Kramers-Kronig transforms of the selected σ_1/σ_N functions required for deriving the associated transmission curves shown in Fig. 2.

[1] R. E. Glover, III, and M. Tinkham, Phys. Rev. **108**, 243 (1957).
[2] Corak, Goodman, Satterthwaite, and Wexler, Phys. Rev. **102**, 656 (1956); W. S. Corak and C. B. Satterthwaite, Phys. Rev. **102**, 662 (1956).
[3] These curves cannot be ruled out by the result of E. Fawcett, [Proc. Roy. Soc. (London) **A232**, 519 (1955)]. He finds, for bulk tin, that $R/R_N \to 0$ at $T=0$ for $\omega = 0.5kT_c/\hbar$, but the surface resistance is so limited by the superconducting penetration depth that it is unlikely that a value of $\sigma_1 < 0.04$ would have been noticeable.
[4] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).
[5] J. Bardeen and D. C. Mattis (to be published).
[6] The difference at low frequencies is essentially the same as that pointed out by Bardeen, Cooper, and Schrieffer (reference 4) in their reference 27.

Superconducting Energy Gap Inferences from Thin-Film Transmission Data*

M. Tinkham,
Department of Physics, University of California, Berkeley, California
AND
R. E. Glover, III, *Department of Physics, University of North Carolina, Chapel Hill, North Carolina*
(Received February 10, 1958)

SINCE publication[1–3] of results on far-infrared transmission experiments with superconducting films, questions have been raised[4] concerning the justification of the conclusion that there is a gap of width $E_g = \hbar\omega_g \approx 3kT_c$ in the electronic excitation spectrum. This conclusion was based on an extrapolation to zero of data on σ_1/σ_N, the real part of the reduced conductivity. For $\hbar\omega > 5kT_c$ the interpretation of the experimental results is straightforward. It was possible to measure σ_1/σ_N in the range from $36kT_c$ to $5kT_c$ where its value fell from 1.0 to about 0.3 (Fig. 6 of reference 3). In the low-frequency end of this range, σ_1/σ_N was found to be falling very rapidly, and extrapolation indicated an intercept between 3 and 4 kT_c. This behavior suggested a region of very sharply diminished density of states (or a gap) having a width between 3 and 4 kT_c. The data which lead to this conclusion were obtained from films of two different metals with varying thickness, purity, and degree of anneal, indicating that the behavior was inherent in superconductors and was not strongly effected by strains, purity, or surface condition of the samples used. To our knowledge, this was the first indication of a gap of $\approx 3kT_c$ rather than $\approx 1.5kT_c$. Our analysis was based directly on the infrared measurements for $\hbar\omega \gtrsim 5kT_c$ where σ_1 can be determined unambiguously. Unlike the analysis suggested in reference 4, it does not depend critically on our microwave measurements, which are less dependable due to the necessity of introducing large corrections for standing-wave effects.

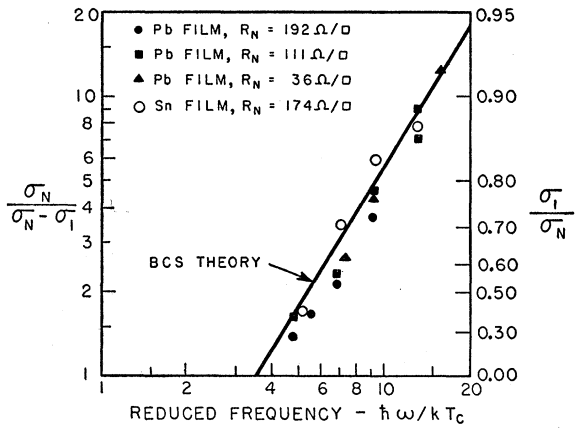

Fig. 1. Logarithmic plot of data (from Fig. 6 of reference 3) showing the onset of absorption expressed in terms of σ_1/σ_N, the real part of the reduced conductivity. This absorption edge is attributed to absorption across an energy gap in the superconducting state. The solid line is the prediction of the theory of Bardeen, Cooper, and Schrieffer (reference 8).

We would like here to reconsider the problem of extrapolating σ_1/σ_N to zero. In making such an extrapolation one could proceed purely empirically to find a way of plotting which would make the data fall on a straight line. This could then be extrapolated to an intercept. However, theoretical considerations help in choosing a suitable form. An elementary argument for a simple gap[2] in the density of states function would suggest $\sigma_1/\sigma_N = 1 - \omega_g/\omega$. The observed rise was found to be faster than this, more like $1 - \omega_g^2/\omega^2$. This was attributed to a humping up of states displaced from the gap to either side. Such humping is also required to explain the observed specific heat[5,6] and the nuclear relaxation data.[7] Subsequently, Bardeen, Cooper, and Schrieffer[8] have proposed a detailed theory which has enjoyed remarkable success in explaining most of the phenomena of superconductivity. This theory predicts the humping of states, and it makes a definite prediction for the shape of the absorption edge, namely

$$\frac{\sigma_1}{\sigma_N} = \left(1 + \frac{\omega_g}{\omega}\right) E(k) - 2\frac{\omega_g}{\omega} K(k),$$

where $\hbar\omega_g = 3.5kT_c$, $k = (\omega - \omega_g)/(\omega + \omega_g)$, and $E(k)$ and $K(k)$ are complete elliptic integrals. This function is fitted within a few percent by the simple approximation

