angular frequency $\omega = kv_s$ (v_s is the velocity of sound), one must now solve

$$(1+i\omega\tau - ik\tau v_z)\psi - \omega_c\tau\left(v_x\frac{\partial\psi}{\partial v_y} - v_y\frac{\partial\psi}{\partial v_x}\right) = \left(\frac{eE_x\tau}{m}+u\right)\frac{v_x}{v} + \left(\frac{eE_y\tau}{m}\right)\frac{v_y}{v} - u(1+i\omega\tau - ik\tau v_z)\frac{v_x}{v}. \quad (11)$$

The solution is of the form

$$\psi = (v_x/v)Q_x(v_z,v,z,t) + (v_y/v)Q_y(v_z,v,z,t). \quad (12)$$

When this is inserted into (11) the coefficients of v_x and v_y may be equated, yielding

$$\begin{aligned}(1+i\omega\tau - ik\tau v_z)Q_x - \omega_c\tau Q_y &= (eE_x\tau/m)+u - u(1+i\omega\tau - ik\tau v_z), \quad (13)\\ (1+i\omega\tau - ik\tau v_z)Q_y + \omega_c\tau Q_x &= eE_y\tau/m.\end{aligned}$$

If a second imaginary unit j is invoked, with $ij = ji$, Eqs. (13) are solved by

$$\bar{Q} = \frac{(e\bar{E}\tau/m)+u-u(1+i\omega\tau - ik\tau v_z)}{1+i\omega\tau + j\omega_c\tau - ik\tau v_z}, \quad (14)$$

where $\bar{Q} \equiv Q_x + jQ_y$ and $\bar{E} \equiv E_x + jE_y$. Subsequent analysis proceeds as in reference 6, with $\bar{E}, \bar{J} \equiv J_x + jJ_y$, and $\bar{T}_z \equiv T_{xz} + jT_{yz}$ replacing E_x, J_x, and T_{xz}, respectively, in the electrodynamic relation and in the transport relations once integration over the angle ϕ in the x-y plane has been performed. For a conductor of density ρ, having N conduction electrons per unit volume and Fermi velocity v_0, one finds

$$\alpha = \frac{Nm}{\rho v_s\tau}\,\mathrm{Re}\left\{\frac{[k_0^2+(1+j\omega_c\tau)k^2](1-G)}{G'k_0^2+k^2}\right\}, \quad (15)$$

where $k_0^2 \equiv 4\pi i\omega Ne^2\tau/mc^2$, $G \equiv \frac{3}{2}[1-(1+a^{-2})(1-a^{-1} \times \tan^{-1}a)]$, $G' \equiv Ga/kl$, $a \equiv kl/(1+i\omega\tau + j\omega_c\tau)$, and $l \equiv v_0\tau$. When $H=0$, the present result reduces to the required value.[4,6]

The frequencies of practical interest are such that $\omega\tau \ll 1$.[5] Since v_0 is generally much larger than v_s, it is nevertheless possible for kl to exceed unity. In the strong-field limit, $\omega_c\tau \gg kl$, the attenuation approaches the limiting value

$$\alpha = \tfrac{1}{5}Nmv_0l\omega^2(1+\omega_c^2\tau^2)^{-1}, \quad (16)$$

for all frequencies $\omega \ll \tau^{-1}$. This reflects the fact that magnetic foreshortening of the mean free path in the direction of transport, rather than finite wavelength disruption of transport, is the limiting factor. When $k^2l^2 \ll 1$, formula (16) is valid at all field strengths, in agreement with (8). As the frequency is increased into the $kl > 1$ regime, the attenuation at the lower field strengths rises less rapidly with frequency than indicated by Eq. (16), due to the effect of finite wavelength. The attenuation falls monotonically with increasing field strength, however, according to

$$\alpha = \frac{Nm}{\rho v_s\tau}\,\mathrm{Re}\left(\frac{1-G}{G'}\right). \quad (17)$$

This formula is valid as long as $|k_0^2/(1+j\omega_c\tau)| \gg k^2$ and $|G'k_0^2| \gg k^2$. However, it goes over into (16) in the strong-field limit. In fact, the structure of (15) is such that the parameter k_0^2 cannot play a significant role as long as $\omega\tau \ll 1$, so that (17) is a good interpolation formula at all such frequencies, and at all field strengths (up to the point where orbit quantization becomes of importance).

