contributions from unwanted processes, and may in this sense be considered upper limits on the desired cross sections. In the section on backgrounds, however, estimates of other processes have been made. Considering the energy sensitivity of the counters, the responses to these processes are found to be small compared with the observed cross sections, with the exception of the contribution of radiative pair production to the 70° data which may amount to 15%.

An additional error of ±8% is assigned to the absolute scale.

DISCUSSION

The results may be compared with theoretical predictions (Fig. 6). The scattering calculated by Powell for a point proton with the static anomalous magnetic moment has the correct amplitude terms independent of and linear in frequency.

With the experimental data is also shown the 0° scattering which is predicted by dispersion theory from the analysis of the photopion production experiments.[3] Several theoretical studies have been made in which the dispersion theory is extended to angles other than 0°.[17] This process contains ambiguities which require the use of a model. Detailed discussions of the problems involved are yet to appear in the literature.

The data may be compared with that of Pugh *et al.*[16] who worked with energy-resolved detection and 135-Mev bremsstrahlung. Their data are in fair agreement with the Powell formula at 90° and 135°, showing a decrease from it at 45° for energies above 100 Mev. Our data show no qualitative disagreement with theirs.

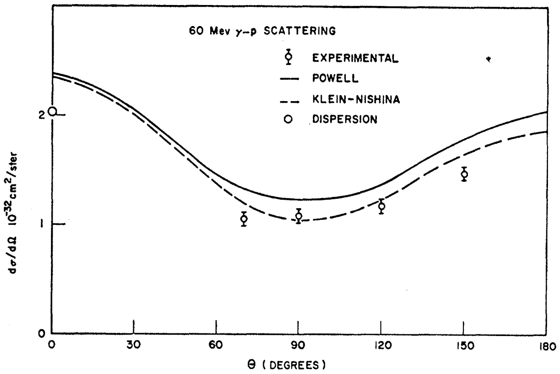

FIG. 6. Experimental and theoretical differential cross sections.

[17] J. Mathews and M. Gell-Mann, Bull. Am. Phys. Soc. Ser. II, 2, 392 (1957); and Watson, Zachariasen, and Karzas, Bull. Am. Phys. Soc. Ser. II, 1, 383 (1956).

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Λ-Nucleon Potential from Hyperfragment Data*

MARC ROSS, *Indiana University, Bloomington, Indiana*

AND

D. B. LICHTENBERG, *Physikalisches Staatsinstitut, Hamburg, Germany*†

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The volume integrals of the Λ-nucleon potentials in the triplet and singlet spin states are deduced from hyperfragment binding-energy data. The effects of tensor forces are neglected in the calculation but are discussed qualitatively. Results are sensitive to the sizes and shapes of the nuclei in which the Λ is bound, but are not very sensitive to Λ binding energies. Results also depend on the range of the Λ-nucleon potential and on the spin configurations of the nuclei. Within the approximations made, the Λ-nucleon potentials are consistent with experiment and agree with theoretical potentials due to pion exchange. A crude determination of the Λ-nucleon potential from the observed lifetime for mesonic decay of hyperfragments is consistent with the binding-energy determination.

I. INTRODUCTION

THE purpose of this work is to interpret hyperfragment data, especially binding energies, in terms of two-body Λ nucleon (hereafter written ΛN) potentials. We assume throughout that the spin of the Λ is $\frac{1}{2}$.

Several analyses of hyperfragment binding energies have appeared recently.[1,2] This new discussion, which is very similar to that of Dalitz,[1] is distinguished by two features:

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† This work was begun while the author was at Indiana University.

[1] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics*, Session V (Interscience Publishers, Inc., New York, 1956), and Midwest Conference on Theoretical Physics, State University of Iowa, 1957 (unpublished); B. W. Downs, Bull. Am. Phys. Soc. Ser. II, 2, 175 (1957); J. T. Jones and J. M. Keller, Nuovo cimento 4, 1329 (1956). G. H. Derrick, Nuovo cimento 4, 565 (1956).

[2] L. Brown and M. Peshkin, Phys. Rev. **107**, 272 (1957).

