If configuration space is divided up into cells of size greater than or equal to a particle wavelength on an edge and if the central limit theorem is then applied to the matrix elements represented as sums over cells, assuming random variation of the integrand within cells, a normal distribution is inferred for the matrix element.[4]

The work of Wigner[3] suggested the exploration of the properties of the eigenvalues and eigenvectors of matrices with random matrix elements. With this in mind, a code was written for the Los Alamos IBM 704 computer to generate random matrices, diagonalize them, and then sort the output spacings and eigenvector components so that histogram plots could be made. Typical results of such calculations are shown in Fig. 1, in which are plotted average differential probabilities $\bar{P}$ as functions of the ratio x of spacing S to mean spacing D, or of eigenvector component a under the assumption that the root-mean-square values of the diagonal and off-diagonal matrix elements are the same. The solid spacing-distribution curves shown are identical and are the result of an analytic calculation for a two-by-two matrix, while the solid eigenvector-component distribution curves are those for the distribution of a component of a randomly oriented unit vector in a vector space of the appropriate number of dimensions (five or ten). In addition, experimental data on four zero-spin target nuclei taken from Fig. 9 of reference 1 on the spacing distribution is shown.

Two features of these plots stand out. The first is that the spacing distribution (including the so-called "repulsion effect") is not very strongly dependent on the dimension of the matrix, while the eigenvector-component distribution is (its width varies as the inverse of the dimension). The second feature is that the results are independent of the choice of basis in the vector space, since the eigenvectors are randomly oriented.

The neutron width distribution can be inferred from the eigenvector-component distribution since the neutron width $\Gamma_{\lambda c}$ for level λ and neutron channel c is proportional to the square of the surface "overlap" integral between the level eigenfunction X_λ and the neutron channel function Φ_c:

$$\Gamma_{\lambda c} \propto \left(\int X_\lambda \Phi_c dS\right)^2. \tag{1}$$

The eigenvector X_λ has the components $a_{\lambda i}$, i.e.,

$$X_\lambda = \sum_i a_{\lambda i}\varphi_i, \tag{2}$$

where the φ_i are a basis choice. The integral of Eq. (1) becomes

$$\int X_\lambda \Phi_c dS = \sum a_{\lambda i} \int \varphi_i \Phi_c dS, \tag{3}$$

to which the central-limit theorem can be applied to infer a Gaussian distribution for the integral which corresponds to the previously suggested distribution.[2]

Examination of the effect of assuming different dispersions for the diagonal and off-diagonal matrix elements is in progress. This is known to have considerable effect in the two-by-two case, for which an analytic formula can be obtained, but preliminary IBM 704 runs indicate that this may not be the case in higher-dimensional vector spaces.

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† On leave of absence from Brookhaven National Laboratory, Upton, New York.

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[4] This argument is parallel to that used previously by Thomas and Porter (reference 2) to motivate the neutron width distribution. It was applied then in a somewhat different way and resulted in only the width distribution.

Beta-Gamma Correlations from Polarized Manganese-52

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson
National Bureau of Standards, Washington, D. C.
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A recent calculation by Curtis and Lewis[1] and by Morita and Morita[2] gives a distribution and correlation function for the beta and the following gamma radiations from oriented nuclei in a $J \rightarrow J$ beta transition. An experimental determination of the parameters in this function can give information on the magnitude and relative phase of the Fermi and Gamow-Teller interactions involved. We have made such experimental determinations on polarized Mn^{52}.

In order to evaluate each parameter uniquely, certain assumptions must be made about the nature of the beta interactions. The recent recoil experiments[3] on A^{35} and Ne^{19} and the balanced recoil experiment[4] on Eu^{152} using resonance fluorescence techniques, together with the re-evaluation of the He^6 recoil experiment,[5] indicate that vector and axial vector are the predominant forms of the Fermi and Gamow-Teller interactions, respectively. Furthermore, nearly all of the recent experiments measuring the longitudinal polarization of electrons and positrons and also their angular distribution in beta decay are in accord with the two-component neutrino hypothesis where neutrinos of negative helicity (left-handed) are emitted in both Fermi and Gamow-Teller decays. We can consider,

