and hence

$$-\tfrac{3}{2}(\mu_p+\mu_n-\tfrac{1}{2})p_D=-0.0078 \text{ nm}.$$

The spin-orbit contribution is

$$(\Delta\mu)_{SL}=-0.0125 \text{ nm},$$

so that

$$\Delta\mu=-0.0203 \text{ nm}. \quad (5)$$

There remains, therefore, a difference of $-0.002$ nm, which is certainly within the limits of the unknown effects.

The new set of triplet potentials, of course, still fits the effective range and deuteron binding energy exactly. It leads to a deuteron quadrupole moment of $2.77\times10^{-27}$ cm$^2$ and a root-mean-square deuteron radius of $2.3\times10^{-13}$ cm. Figure 1 shows the three components of the potential. Compared to those of reference 2, they are somewhat deeper and narrower; the repulsive centers have become smaller and stronger, while the repulsive "humps" have disappeared. The spin-orbit part is practically zero from about $1.2\times10^{-13}$ cm on.

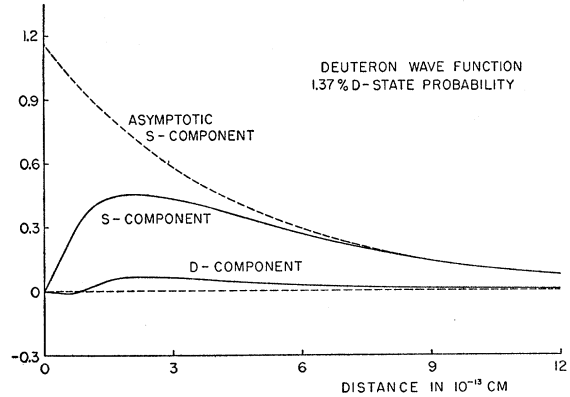


FIG. 2. Deuteron wave function corresponding to the potentials of Fig. 1.

The deuteron wave function is shown in Fig. 2. The $S$ component is much more rounded than it had been previously, and it does not become equal to its asymptotic value until about $8\times10^{-13}$ cm. The negative part of the $D$ component at the origin has become quite small. Scattering wave functions were not plotted but can be easily obtained.[3]

It is perhaps worthy of note that we have here a set of triplet potentials which fits all the low-energy data, and for which the $D$-state probability in the deuteron contributes much less to the magnetic moment than does the spin-orbit force.

* Supported in part by the National Science Foundation.
† National Science Foundation Predoctoral Fellow.
[1] Herman Feshbach, Phys. Rev. **107**, 1626 (1957).
[2] R. G. Newton and T. Fulton, Phys. Rev. **107**, 1103 (1957).
[3] Anyone who wishes to use it is welcome to a copy of the IBM-650 deck for the rapid computation of scattering wave functions of any energy.

## Nuclear Fine Structure Widths and Spacings*

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WITHIN the past few years, high-resolution measurements[1] have led to a rather complete qualitative understanding of the statistical properties of nuclear fine-structure levels from an empirical point of view. There have been proposals suggested to provide justification for the width distributions[2] and the spacing distributions,[3] but the connection between the two has so far not been emphasized. It is the purpose of this note to present some recently obtained results which show that *both* distributions can be derived from a single statistical hypothesis: *the matrix elements of the Hamiltonian operator which defines the eigenstates of the compound nucleus follow normal distributions.* These normal distributions are not entirely unmotivated.

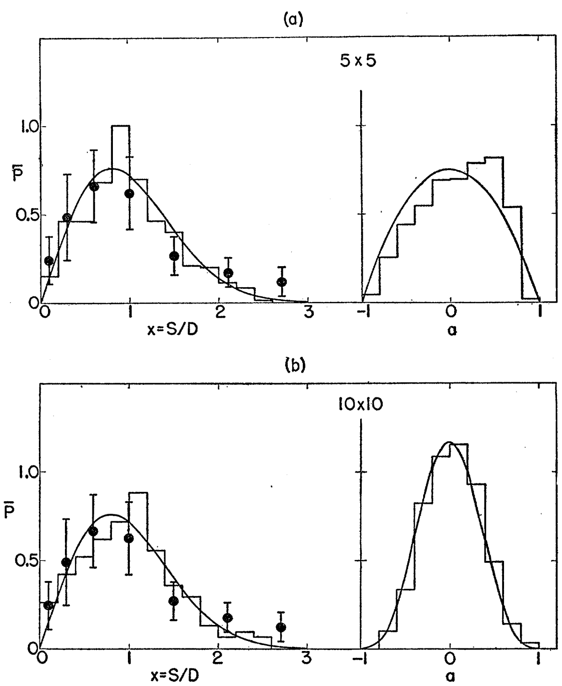


FIG. 1. Plots of the differential probability $\bar{P}$ as a function of the ratio $x$ of spacing $S$ to average spacing $D$ or as a function of eigenvector component $a$. The histograms are the results of runs on the Los Alamos IBM 704. The solid spacing-distribution curves are analytic results for a two-by-two matrix while the solid eigenvector-component curves are the predictions for the component of a randomly oriented unit vector. The experimental points are the "corrected" points from Fig. 9 of reference 1, divided by 37 to convert them to relative frequency. Parts (a) and (b) of the figure show the results for five-by-five and ten-by-ten matrices. In part (a) the spacing histogram is the result of 300 total counts, and the component histogram is the result of 1875 total counts. In part (b) the spacing histogram is the result of 153 total counts, and the component histogram is the result of 1700 total counts.