(1) Accurate information on the nuclei underlying the hyperfragments in question is essential to interpreting the binding energies. New data[3] on electron scattering from lithium have just become available. The new data enable a simple consistent explanation of the several hyperfragment binding energies which can be reliably treated.

(2) The interpretation of binding energies in terms of a ΛN potential depends sensitively on the volume integral of the potential, but also depends significantly on the range of that potential. There is not yet any direct empirical basis for determining that range. We feel that we can reliably determine the range by field-theoretical arguments. We have considered, in particular, π- and K-meson theories of hyperon-nucleon forces.[4] The theory in which the π plays the major role appears to be more readily successful; but, in either case, the theoretical potential (including an added hard core at small distances) has certain general features of shape. That is, the ΛN potential has a hard core plus an attractive tail of characteristic range $\hbar/2m_\pi c$ or $\hbar/m_K c$. This singular potential is not readily amenable for use in calculation of hyperfragment binding energies. Instead, because of the core, it is simpler to construct an equivalent smooth and monotonic potential which leads to the same low-energy scattering behavior as the singular potential. We feel it is this behavior which is principally involved in the hyperfragment binding-energy problem. If the singular potential is adjusted in strength to yield very roughly the correct ΛN low-energy interaction, then the associated equivalent smooth potential has a characteristic range: For example, if the potential is a Gaussian $[\exp(-\alpha r^2)]$, its mean square radius is $r_0^2=3/2\alpha=(1.1\times10^{-13}\text{ cm})^2$ if pions are responsible for the forces. If K particles are responsible, the range is probably similar. The range of this equivalent potential is certainly more significant to the problem at hand than the range of the tail of the actual potential. This point is discussed further, and some numerical details exhibited in reference 4.

We shall limit our calculations at present to fragments with good binding-energy measurements and good information on associated nuclei. Good binding-energy data have been obtained by the Wisconsin group,[5] Chicago group,[6] and others for ${}_\Lambda H^3$ (i.e., $\Lambda+H^2$), ${}_\Lambda H^4$, ${}_\Lambda He^4$, ${}_\Lambda He^5$, ${}_\Lambda Li^7$, ${}_\Lambda Li^8$, and ${}_\Lambda Be^9$. We shall do calculations for ${}_\Lambda H^3$, ${}_\Lambda He^5$, ${}_\Lambda Li^7$, and ${}_\Lambda Li^8$.

We omit the four-body fragments and ${}_\Lambda Be^9$ because the sizes of the corresponding underlying nuclei H^3, He^3, and Be^8 have not been well determined, as have the sizes of H^2, He^4, Li^6, Li^7. For the latter nuclei very fine electron-scattering data exist[3,7] (in addition to further detailed information on H^2). We shall base our numerical work on these data (using the shapes suggested by Hofstadter),[7] after unfolding (subtracting out) a Gaussian distribution corresponding to the finite proton size. The conclusions are sufficiently sensitive to size and shape to make it unprofitable to consider the cases where good experimental information is not yet available.

In Sec. II we incorporate the new material above into the simplest calculational method for deducing a two-body potential from the binding energies. (The three-body system has already been treated by somewhat more elaborate variational methods. The results of these treatments will be adopted without being repeated here.) That is, we assume that the nucleus, to which the Λ is attached, is undistorted in the presence of the Λ. Given a binding energy and knowing the nuclear density distribution and range of the ΛN potential, one directly finds the Λ-nucleus potential. Knowing the probability of different nucleon spin orientations in the nucleus, and neglecting complications such as many-body effects and tensor forces, we can immediately interpret the Λ-nucleus potential in terms of ${}^1S$ and ${}^3S$ ΛN potentials. We will briefly discuss in Sec. III possible effects of nuclear distortion on the results; their dependence on nuclear range and shape, and on the various other approximations mentioned.