Calculations are in progress on attenuation at finite frequencies for cases II and III, and for longitudinal waves with a transverse magnetic field. One expects to find resonant attenuation for special values of $kl/\omega_c\tau \sim 1$. Data indicating this effect have recently been reported.[7]

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Isotopic and Other Types of Thermal Resistance in Germanium*

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SEVERAL authors[1] have recently called attention to a 1942 comment of Pomeranchuk that naturally occurring isotopes in a crystal will disturb the periodicity of the lattice and thus produce thermal resistance. They have pointed out that the failure to observe the exponential increase in thermal conductivity with decreasing temperature in seemingly very perfect crystals of Ge, predicted by Peierls' theory, could be accounted for by the random mixture of the 5 isotopes in normal germanium.

We obtained seventeen grams of 95.8% Ge^{74} from the Stable Isotope Division of the Union Carbide Nuclear Company, Oak Ridge, Tennessee. The isotopic analysis furnished with the Ge^{74} is given in Table I. This was purified by careful zone refining and used to grow a single crystal in the [100] direction using a

TABLE I. Oak Ridge isotopic analysis of enriched Ge^{74}.

Isotope	Atomic percent	Precision
78	0.691	±0.010
72	1.135	±0.016
73	1.56	±0.01
74	95.80	±0.03
76	0.818	±0.008

modified Teal-Little crystal puller.[2] From this crystal the sample, of dimensions shown in Fig. 1, was prepared. Encapsulated Ge thermometers[3] soldered to the arms indicated were used below 25°K for measuring temperatures and copper-constantan thermocouples were used above. Measurements of electrical conductivity and Hall, thermoelectric, and Nernst effects show the sample to be very high purity *n*-type material. There are about 1.2×10^{13} excess donor atoms per cc.

The results are shown in Fig. 1 where κ^{74}, the thermal conductivity of the enriched sample, is compared with κ^{normal} for a control sample. The samples are comparable physically and electrically in all ways that can be detected except for the difference in isotopic content which, we thus conclude, has a profound effect on thermal conduction. Below 5°K, $\kappa^{74}=0.060T^3$ (dotted line, Fig. 1). This agrees well with Casimir's theory for thermal resistance caused by boundary scattering. Using McSkimin's[4] data to estimate the average velocity of sound as 3.50×10^5 cm/sec, and the heat capacity data of Keesom and Pearlman,[5] one calculates a boundary-scattering mean free path of 1.80 mm. This is to be compared with a mean free path of 1.57 mm calculated from the cross-sectional area. The $T^{2.4}$ behavior of the normal sample below 5°K, and the similar results of other research[6] above 2°K, must thus be due to the existence of isotopic scattering. Such an effect has been predicted by Slack.[7] He shows, using the theoretical results of Klemens,[8] that when the two scattering mechanisms, namely boundary and isotopic scattering, are important, the latter will be observable down to one-tenth the temperature at which the conductivity maximum occurs. In fact, the measured difference in resistivity between the two samples has a T^{-1} dependence between 4° and 15°K which is the result of the combined effect of isotopic and boundary scattering. An accurate formulation of this problem has been made by Herring.[9] An approximate analytical solution gives values for the ratio κ/κ^{74} which agree well with experiment throughout the temperature range below the maximum; moreover, no adjustable constants are involved.

The considerations of Slack[1] would lead one to expect an almost 15-fold increase of κ^{74} over κ^{normal} at temperatures slightly above the conductivity maximum, corresponding to the 15-fold decrease in the mean square deviation in mass if only isotopic scattering is important in this region. The observed ratio of only about 3 therefore indicates the presence of additional scattering. Etch-pit counts, made with the help of F. Vogel, show only about 10^4 dislocations/cm^2—a concentration insufficient to be noticeable. Our initial search was for some other electrically inactive defect probably present in the normal as well as the enriched sample. The extra resistance is about 0.022 cm deg per watt at 20°K, an amount sufficiently great so that its removal from normal Ge should be clearly evident. Dissolved gases which, under some circumstances, can be present in rather surprisingly large concentrations,[10] were eliminated by measuring κ for a crystal prepared in vacuum by the floating-zone technique.[11] The results agree with the control sample. Other experiments,[6] using crystals prepared differently, also agree. The elusive defect, if it were to exist, would be present in about the same concentration regardless of the method of crystal preparation. This is a rather unlikely hypothesis.

