, he finds the amplitude for nuclear resonance scattering to be small compared with that for nuclear Thomson scattering; at large angles, where nuclear Thomson scattering predominates, the total cross section is reduced by about 10% due to interference between the two processes.

The process most probably responsible for the large observed scattering cross sections at 2.62 Mev is Delbrück scattering. Unfortunately, as was stated earlier, there exists no theoretical prediction of the angular distribution and, in its absence, the amplitudes for the process cannot be elicited from the experimental results; there are four unknown Delbrück amplitudes at each angle of scattering and one cannot proceed from the data to these amplitudes.

At zero angle the real Rayleigh and Thomson amplitudes are in phase with each other and out of phase with the real Delbrück amplitude.[10] These predictions are obtained unambiguously from dispersion relations and the energy dependence of the total cross section for pair production and the photoelectric effect. Since the spin-flip amplitudes vanish at zero degrees, the phase relations apply only to the non-spin-flip amplitudes. For nonforward scattering, the phase relations are not fixed by any general argument but must be determined from detailed calculation.

At 2.62 Mev there are several possibilities involving relative phases and magnitudes of the various amplitudes that will lead to scattering cross sections larger than those from the combined Rayleigh and Thomson processes. Figure 7 indicates that, for angles up to about 15°, the real non-spin-flip Rayleigh amplitude, a_{1K}, makes the dominant contribution to the observed cross section; this is consistent with the calculated amplitudes at zero degrees where $a_{1K}=1.6$ and the real and imaginary Delbrück amplitudes are 0.33 and 0.087, respectively, all in units of r_0. In the region of intermediate angles (45°–75°), the Rayleigh amplitudes have decreased sufficiently so that at least one of the Delbrück amplitudes has a value, depending on the phase relations, in the range from about 1 to 5 times larger than the largest combined Rayleigh-Thomson amplitude which is about 0.01 in this angular region. For angles greater than 90°, the Rayleigh and Delbrück amplitudes are small compared to the nuclear Thomson amplitude.

[38] J. S. Levinger, Phys. Rev. 84, 523 (1951).