II. CALCULATION OF THE ΛN POTENTIAL FROM BINDING ENERGIES

Consider that a two-body ΛN potential $U(r)$, including spin dependence, is responsible for hyperfragment binding. The potential the Λ sees at a nucleus (by nucleus we will mean the A nucleons in a hyperfragment of mass $A+1$) is then

$$v=\sum_i U(r_\Lambda-r_{ni}),$$

where r_Λ is the position of the Λ and r_{ni} are the positions of the nucleons. We assume, as stated in the introduction, that the hyperfragment wave function has the form

$$\Psi=\psi(r_{ni},\cdots,r_{nA})\phi(r_\Lambda),$$

where ψ is the nuclear wave function and ϕ is the Λ wave function. The Schrödinger equation in the Λ variable alone is

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)\phi=E\phi,$$

where $-E$ is the binding energy of the Λ, μ is its reduced mass, r is the distance of the Λ from the center of mass of the nucleus, and

$$V(r)=(\psi,v\psi)=\int d^3r_n\langle U(r-r_n)\rangle\rho(r_n).$$

[3] R. Hofstadter and G. R. Burleson, Bull. Am. Phys. Soc. Ser. II, 2, 390 (1957).

[4] D. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957); **109**, 2163 (1958).

[5] Schneps, Fry, and Swami, Phys. Rev. **106**, 1062 (1957).

[6] Levi Setti, Slater, and Telegdi, reported by V. Telegdi, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), Session VIII. A Filipowski *et al.*, Acta Phys. Polon. **16**, 139 (1957).

[7] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

Here ⟨ ⟩ denotes the average over nucleon spin orientations and $\rho(r)$ is the nuclear density distribution. We shall refer to $V(r)$ as the Λ-nucleus potential. In practice we shall start by determining V. From V we deduce the nuclear spin average over the volume integral of U, i.e.,

$$\langle\Omega\rangle \equiv -\int d^3r \langle U(r)\rangle = -(1/A)\int d^3r\, V. \qquad (1)$$

On determining ⟨Ω⟩ for two nuclei, we obtain the volume integrals of the potentials in the ^1S and ^3S ΛN states. On comparing the volume integrals with those obtained from other nuclei, we determine whether the method gives consistent results.

In the following, we shall determine V (and ⟨Ω⟩) for simplicity by variational methods (this approximation leads to an overestimation of ⟨Ω⟩ of the order of 1%). We will write down general analytical expressions for ⟨Ω⟩ for two general shapes for $\rho(r)$ so that results for additional hyperfragments can be found immediately when sufficient data become available.

The ΛN potential is assumed to be of the form $U=U_0\exp(-\alpha r^2)$ where U_0 is spin dependent and $\alpha=1.24\times(10^{13}\text{ cm}^{-1})^2$. We shall assume a trial wave function for the Λ of the form

$$\phi=N'^{\frac{1}{2}}\exp(-\delta r^2/2), \quad \text{with} \quad N'=(\delta/\pi)^{\frac{3}{2}} \qquad (2)$$

(except for $_\Lambda H^3$ where a Hulthén trial function is used). Now consider two nuclear density distributions:

$$(i) \quad \rho=AN\exp(-\beta r^2) \qquad (3)$$

where $N=(\beta/\pi)^{\frac{3}{2}}$ and $r_0=3/2\beta$. Then $V\sim\exp(-\gamma r^2)$ with $\gamma=\alpha\beta/(\alpha+\beta)$. One determines δ from

$$E=\frac{3}{4}\frac{\hbar^2}{\mu}\delta\left(1-\frac{2}{3}\frac{\delta+\gamma}{\gamma}\right), \qquad (4)$$

and then ⟨Ω⟩ from

$$\langle\Omega\rangle=\frac{\pi^{\frac{3}{2}}\hbar^2}{2\mu A\delta^{\frac{1}{2}}}\left(\frac{\delta}{\gamma}+1\right)^{\frac{5}{2}}. \qquad (5)$$

We use this Gaussian shape for He^4.