Evidence from three independent experiments[12–14] has recently accrued which gives detailed information about the vibrational spectrum of germanium. In particular, along the [111] axis of the Brillouin zone the transverse acoustical branches cross the zone boundary with an energy of only about 0.007 ev. It has been called to our attention[9] that this surprisingly low value, only something like one-quarter of the Debye temperature, θ_D, means that umklapp-scattering can persist as an important mechanism below one-tenth θ_D and indeed is the additional scattering mechanism we had been searching for. Thus no temperature region exists where the temperature dependence of the thermal conductivity is exponential even in the absence of isotopic or other defect scattering other than boundary

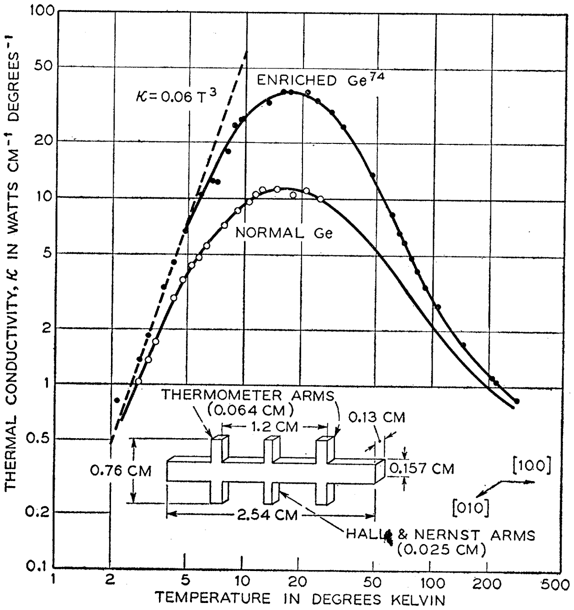

FIG. 1. Isotope effect on thermal conduction in Ge.

scattering. The use of the Debye temperature in estimating umklapp-scattering[1] is risky unless details of the vibrational spectrum are known. The data of Haynes[12] indicate that the situation for silicon should be similar to that of germanium; furthermore the temperature dependence of θ_D [15] for lead and gray tin indicates roughly a similar behavior, while that of diamond is probably more nearly normal.

The high-temperature results (not corrected for radiation losses in Fig. 1) approach a difference corresponding to a constant isotopic thermal resistance of roughly 0.15 watt^{-1} cm deg. This is the kind of behavior expected and should be helpful in connection with other work in determining phonon-phonon scattering times.

We wish particularly to acknowledge the skillful contribution of those[2] responsible for the purification and growth of the crystal. We are indebted to Dr. N. B. Hannay for encouragement and stimulation in starting this research. During the course of the work we have profited from informative discussions with Dr. C. Herring and Dr. G. A. Slack.

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Structure of the α and β Forms of Solid He^3†

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Two solid forms of He^3, designated α and β, were recently reported[1] to exist below and above, respectively, a transition line which intersects the melting curve at 3.15°K and 141 kg/cm^2. To gain some insight into the nature of this transition, we have undertaken to investigate the structure of these two forms by x-ray diffraction. The beryllium capillary into which the helium was solidified and the associated diffraction and cryogenic apparatus were those that have been used in the study[2] of the structure of He^4.

The structures were determined by using the Laue and the Debye-Scherrer-Hull methods. He^3, like He^4, readily forms large crystals, and in several runs we were able to obtain a Laue diffraction pattern from only a single crystal. It is much more difficult to grow helium crystals small enough for a suitable powder pattern. The powder lines produced were always spotty and ill-defined. Also, because of the large zero-point vibrations of the atoms, the decline of the intensity of the diffraction lines with increasing angle was so steep that not many lines could be observed.

The Laue reflections of the α form of He^3 were indexed on the basis of a cubic lattice. The reflections observed were of the forms of $\{110\}$, $\{200\}$, and $\{121\}$. There were no reflections for which the sum of the indices was odd. It was therefore concluded that the α form has the body-centered cubic structure. In the diffraction photographs of coarse powders at 1.9°K and under a pressure of 100 kg/cm^2, the reflection from (110), the only one that could be observed, gave the length of the cube axis $A=4.01\pm0.03$ A. The density of the solid calculated from this size unit cell is 0.154 ± 0.004 g/cm^3. The value derived by the extrapolation to these conditions from the directly measured densities[1] is 0.1515 ± 0.0002 g/cm^3.

The β form was found to have the hexagonal closest-packed structure. The Laue reflections that were obtained were of the forms of $\{100\}$, $\{002\}$, $\{101\}$, $\{102\}$, $\{103\}$, $\{110\}$, $\{112\}$, and $\{201\}$. No reflections could be seen for which $h+2k$ was equal to $3n$ when l was odd. The lengths of the axes of the unit cell were determined from the (100), (002), and (101) reflections from a powder at 3.3°K and under a pressure of 183 kg/cm^2 to be $A=3.46\pm0.03$ A and $C=5.60\pm0.03$ A. The density computed from these dimensions is 0.172 ± 0.004 g/cm^3. The density extrapolated from the directly measured values[1] is 0.1694 ± 0.0003 g/cm^3.

A more complete and extensive report will be submitted later.

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