

FIG. 4. Dependence of the lifetime on the ratio, R, of yields in the two exposures. Curve a is computed with the use of a spectrum of  $K^{0}$ 's derived from an energy-dependent matrix element of the form  $\gamma^2 - 1$ , Gaussian momentum distribution in the target nucleus, and isotropic angular distribution in the center-of-mass system. Curve b has isotropic and energy-independent matrix element, and Fermi momentum distribution. Curve c has energyindependent matrix element, Gaussian distribution, and  $\cos^2\!\theta$ angular dependence in the center-of-mass system.

This ratio is given as a function of  $\tau$ , the mean life of the  $K_{2^{0}}$ , by

$$R = \frac{\int P(\beta\gamma) \exp[-t_L(\beta\gamma)/\tau] \{1 - \exp[\Delta t(\beta\gamma)/\tau]\} d(\beta\gamma)}{\int P(\beta\gamma) \exp[-t_S(\beta\gamma)/\tau] \{1 - \exp[\Delta t(\beta\gamma)/\tau]\} d(\beta\gamma)},$$
(2)

where  $t_L(\beta\gamma)$ ,  $t_S(\beta\gamma)$  are the flight times for the longand the short-distance runs respectively for  $K_2^0$  mesons of momentum  $m\beta\gamma c$  which have a spectral distribution  $P(\beta\gamma)$  at 68°;  $\Delta t(\beta\gamma)$  is the time of traversal of the chamber. In Fig. 4, we have plotted the relation between R and  $\tau$  as given by Eq. (2), using an intermediate and two extreme shapes for  $P(\beta\gamma)$  as computed by Sternheimer<sup>6</sup> for associated production of Y-Kpairs in a complex nucleus by incident 3-Bev protons. Applying Eq. (1) to Fig. 4, using curve b, we obtain:

$$\tau = (9.0_{-2.5}^{+3.5}) \times 10^{-8}$$
 second.

An estimate of the  $K_2^0$  lifetime can be obtained from the known  $K^+$  branching ratios<sup>7,8</sup> and lifetime<sup>9</sup> by application of charge independence and the  $\Delta T = \frac{1}{2}$ selection rule<sup>10</sup> (T = total isotopic spin). The  $K_2^0$ lifetime is the reciprocal of the sum of the partial rates for the decay modes  $(\pi^{\pm}\mu^{\mp}\nu)$ ,  $(\pi^{\pm}e^{\mp}\nu)$ ,  $(\pi^{+}\pi^{-}\pi^{0})$ , and  $(\pi^0\pi^0\pi^0)$ . The partial rates for the decays involving three  $\pi$  mesons are evaluated as in reference 10. For the decay modes involving leptons, we have assumed that the partial rates in the  $K_{2^{0}}$  decay are equal to those in  $K^{+}$ decay, and that there is no asymmetry in charge, as is to be expected<sup>11</sup> with the large  $K_2^0/K_1^0$  lifetime ratio measured. This leads to an estimated  $K_{2^0}$  lifetime of  $\tau \sim 5 \times 10^{-8}$  second. A similar result has been obtained by  $Okun^{12}$  using a specific model for the K-meson decay interaction.

No decays have been found which do not fit one of the modes  $(\pi^{\pm}\mu^{\mp}\nu)$ ,  $(\pi^{\pm}e^{\mp}\nu)$ ,  $(\pi^{+}\pi^{-}\pi^{0})$ . Limits of the order <1% may be placed on the existence of the two-body modes  $(\pi^+\pi^-)$ ,  $(\mu^+\mu^-)$ ,  $(e^+e^-)$ ,  $(\mu^\pm e^\mp)$  for  $K_{2^0}$ decay. The absence of the lepton modes is in agreement with current ideas on the universality of the weak interactions. The absence of 2-pion decay is to be expected on the basis of CP invariance.<sup>11,13,14</sup> A discussion of the reverse argument-what do we learn about time reversal from the data contained herein-is given in the accompanying Letter by Weinberg.<sup>15</sup> He concludes that the existence of the reaction<sup>16</sup>  $\theta^0 \rightarrow 2\pi^0$  implies a close identity of the  $\pi^+ - \pi^-$  and  $2\pi^0$ phases, a result which is difficult to understand on any other grounds than *CP* invariance or accident.

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## Time-Reversal Invariance and $\theta_2^0$ Decav<sup>\*</sup>

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 $R^{\rm ECENT}$  experiments<sup>1</sup> show that the  $\theta_{2^0}$  decays much more slowly than the  $\theta_{1^0}$   $( au_2/ au_1 \sim 900)^2$ and that the mode  $\theta_2^0 \rightarrow \pi^+ + \pi^-$  occurs infrequently  $(\leq 0.6\%)$  if at all. This is just what one would expect if CP (or C) were conserved in  $K^0$  decay<sup>3</sup>; conversely, we may ask how much support is given to CP invariance by these experiments,<sup>4</sup> and what additional support may be gained by similar experiments in the near future.