$$(ii) \quad \rho=AN(1+p\beta r^2)\exp(-\beta r^2), \qquad (6)$$

where

$$N=\frac{2}{2+3p}\left(\frac{\beta}{\pi}\right)^{\frac{3}{2}}, \quad r_0=\frac{2+5p}{2+3p}\left(\frac{3}{2\beta}\right),$$

and $\frac{3}{2}p$ is the relative number of P-shell nucleons [with the $r^2\exp(-\beta r^2)$ distribution] to S-shell nucleons [with the $\exp(-\beta r^2)$ distribution]. Then $V\sim(1+cp\gamma r^2)\exp(-\gamma r^2)$ where $\gamma=\alpha\beta/(\alpha+\beta)$, and $c=1/(1+3p\gamma/2\alpha)$. One determines δ from

$$E=\frac{3}{4}\frac{\hbar^2}{\mu}\delta(1-D^{-1}), \qquad (7)$$

where

$$D=\frac{3}{2}-\frac{5\delta}{2(\delta+\gamma)}+\frac{\delta}{\delta+\gamma+\frac{3}{2}cp\gamma},$$

and then ⟨Ω⟩ from

$$\langle\Omega\rangle=\frac{3\pi^{\frac{3}{2}}\hbar^2}{4\mu A\delta^{\frac{1}{2}}}\left(\frac{\delta}{\gamma}+1\right)^{\frac{5}{2}}\frac{(1+\frac{3}{2}p)c}{\delta/\gamma+1+\frac{3}{2}pc}D^{-1}. \qquad (8)$$

We use this modified Gaussian shape for P-shell (Li) nuclei.

TABLE I. Values of parameters for several hyperfragments. Here V is the Λ-nucleus potential, $-E$ is the Λ binding energy, δ is a measure of the Λ wave function [see Eq. (2)], and ⟨Ω⟩ is the average volume integral of the Λ-nucleus potential (per nucleon).

Hyper-fragment	Shape of nucleus and of V	Rms radius of nucleus[a] (10^{-13} cm)	Rms radius of V (10^{-13} cm)	$-E$[b] Mev	δ (10^{-13} cm^{-1})^2	⟨Ω⟩ Mev ×cm^3 ×10^{-39}
$_\Lambda H^3$	Hulthén	1.96	2.10[c]	0.6[d]	⋯	340[e]
$_\Lambda He^5$	Gaussian	1.61	1.81	2.5	0.372	220
$_\Lambda Li^7$	Mod. Gauss.	2.20	2.37	4.2	0.286	220
$_\Lambda Li^8$	Mod. Gauss.	2.15	2.32	5.2	0.318	190

[a] References 3 and 7. These numbers are, as yet, uncorrected for finite proton size.
[b] See references 5 and 6.
[c] The ΛN potential was approximately folded in, assuming that V is of Hulthén shape as well as the nucleus H^2.
[d] These binding energies are not too well determined. For example, since this $_\Lambda H^3$ calculation was completed, the accepted value of $-E$ dropped to 0.25 Mev, due to more hyperfragment measurements, and now it has risen by perhaps 0.5 Mev due to new and larger Q values for the Λ decay. (See, for example, D'Andlau *et al.*, *Padua-Venice Conference on Mesons and Recently Discovered Particles, 1957* (Nuovo cimento, to be published). The numerical calculation of ⟨Ω⟩ was not repeated, as ⟨Ω⟩ is insensitive to E.
[e] This number becomes 300 after correction for deuteron distortion.

The Λ binding energies $-E$, and the volume integrals ⟨Ω⟩, along with various other parameters involved in the above relations, are presented in Table I for $_\Lambda He^5$, $_\Lambda Li^7$, and $_\Lambda Li^8$. The result for $_\Lambda H^3$, assuming no deuteron distortion, as above, but using a Hulthén density distribution ρ, is also presented.

The ⟨Ω⟩ for $_\Lambda H^3$, assuming no deuteron distortion, will not be used directly. Instead we use the previously calculated reduction in ⟨Ω⟩ when the no-distortion approximation is improved by solution of the three-body problem by variational methods.[1,2] This reduction is about 10%. Thus the value of ⟨Ω⟩ we will use for $_\Lambda H^3$ is 300 Mev cm$^3\times10^{-39}$.‡

The remaining step is to note how many nucleons are in the triplet state and how many in the singlet state with respect to the Λ. This depends on whether singlet or triplet forces are more attractive. The information for these two cases is presented in Table II. It is noted that there is some uncertainty in the distribution of spins in Li, depending on whether jj coupling or LS coupling is assumed. It is most probable that the actual situation lies between these two cases. Our procedure is to use the ⟨Ω⟩ for $_\Lambda H^3$ and $_\Lambda Li^7$ shown in Table I to compute the ΛN triplet potential Ω_t and the singlet potential Ω_s. In doing this, we make use of the distribution of spins shown in Table II. These values of Ω_t and Ω_s are then used to calculate ⟨Ω⟩ for $_\Lambda He^5$ and $_\Lambda Li^8$.