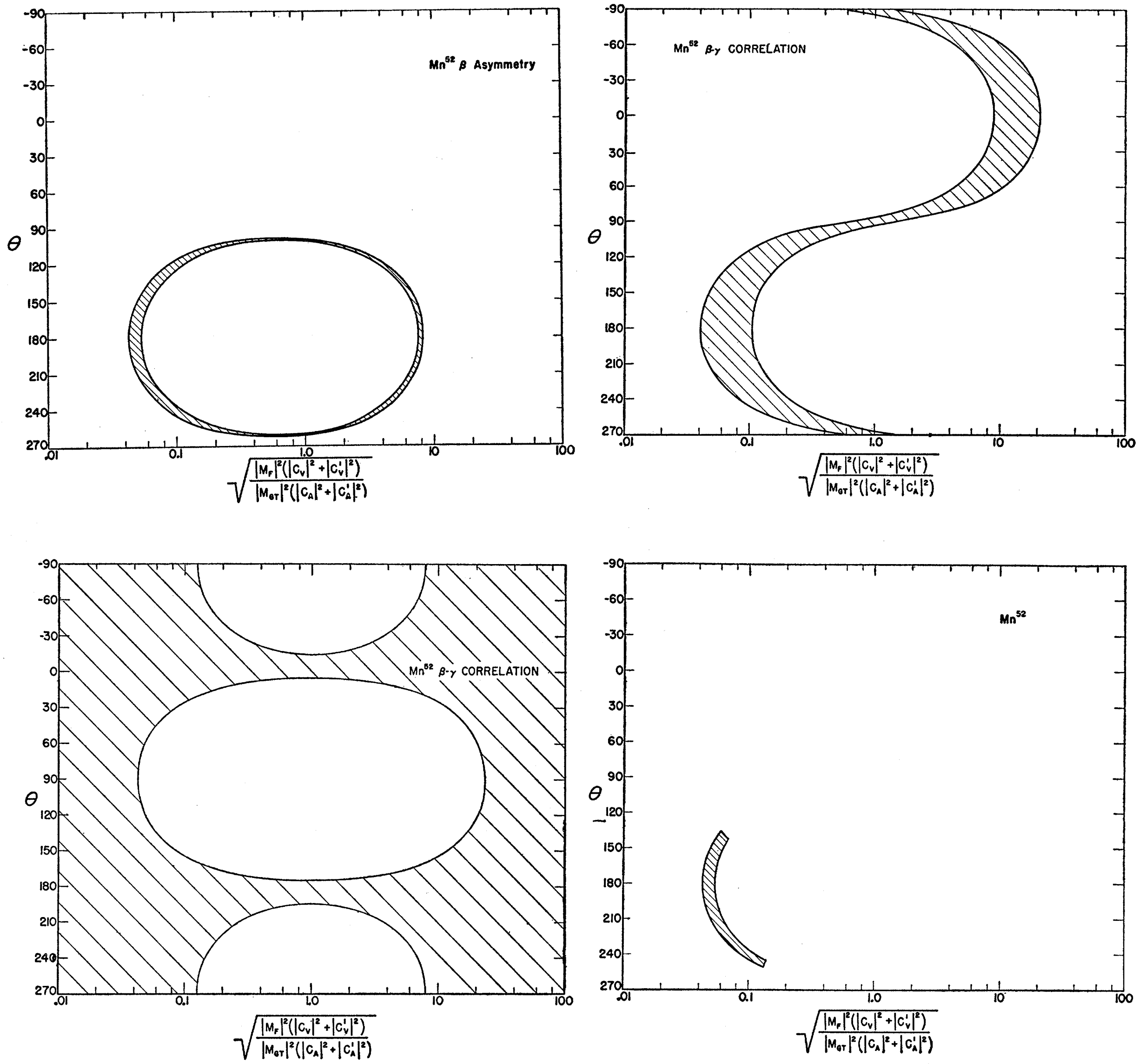

FIG. 1. The relative phase θ *vs* the square root of the ratio of Fermi to Gamow-Teller intensities as determined by (a) the beta asymmetry, (b) the beta-gamma correlation with the gamma detectors in the plane of **J** and **p**, (c) the beta-gamma correlation with the gamma detectors in the plane of **J** but perpendicular to **p**, and (d) a combination of all three experiments. The cross-hatched region encompasses the values lying between the limits of experimental error.

therefore, for the analysis of our results that $C_V=C_V'$, $C_A=C_A'$, and all other coupling constants are zero.

The 5.7-day isomer of Mn^{52} decays by 33% positron emission with a maximum energy of 0.580 Mev and the remaining 67% by orbital electron capture to the third excited state of Cr^{52}, which in turn decays by the cascade emission of 0.73-, 0.94-, and 1.45-Mev gamma rays. The spin sequence[6] is $6^+(\beta)6^+(\gamma)4^+(\gamma)2^+(\gamma)0^+$.

Each of the parameters in the distribution and correlation function depends on the coupling coefficients and the nuclear matrix elements in different combinations, with multiplying constants which depend on the angular momenta involved. The beta-emission probability is proportional to $[(|C_V|^2+|C_V'|^2)|M_{\mathrm{F}}|^2+(|C_A|^2+|C_A'|^2)|M_{\mathrm{GT}}|^2]$. This term can be used, of course, to normalize each of the other parameters to unit disintegration rate. The angular distribution of the beta emission with respect to the nuclear spin direction involves the matrix elements

$$\mathrm{Re}[C_A C_A'^*|M_{\mathrm{GT}}|^2-(42)^{\frac{1}{2}}(C_V C_A'^*+C_V' C_A^*)M_{\mathrm{F}}M_{\mathrm{GT}}^*],$$

and when normalized is identical to that given by Lee and Yang.[7] The gamma-ray angular distribution with respect to the nuclear spin axis involves the combinations $[(|C_V|^2+|C_V'|^2)|M_F|^2+(13/14)(|C_A|^2+|C_A'|^2)|M_{GT}|^2]$, and is rather insensitive to the degree of Fermi to Gamow-Teller mixing. This insensitivity is an advantage here since the measured gamma-ray anisotropy is used as a basis for calculating the nuclear orientation parameters, f_1, f_2, etc., from the measured hfs.[8]