Since we do not assume CP invariance, we must define  $\theta_1^0$  and  $\theta_2^0$  as linear combinations of  $\theta^0$  and  $\bar{\theta}^0$  for which the decay curve is a simple exponential. Now, if there were just one rapid  $K^0$  decay mode (e.g.,  $K^0 \rightarrow \pi^+ + \pi^-$ ), and all others  $(K^0 \rightarrow \pi^+ + e^- + \nu,$  $\gamma + \gamma$ , etc.) were much slower for phase-space or other reasons, then we could understand the experimental situation without reference to CP conservation. (Proof: Define  $\theta_3^0 = a\theta^0 + b\bar{\theta}^0$  as that linear combination which cannot decay into the rapid mode. If no other modes existed  $\theta_3$  would be stable, so  $\theta_2$  would be identical to  $\theta_3$ . Since the other modes do exist, but are very slow,  $\theta_2 \simeq \theta_3$ . Thus  $\theta_2$  decays into the rapid mode with small branching ratio, and has a long lifetime. It is implicit in this reasoning that mass splitting is negligible, since a transition  $\theta^0 \rightarrow \overline{\theta}^0$  with matrix element M would lead to  $\theta_3^0 \rightarrow bM^*\theta^0 + aM\bar{\theta}^0$ , and this linear combination may decay into the rapid mode.)

However, there are two rapid  $K^0 \mod s$ ,  $K^0 \longrightarrow \pi^+ + \pi^$ and  $K^0 \longrightarrow 2\pi^0$ , so the above argument fails unless there is a special phase relation between the two modes.<sup>5</sup> This phase relation would follow from *CP* invariance, but is difficult to understand on any other basis.

To make this more quantitative, we use the Wigner-Weisskopf method, and obtain for the decay rate of  $\theta_1^{0}$  or  $\theta_2^{0}$  into mode  $j,^6$ 

$$\Gamma_{1 \text{ or } 2, j} = \frac{1}{2} (1+\alpha) \Gamma_{\theta j} + \frac{1}{2} (1-\alpha) \Gamma_{\overline{\theta} j} \pm \left[ (1-\alpha^2) \Gamma_{\theta j} \Gamma_{\overline{\theta} j} \right]^{\frac{1}{2}} \cos(\delta_j - \delta),$$
 (1)

where  $\delta \equiv \arg(pq^*)$ . Let us suppose that an experimental upper limit  $\epsilon_j < 1$  can be placed on  $\Gamma_{2j}/\Gamma_{1j}$ . Then from (1) we have

$$\left|\tan^{1}_{2}(\delta_{j} - \delta)\right| \leq \epsilon_{j}.$$
 (2)

Experimentally<sup>1,2</sup>  $\epsilon_{\pi^+\pi^-} \leq 8 \times 10^{-6}$ , so  $|\delta_{\pi^+\pi^-} - \delta| \leq 0.006$ . The experimental situation for the  $2\pi^0$  mode is more uncertain. No  $\gamma$ -ray search has been made in  $\theta_{2^0}$  decay, and the  $\theta_{2^0}$  flux is unknown, so no experimental upper limit has been set on the  $\theta_{2^0} \rightarrow 2\pi^0$  branching ratio. There are, however, two reasons for believing this ratio to be small.

(a) The sum of the decay rates for  $K^0 \rightarrow \pi + \mu + \nu$ ,  $\pi + e + \nu$ ,  $3\pi$  can be calculated<sup>7</sup> on the assumption of a single connection with  $K^+$  decay, and comes out comparable to the experimental  $\theta_2^0$  decay rate. Thus a large branching ratio for  $\theta_2^0 \rightarrow 2\pi$  would leave inadequate room for the 3-particle modes.