‡ *Note added in proof.*—Recent work by Dalitz and Downs on $_\Lambda H^3$ (Phys. Rev., to be published) shows that this value for ⟨Ω⟩ is, rather fortuitously, not too bad. The problem is complicated by the fact that one uses different shape ΛN potentials in this nucleus than in other hypernuclei.

TABLE II. Fractions of nucleon spin directions which are triplet or singlet with respect to the Λ, depending on whether the Λ spin lines up with the nucleon spin, or not (i.e., whether Λ*N* forces are most attractive in the triplet state, or singlet state, respectively).

	Triplet favored		Singlet favored	
	Singlet	Triplet	Singlet	Triplet
$_{\Lambda}H^3$	0	1	$\frac{3}{4}$	$\frac{1}{4}$
$_{\Lambda}He^5$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$
$_{\Lambda}Li^7$ [a]	0.22	0.78	0.31	0.69
$_{\Lambda}Li^8$ [b]	0.21	0.79	0.27	0.73
$_{\Lambda}Li^7$ [c]	0.17	0.83	0.42	0.58

[a] These numbers were calculated by using a $(p_{\frac{3}{2}})^2$ configuration.
[b] These numbers were calculated by using a configuration (two particles with $J=0$) $p_{\frac{3}{2}}$.
[c] These numbers were calculated using *a* $L=0$, $S=1$ configuration.

Results are shown in Table III assuming *jj* coupling for the *P*-shell nucleons, and in Table IV assuming *LS* coupling. The values of $\langle\Omega\rangle$ for $_{\Lambda}He^5$ and $_{\Lambda}Li^8$ in Tables III and IV should be compared to the values for these hyperfragments deduced directly from the binding energies (shown in Table I).§

The results can be fit about equally well in the singlet-favored and triplet-favored cases, as is shown in Table III for *jj* coupling. Table IV shows that the fit is not so good for *LS* coupling. One can remark that in the triplet-favored case Ω_s is not accurately determined while in the singlet-favored case both Ω_t and Ω_s are fairly reliably determined.

It should be noted that the value of $\langle\Omega\rangle$ for $_{\Lambda}He^5$ (calculated from Ω_t and Ω_s) must be smaller than for any other hyperfragment we have considered. This is because the α particle, alone of all the nuclei considered here, cannot make any adjustment to take advantage of the spin dependence of the forces. This is not in agreement with the values of $\langle\Omega\rangle$ deduced directly from the hyperfragment binding energies (last column of Table I), but the discrepancy is not large. A somewhat smaller range for the Λ*N* force would improve the agreement by decreasing $\langle\Omega\rangle$ for $_{\Lambda}He^5$ more than for other hyperfragments. One may argue that either or both

TABLE III. Fit to the Λ-nucleus potential strength $\langle\Omega\rangle$ with either singlet-favored or triplet-favored Λ*N* potentials of strength Ω_t and Ω_s in triplet and singlet states, respectively. Units are Mev cm$^3\times10^{-39}$. The hyperfragments $_{\Lambda}H^3$ and $_{\Lambda}Li^7$ were used to determine Ω_t and Ω_s. The values of $\langle\Omega\rangle$ for $_{\Lambda}He^5$ and $_{\Lambda}Li^8$ were calculated from Ω_t and Ω_s and should be compared with the values of $\langle\Omega\rangle$ deduced directly for these latter two nuclei (shown in Table I).