Two angular correlation terms occur relating positrons, emitted along a unit vector **p** perpendicular to the nuclear polarization defined by a unit vector **J**, to the following gamma rays, emitted along a unit vector **k**. The correlation function that is nonzero for the case where **k** is in the plane determined by **p** and **J** involves the combinations

$$\mathrm{Re}[-C_AC_A'^*|M_{GT}|^2-\tfrac{1}{2}(42)^{\frac{1}{2}}(C_VC_A'^*+C_V'C_A^*)M_FM_{GT}^*].$$

The other correlation function, where **k** is in a plane containing **J** but perpendicular to **p**, involves the matrix elements $\mathrm{Im}[C_VC_A'^*+C_V'C_A^*)M_FM_{GT}^*]$.

The methods of polarization of the nuclei and detection of the radiations are essentially the same as those used in the previous experiments.[9,10] In the correlation experiments, where the positrons are detected at right angles to the polarizing field, allowance is made for the curvature of path of the positrons by making the source-to-detector distance small compared to the mean radius of curvature for the momentum band of positrons detected, and by suitably orienting the plane of the gamma counters.

The usual corrections have been made for attenuation due to electron backscattering, the finite solid angles of the detectors, and presence of Compton electrons; this last correction is a major one in the case of Mn^{52} where the possibility of a coincidence between a Compton electron and a gamma ray is appreciable.

The experimental value for the beta asymmetry coefficient α is $(0.232\pm0.010)(v/c)f_1$. This value is slightly smaller than that which can be deduced from a measurement by Boehm[11] of the beta-gamma circular polarization correlation from Mn^{52}. Nevertheless it, too, is much larger than would be obtained for the case of a pure Gamow-Teller transition.

The beta-gamma correlation measurements from polarized Mn^{52} were performed with a geometry in which gammas in two channels, one with two counters located diametrically opposite at angles of 45° and 225° with respect to the nuclear polarization and the other with counters at 135° and 315°, are each in coincidence with positrons detected at 90° with respect to the nuclear polarization. The measured correlation

$$[W(\mathbf{p},\mathbf{k},\mathbf{J})-W(-\mathbf{p},\mathbf{k},\mathbf{J})]/[W(\mathbf{p},\mathbf{k},\mathbf{J})+W(-\mathbf{p},\mathbf{k},\mathbf{J})]$$

equals $(0.040\pm0.016)(v/c)f_1$ for the case where the gamma counters are in the plane of **p** and **J** and equals $(0.012\pm0.022)(v/c)f_2$ for the case where the plane containing the gamma counters contains **J** but is perpendicular to **p**.

We may interpret these results by expressing the parameters in terms of two variables, the square root of the ratio of the Fermi to Gamow-Teller intensities,

$$[|M_F|^2(|C_V|^2+|C_V'|^2)]/[|M_{GT}|^2(|C_A|^2+|C_A'|^2)]^{\frac{1}{2}},$$

and the relative phase of the Fermi and Gamow-Teller interactions, θ, where θ is defined by

$$C_VM_F=|a|C_AM_{GT}e^{i\theta}.$$

In Fig. 1, (a), (b), and (c) portray the experimental results for the beta asymmetry and the two beta-gamma correlation measurements, respectively. In each figure the cross-hatched region encompasses the values lying between the limits of experimental error for the particular experiment.

It is seen that no single measurement significantly limits the range of either the mixing ratio or the phase angle. If, however, these three figures are superimposed, the region common to all three measurements is somewhat restricted. This region is shown in (d) in Fig. 1, and is seen to define rather closely the degree of Fermi to Gamow-Teller mixing.

For time-reversal invariance to be valid, the phase angle must be precisely 0° or 180°. The rather wide limits of phase angle deduced from the present experiments do not allow any conclusions other than that time-reversal invariance is consistent with the present results, while a maximum breakdown is not, in agreement with the findings of Boehm and Wapstra.[12] It should be pointed out, however, that the ratio of Fermi to Gamow-Teller transition amplitudes is determined for the first time from a direct dynamical measurement and without recourse to any knowledge of comparative half-lives. Once the values of the coupling constants have been fixed, the experiments then determine—in a way independent of all nuclear models—nuclear matrix elements for beta decay.

Further experiments are contemplated in order to improve upon the accuracy of the results contained herein, but owing to the long-term nature of the work this short account is being published at the present time.

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