(b) It is easy to show that

$$\sum_{j} (\Gamma_{\theta_{j}} \Gamma_{\overline{\theta}_{j}})^{\frac{1}{2}} \sin(\delta_{j} - \delta) = \alpha \Delta / (1 - \alpha^{2})^{\frac{1}{2}}, \qquad (3)$$

$$(\Gamma_{\theta_{j}} \Gamma_{\overline{\theta}_{j}})^{\frac{1}{2}} |\sin(\delta_{j} - \delta)| = \{ [\Gamma_{1j} \Gamma_{2j} - (\frac{1}{2}(1 + \alpha)\Gamma_{\theta_{j}} - \frac{1}{2}(1 - \alpha)\Gamma_{\overline{\theta}_{j}})^{2}] / (1 - \alpha^{2}) \}^{\frac{1}{2}}. \qquad (4)$$

If we ignore mass splitting (so  $\alpha = 0$ ) and final-state interactions in the  $2\pi^0$  mode (so  $\Gamma_{\theta, 2\pi^0} = \Gamma_{\overline{\theta}, 2\pi^0}$ ), we can use (3) and (4) to show that

$$(\Gamma_{1,\,2\pi^{0}}\Gamma_{2,\,2\pi^{0}})^{\frac{1}{2}} \leq (\Gamma_{1,\,\pi^{+}+\pi^{-}}\Gamma_{2,\,\pi^{+}+\pi^{-}})^{\frac{1}{2}} + (\Gamma_{1}{}'\Gamma_{2}{}')^{\frac{1}{2}}, \quad (5)$$

where  $\Gamma_{1,2}'$  are total decay rates for  $\theta_{1,2}^{0}$  into all except

the  $2\pi$  modes. Now  $\Gamma_1 \leq 0.02\tau_1^{-1}$ , and  $\Gamma_2 \leq \tau_2^{-1} - \Gamma_{2, 2\pi^0}$ , so we obtain

$$\Gamma_{2, 2\pi^{0}} \lesssim 0.26 \tau_{2}^{-1}.$$
 (6)

Accepting this upper limit, we have  $\epsilon_{2\pi^0} \leq 0.002$  so that  $|\delta_{2\pi^0} - \delta| \leq 0.09$ . Now, combining our results for the two modes, we have finally

$$\delta_{2\pi^{0}} - \delta_{\pi^{+} + \pi^{-}} \big| \lesssim 6^{\circ}. \tag{7}$$

This phase relation can be put in an isotopic spin language; if the states I=0 and I=2 are created with amplitudes  $a_I$  and scatter with phase shifts  $\eta_I$ , then the quantity  $a_2e^{-i\eta_2}/(a_0e^{-i\eta_0})$  is almost real. This would follow if T or C were conserved (since then numerator and denominator would be separately real), but it is difficult to understand otherwise. (Of course  $a_2=0$ would make the ratio real, but this seems inconsistent with the  $\theta_1$  branching ratios.)

Thus we see that at least one prediction of time reversal invariance is very well fulfilled. It has been suggested that the absence of charge asymmetries in  $\theta_2^0$  decay may also serve as a test of T invariance. We wish to point out that because of the absence of  $\theta_2^{0}\rightarrow 2\pi$  decays, such charge asymmetries must be expected to be very small (typically  $\leq 10\%$ ) independently of whether T is conserved.

Using *CPT* invariance and the unitarity of  $S_{\text{strong}}$  (and taking final-state interactions, which may be large,<sup>8</sup> properly into account), we may prove that

$$\sum_{j \in G} \Gamma_{\theta j} = \sum_{j \in G^*} \Gamma_{\overline{\theta} j};$$

$$\sum_{j \in G} (\Gamma_{\theta j} \Gamma_{\overline{\theta} j})^{\frac{1}{2}} e^{i\delta j} = \sum_{j \in G^*} (\Gamma_{\theta j} \Gamma_{\overline{\theta} j})^{\frac{1}{2}} e^{i\delta j}.$$
(8)

Here G is any set of final states with the property that j cannot scatter strongly into j' if j is in G and j' is not. The set  $G^*$  consists of final states charge conjugate to those in G. By using (8), it is possible to prove that

$$|\alpha| \leq \sum_{j \in G, G^*} \epsilon_j^{\frac{1}{2}} (\Gamma_{1j} + \Gamma_{2j}) / \sum_{j \in G, G^*} (\Gamma_{1j} + \Gamma_{2j}), \quad (9)$$

$$A_G \leq |\alpha|/\epsilon_G^{\frac{1}{2}} + \frac{1}{2}|\alpha|^2(1+1/\epsilon_G), \tag{10}$$

where  $A_G$  is the charge asymmetry,

and

$$A_{G} \equiv \left| \left( \sum_{j \in G} \Gamma_{2j} - \sum_{j \in G^{*}} \Gamma_{2j} \right) / \sum_{j \in G, G^{*}} \Gamma_{2j} \right|, \qquad (11)$$

$$\epsilon_G \equiv \sum_{j \in G, G^*} \Gamma_{2j} / \sum_{j \in G, G^*} \Gamma_{1j}.$$

If G consists of  $\pi^+\pi^-$  and  $\pi^0\pi^0$ , then using our previous estimates of  $\epsilon_{\pi^+\pi^-}$  and  $\epsilon_{\pi^0\pi^0}$ , we have from (9)

$$|\alpha| \lesssim 0.009 \tag{12}$$

[using the lifetime ratio alone, we would have<sup>6</sup>  $|\alpha| \leq 2/(900)^{\frac{1}{2}} = 0.07$ ]. Now for example, suppose that G consists of  $\pi^-e^+\nu$  alone, so that  $G^*$  consists of  $\pi^+e^{-\bar{\nu}}$ .