$$\frac{\sigma_1}{\sigma_N} = 1 - \left(\frac{\omega_g}{\omega}\right)^{1.65}.$$

These considerations suggest that the data be plotted in the form $\log[(1 - \sigma_1/\sigma_N)^{-1}]$ *vs* $\log(\hbar\omega/kT_c)$. In such a plot, any function of the form $1 - (\omega_g/\omega)^n$ will result in a straight line, from which ω_g and n may be read off. Data from Fig. 6 of reference 3 are plotted in this way in Fig. 1, and the straight line is the prediction of the BCS theory. Within the scatter (which is of about the same magnitude as the statistical error of the points) the experimental data would fit a straight line with the BCS slope ($n=1.65$), and an intercept of $\hbar\omega_g \approx 3\text{–}4\ kT_c$. The quantitative agreement with the prediction of the BCS theory is striking, and strongly supports that theory. However, the extrapolation is plausible simply from the goodness of fit to a straight line.

Such a conclusion might be challenged by assuming that the σ_1/σ_N curve breaks off its essentially straight path in the short distance over which the extrapolation is made. It seems likely, however, that any such break might be attributed to a small residual strain-induced normal conductivity. Such an effect would not change the conclusion that at low temperature there is a sudden onset of additional absorption in the region near 3–4 kT_c, which is most reasonably attributed to an energy gap of this width. Subsequent experiments[9,10] are consistent with this conclusion, which differs from that ($E_g \approx 1.5kT_c$) which had been drawn earlier[5] from the specific heat data alone.

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Cross Sections for Double and Triple Meson Production in Hydrogen by Photons with Energies up to 1 Bev*

J. M. Sellen, G. Cocconi, V. T. Cocconi, and E. L. Hart
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received March 5, 1958)

Diffusion cloud chamber pictures taken at the Cornell synchrotron were analyzed for the three-pronged events ascribable to multiple meson production:

$$\gamma+p\rightarrow p+\pi^++\pi^- \quad \text{(reaction 1)}$$
$$\rightarrow p+\pi^++\pi^-+\pi^0 \quad \text{(reaction 2)}$$
$$\rightarrow n+\pi^++\pi^++\pi^- \quad \text{(reaction 3).}$$

This is a report on the results concerning cross sections. The data on angular and momentum distributions will be published later.

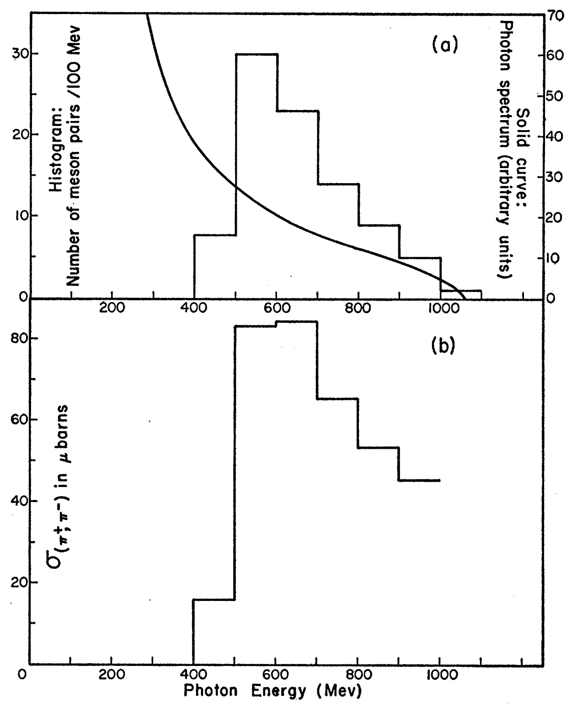

FIG. 1. (a) The histogram represents the energy distribution of the photons responsible for the events of double meson production observed. The curve gives the spectrum of the photons entering the chamber, as deduced from the analysis of ~1000 electron pairs and triplets. (b) Total cross section for the double meson production of reaction (1), as a function of the primary photon energy. The absolute values are determined as described in the text.

The bremsstrahlung beam of 1.05-Bev peak energy was passed through 4 meters of LiH to eliminate most of the photons below ~20 Mev, then collimated to $\frac{1}{8}$ in.$\times\frac{5}{8}$ in. and made to enter a diffusion chamber (24-in. useful diameter; sensitive layer ~3 in.), filled to 20 atmos of H_2, operating in a magnetic field of 6 kilogauss. The beam intensity was adjusted to produce on the average 8 electron pairs/picture.

In 40 000 pictures we have observed 94 events of multiple meson production on protons. Of these, 90 were classified as due to reaction (1), 3 to reaction (2), and 1 to reaction (3).

Contaminations due to the presence of the methyl alcohol vapor or impurities in the chamber proved to be negligible, since of about 40 stars observed in the same pictures, only ~$\frac{1}{5}$ contained a single π meson, and none a meson pair. Eight cases of $p+e^++e^-$ were observed, and interpreted either as events $\gamma+p\rightarrow p+\pi^0$, with the π^0 undergoing Dalitz decay, or as electron pairs with high proton recoil momentum. The expected number of cases of double μ-meson production is less than one.

A major fraction of the film has been scanned twice. The second scanning yielded all—and no more than—the events found in the first scanning.

For reaction (1) the complete analysis of the events