	Triplet favored	Singlet favored
Ω_t	300	165
Ω_s	−65	345
$_{\Lambda}H^3$	300	300
$_{\Lambda}Li^7$	220	220
$_{\Lambda}He^5$	210	210
$_{\Lambda}Li^8$	220	220

§ *Note added in proof.*—New absolute cross section measurements of electron scattering from Li indicate a longer tailed distribution than the harmonic well function used above (R. Hofstadter, private communication). The value of $\langle\Omega\rangle$ shown for Li in Table I will be increased, very roughly, by 10%.

$_{\Lambda}He^5$ and $_{\Lambda}Li^8$ are less reliable cases than $_{\Lambda}H^3$ and $_{\Lambda}Li^7$. The very high nucleon density in $_{\Lambda}He^5$ raises some difficult questions which are briefly discussed in the next section. In the case of $_{\Lambda}Li^8$ we have the simple worry that the proton-density radius employed may be significantly smaller than the nucleon-density radius which should have been used. If so, the calculated value of $\langle\Omega\rangle$ in Table I is smaller than it should be for $_{\Lambda}Li^8$. This effect would improve agreement with experiment. A quantitative estimate of the sensitivity of results to nuclear radius can be easily made. For a potential well of radius r, if the binding energy is small compared to the well depth, the calculated volume integral of the potential, $A\langle\Omega\rangle$, varies linearly with r. The results are insensitive to another source of error, the experimental error in the binding energies, since the binding energies are small compared to the Λ-nucleus well depth. For example, in the case of $_{\Lambda}He^5$, a 20% decrease in the experimental binding energy will decrease the volume integral $\langle\Omega\rangle$ by only about 1%. On the other hand, a 10% decrease in r (i.e., a 20% increase in β) will decrease $\langle\Omega\rangle$ by about 6%.

TABLE IV. Fit to the Λ-nucleus potential strength $\langle\Omega\rangle$ with either singlet-favored or triplet-favored Λ*N* potentials of strength Ω_t and Ω_s, assuming *LS* coupling of the nucleons in Li. See caption for Table III.

	Triplet favored	Singlet favored
Ω_t	300	115
Ω_s	−170	360
$_{\Lambda}He^5$	180	175
$_{\Lambda}Li^8$	200	180

It may be of interest to compare these results with the *NN* potentials, and to check whether the Λ*N* system itself can be bound. It is readily seen that the values of $\langle\Omega\rangle$ above are not *very* sensitive to the assumed range of the Λ*N* potential because that range is always folded in with a (larger) range of the nucleus. The question of binding the two-body system is sensitive to this range. Following Blatt and Weisskopf,[8] let us consider the well-depth parameter s. For the Gaussian well, $s=0.00216\langle\Omega\rangle/(\langle r^2\rangle)^{\frac{1}{2}}\sim Vr^2$, using units of Mev and 10^{-13} cm. For the singlet well in the singlet-favored case, $s=0.68$. For binding, one needs $s=1$; therefore the Λ*N* system is not bound. Meanwhile the neutron-proton singlet and triplet forces have $s=0.90$ and $s=1.45$, respectively. This comparison of values of s is more meaningful in the two-body system than a comparison of Ω's (one finds that the values of Ω for singlet and triplet two-nucleon states are $\Omega_s\approx1000$ and $\Omega_t\approx2000$, respectively, using standard Yukawa potentials; but these values are misleadingly large because of the long tail of the Yukawa potential). On the other hand, the proper comparison in heavy nuclei involves a comparison of Ω's. In nuclear matter, where the depth

[8] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 55, 201.

$V_0=40$ Mev and the radius $R=1.45A^{\frac{1}{3}}\times10^{-13}$ cm for the nucleon-nucleus square well,[8a] we find $\langle\Omega\rangle=510$, compared with $\langle\Omega\rangle=210$ for the Λ-nucleus potential (per nucleon). The depth of the Λ-nucleus potential, as compared to 40 Mev, is then 17 Mev.

III. RELIABILITY OF BINDING-ENERGY CALCULATIONS

Let us briefly consider several effects beyond the scope of the above calculation: (1) possibility of three-body forces,[9] (2) exchange properties of the ΛN potentials, (3) distortion of the nucleus due to the presence of the Λ, and (4) tensor forces in the ΛN interaction.