If configuration space is divided up into cells of size greater than or equal to a particle wavelength on an edge and if the central limit theorem is then applied to the matrix elements represented as sums over cells, assuming random variation of the integrand within cells, a normal distribution is inferred for the matrix element.[4]

The work of Wigner[3] suggested the exploration of the properties of the eigenvalues and eigenvectors of matrices with random matrix elements. With this in mind, a code was written for the Los Alamos IBM 704 computer to generate random matrices, diagonalize them, and then sort the output spacings and eigenvector components so that histogram plots could be made. Typical results of such calculations are shown in Fig. 1, in which are plotted average differential probabilities $\bar{P}$ as functions of the ratio $x$ of spacing $S$ to mean spacing $D$, or of eigenvector component $a$ under the assumption that the root-mean-square values of the diagonal and off-diagonal matrix elements are the same. The solid spacing-distribution curves shown are identical and are the result of an analytic calculation for a two-by-two matrix, while the solid eigenvector-component distribution curves are those for the distribution of a component of a randomly oriented unit vector in a vector space of the appropriate number of dimensions (five or ten). In addition, experimental data on four zero-spin target nuclei taken from Fig. 9 of reference 1 on the spacing distribution is shown.

Two features of these plots stand out. The first is that the spacing distribution (including the so-called "repulsion effect") is not very strongly dependent on the dimension of the matrix, while the eigenvector-component distribution is (its width varies as the inverse of the dimension). The second feature is that the results are independent of the choice of basis in the vector space, since the eigenvectors are randomly oriented.

The neutron width distribution can be inferred from the eigenvector-component distribution since the neutron width $\Gamma_{\lambda c}$ for level $\lambda$ and neutron channel $c$ is proportional to the square of the surface "overlap" integral between the level eigenfunction $X_\lambda$ and the neutron channel function $\Phi_c$:

$$\Gamma_{\lambda c} \propto \left( \int X_\lambda \Phi_c dS \right)^2. \tag{1}$$

The eigenvector $X_\lambda$ has the components $a_{\lambda i}$, i.e.,

$$X_\lambda = \sum_i a_{\lambda i} \varphi_i, \tag{2}$$

where the $\varphi_i$ are a basis choice. The integral of Eq. (1) becomes

$$\int X_\lambda \Phi_c dS = \sum a_{\lambda i} \int \varphi_i \Phi_c dS, \tag{3}$$

to which the central-limit theorem can be applied to infer a Gaussian distribution for the integral which corresponds to the previously suggested distribution.[2]

Examination of the effect of assuming different dispersions for the diagonal and off-diagonal matrix elements is in progress. This is known to have considerable effect in the two-by-two case, for which an analytic formula can be obtained, but preliminary IBM 704 runs indicate that this may not be the case in higher-dimensional vector spaces.

We would like to thank Dr. Kenneth Ford and Professor J. A. Wheeler for valuable comments.

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† On leave of absence from Brookhaven National Laboratory, Upton, New York.

[1] J. A. Harvey and D. J. Hughes, Phys. Rev. **109**, 471 (1958), and references cited therein.

[2] R. G. Thomas and C. E. Porter, Phys. Rev. **104**, 483 (1956).

[3] E. P. Wigner, Gatlinburg Conference on Neutron Physics by Time-of-Flight, Oak Ridge National Laboratory Report ORNL-2309, 1957 (unpublished), p. 59. See also E. P. Wigner, Proceedings of the International Conference on the Neutron Interactions with the Nucleus, 1957 (unpublished).

[4] This argument is parallel to that used previously by Thomas and Porter (reference 2) to motivate the neutron width distribution. It was applied then in a somewhat different way and resulted in only the width distribution.

## Beta-Gamma Correlations from Polarized Manganese-52

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A RECENT calculation by Curtis and Lewis[1] and by Morita and Morita[2] gives a distribution and correlation function for the beta and the following gamma radiations from oriented nuclei in a $J \rightarrow J$ beta transition. An experimental determination of the parameters in this function can give information on the magnitude and relative phase of the Fermi and Gamow-Teller interactions involved. We have made such experimental determinations on polarized $Mn^{52}$.

In order to evaluate each parameter uniquely, certain assumptions must be made about the nature of the beta interactions. The recent recoil experiments[3] on $A^{35}$ and $Ne^{19}$ and the balanced recoil experiment[4] on $Eu^{152}$ using resonance fluorescence techniques, together with the re-evaluation of the $He^6$ recoil experiment,[5] indicate that vector and axial vector are the predominant forms of the Fermi and Gamow-Teller interactions, respectively. Furthermore, nearly all of the recent experiments measuring the longitudinal polarization of electrons and positrons and also their angular distribution in beta decay are in accord with the two-component neutrino hypothesis where neutrinos of negative helicity (left-handed) are emitted in both Fermi and Gamow-Teller decays. We can consider,