Experimentally<sup>1,2</sup>  $\epsilon G \gtrsim 0.012$  so, from (10) and (12),

$$A_{\pi^- e^+ \nu} \lesssim 8.5\%$$
 (13)

Probably it will be some time before experiments are performed which are capable of detecting such small charge asymmetries.

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<sup>5</sup> I have learned by private communication from Professor T. D. Lee that this fact was also known to him.

<sup>6</sup> Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957). Where not explained, our notation is the same as in this reference. However, we use outgoing- instead of standing-wave states to define  $H_{j\theta}$ ,  $H_{j\overline{\theta}}$ , and use j to label observed decay modes, which are taken with definite parity, but are not necessarily eigenstates of  $H_{\text{strong}}$ , CP, or T. We assume that  $K^0$  has spin zero and definite parity, and make use of the CPT theorem.

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## Theoretical Angular Distribution of Nucleon-Antinucleon Scattering at 140 Mev\*

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A POSSIBLE explanation of the "large" value of the nucleon-antinucleon  $(N-\bar{N})$  cross section,<sup>1</sup> in the intermediate energy range, has been given by the work of Ball and Chew<sup>2</sup> on the basis of the Yukawa interaction with a "black central hole" to account for the annihilation. Using the Gartenhaus potential,<sup>3</sup> with the spin-orbit term added by Signell and Marshak,<sup>4</sup> in the WKB approximation, they obtained results in satisfactory agreement with the experimental data available at the time.

In recent months, experiments both with bubble chambers and with emulsions have been planned and are now being carried out in order to obtain a more complete knowledge of the  $p-\bar{p}$  interaction. In connection with this program it has seemed worthwhile to perform the calculation of the angular distribution of antinucleon-nucleon scattering, using the transmis-



FIG. 1. Scattering cross section of  $\bar{p}$ -p (neglecting Coulomb scattering) and  $\bar{n}$ -n, at  $E_{\rm lab}$ =140 Mev (in the c.m. system).

sion coefficients and the phase shifts given in reference 2. This is a report of the results of the calculation.

There are two important differences between the N-N and  $N-\bar{N}$  interactions. One is the annihilation process and the other the possibility of charge exchange:  $p+\bar{p}\leftrightarrow n+\bar{n}$ . The first is easily considered by using diagonal scattering matrix elements of the form  $S_{\alpha}=R_{\alpha}e^{2i\delta_{\alpha}}$ , where  $R_{\alpha}$  are the reflection coefficients and  $\delta_{\alpha}$  the real scattering phase shifts for the  $\alpha$  eigenstate. The second involves isotopic-spin considerations.<sup>5</sup> If the amplitudes for scattering in isotopic-spin singlet and triplet states are represented by  $f^1$  and  $f^3$ , respectively, then the amplitudes for ordinary (o) and exchange (e)  $p-\bar{p}$  and  $n-\bar{n}$  scattering are given by

$$f^{o} = \frac{1}{2}(f^{1} + f^{3}), \quad f^{e} = \frac{1}{2}(f^{1} - f^{3})$$

For the  $p-\bar{n}$  and  $n-\bar{p}$  systems, which are pure isotopic triplets, there is no exchange scattering and the ordinary scattering amplitude is simply  $f^3$ . Once these complications are recognized, the formalism developed by Blatt and Biedenharn<sup>6</sup> can be consistently used.

Since in the WKB approximation there is no way to calculate "mixture parameters," interchange between waves with the same total angular momentum and parity but with different orbital angular momentum has not been considered in reference 2. Therefore it must be assumed that the mixture parameters are small if this method is expected to give good accuracy. With mixing neglected, the scattering cross sections were calculated; the results are plotted in Figs. 1 and 2.

The forward peak in the ordinary scattering is largely a diffraction effect, since the S and P waves are mostly absorbed, leading to annihilation. The forward peak in the exchange scattering is much weaker.

Notwithstanding the very limited experimental data available up to now,<sup>7</sup> a crude test of the theory can already be made by integrating the  $p-\bar{p}$  elastic differential cross section of Fig. 1 over the forward