The effect of three-body forces has been examined by Brueckner, Levinson, and Mahmoud[10] in the case of nuclear forces, and they find the contribution to the nuclear potential is very small. If pions are mainly responsible for the ΛN interaction, three-body forces are *relatively* more important for hyperfragments than for ordinary nuclear matter. But since the effect is small to begin with, we do not believe that it is important even for hyperfragments. If K mesons are mainly responsible for the forces, the effects will be smaller than in the nuclear-force problem. This is because the shorter range of the K-particle forces makes it relatively less likely that the Λ will find itself within the range of two nucleons at the same time. The effect will be most important for $_{\Lambda}He^5$, since this is the densest nucleus we have considered.‖

We next mention exchange properties of the ΛN potential. If the potential is due to pions, exchange forces are absent. To the extent that K mesons contribute, exchange forces are present, but are probably less important than in the nuclear-force problem, since the range is shorter. (It does not matter whether forces are ordinary or exchange, if they have zero range.) Thus, compared to nonexchange forces, K-meson forces would lead to slightly smaller values of $\langle\Omega\rangle$ for the Li hyperfragments.

The effect of nuclear distortion in the presence of the Λ was included for the loosely bound $_{\Lambda}H^3$ fragment where it should be most important. A 10% reduction in the volume integral of the ΛN potential $\langle\Omega\rangle$ resulted. An estimate of the effects of compression of the He^4 nucleus can also be made. (A simple compression is probably the most important distortion to consider.) A crude α-particle wave function ψ can be determined by choosing a suitably adjustable central potential for the interaction Hamiltonian and performing a variational calculation (adjusting the well to obtain the correct binding energy and radius). This calculation yields a compressibility:

$$\Delta B_\alpha\approx-100(\Delta r_0/r_0)^2\text{ Mev.}\qquad(9)$$

Here B_α is the binding energy, $-E_\alpha$, of He^4 and r_0 is the rms radius of $|\psi|^2$. A nuclear compressibility can also be determined in the heavy-nucleus region merely by examination of the semiempirical mass formula. Extrapolating this result to He^4 gives essentially the same result. It is clear that $\Delta B_\alpha\sim(\Delta r_0)^2$, since r_0 is just that value of the radius which makes $dB_\alpha/dr_0=0$. On the other hand, $dB_\Lambda/dr_0\neq0$, where B_Λ is the binding energy of the Λ to He^4. The change in binding energy with radius can be obtained for a square well of depth V_0 and radius R as follows: if $B_\Lambda\ll V_0$, $B_\Lambda\approx V_0\times[R(2\mu V_0)^{\frac{1}{2}}-\pi/2]^2$ so that, letting $\langle\Omega\rangle$, i.e., V_0R^3, be constant, we obtain

$$\Delta B_\Lambda\approx-\frac{\Delta R}{R}[3B_\Lambda+(2\mu B_\Lambda)^{\frac{1}{2}}V_0R]\approx-20\frac{\Delta r_0}{r_0}\text{ Mev.}\qquad(10)$$

Thus, keeping $\langle\Omega\rangle$ constant, we find that $(B_\alpha+B_\Lambda)$ increases by a maximum of 0.9 Mev, associated with a compression of the α particle $\Delta r_0/r_0\approx-0.1$. To offset this increased binding energy, we must decrease $\langle\Omega\rangle$:

$$\Delta\langle\Omega\rangle/\langle\Omega\rangle\approx-3\%.$$

Actually the situation in the interior of $_{\Lambda}He^5$ may not be so simple. One might argue, for example, that the simple approximation of the true ΛN potential by a smooth one with the same low-energy scattering behavior is not valid when the Λ is in the midst of nuclear matter of saturated density. The hard core in the ΛN potential may require that the density at the center of $_{\Lambda}He^5$ not be greater than the corresponding density of He^4. If one simply asks what loss in binding would follow from *expanding* He^4 so that the central density of $_{\Lambda}He^5$ is not higher than that of normal He^4, then one finds roughly $\Delta\langle\Omega\rangle/\langle\Omega\rangle\approx\frac{1}{6}$. This probably overestimates the effect of saturation. In ordinary nuclear matter, the Pauli principle, exchange forces, and repulsive cores combine to produce saturation. Therefore, it is not really clear that the α particle is of saturated density—i.e., that if another particle could be added in an S state, that the total volume would increase. The effect of saturation, if present for $_{\Lambda}He^5$, can also be expected for the lithium hyperfragments. But here the increase in $\langle\Omega\rangle$ would be smaller percentage-wise, since the P-shell nucleons in Li are not very closely packed.

It is rather difficult to determine a rough correction due to tensor forces to the results found in Sec. II. We shall just make a few qualitative remarks here by examining the comparable problem in nuclear binding. The main question to ask is whether the triplet ΛN force, when of mixed central and tensor character, will be more important for $_{\Lambda}H^3$ than for heavier fragments, relative to a pure central force. The role of tensor forces in the light nuclei has been discussed extensively

[8a] R. K. Adair, Phys. Rev. **94**, 737 (1954).
[9] E. Henley, Phys. Rev. **106**, 1083 (1957).
[10] Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 217 (1954).
‖ *Note added in proof.*—R. Spitzer (to be published) has recently found that 3-body forces calculated (π) meson theoretically may be significant. Although we feel the correction is not important in $_{\Lambda}H^3$, it could be large in $_{\Lambda}He^5$. For the time being, however, it seems sensible to try to base an empirical analysis on 2-body forces.

by Feingold.[11] One fact seems to be established: the strong nucleon-nucleon tensor force is essential for the binding of H^2 and somewhat less important for He^4, Li^6, Li^7, etc. This change is probably associated with some kind of angular averaging. Yet it should be kept in mind that the tensor force is still prominent in these heavier nuclei, with $\sim\frac{1}{4}$ the binding being contributed by it. The effect of tensor forces for H^2 has been considered in detailed variational calculations by many authors.[12] Here the shape and range of the tensor force are very important for the binding. Only if the tensor force is given a Yukawa tail and is chosen of longer range than the central force, can the binding energy of H^2 be explained. In this case, the tensor force contributes nearly half of the binding energy. There are good theoretical reasons why the tensor force should be longer-ranged than the central in the two-nucleon problem. The tensor force is associated primarily with the exchange of one pion, while the central force is primarily associated with the exchange of two pions. But the ΛN force cannot be associated with the exchange of one pion, so that the tensor force cannot be long-ranged. Thus, we would expect that even if the ΛN tensor force is strong, it will not be very effective in contributing to the binding of ${}_{\Lambda}H^3$, and will be even less important for the binding of the heavier hyperfragments. (Of course, since ${}_{\Lambda}H^3$ is so lightly bound, the tensor force may be necessary for its binding even though its relative effectiveness compared to a central force of the same volume integral is less than in the ordinary nuclear-force problem.)

Another property of the tensor force is that its effectiveness relative to a central force of the same volume integral is not linear in its strength. This is meant in the sense that even if a Λ (or a nucleon) is in an S state in a nucleus, there is tensor coupling via the D state, but that this coupling is a "second-order effect." A qualitative idea of the effectiveness of the tensor force may be obtained by considering standard Yukawa potentials for the deuteron. The volume integral of the tensor potential (the coefficient of S_{12}) plus the central potential is slightly more than the corresponding quantity for a pure central potential giving rise to the same scattering length and effective range. If the tensor force were weaker, it would be substantially less effective than a central force with the same volume integral.

In light of the above remarks we may say the following: in the case of triplet-favored ΛN forces, if there is a strong tensor force, it will be more important for ${}_{\Lambda}H^3$ than for the heavier fragments. (However, it will not be so important as in the nuclear-force problem because of the shorter range.) Thus, the volume integrals $\langle\Omega\rangle$ should be increased more for the heavier fragments than for ${}_{\Lambda}H^3$. The spin dependence of the forces is thereby reduced. Thus, we would predict a more attractive singlet potential than shown in Table II. In the singlet-favored case, the tensor force will be rather ineffective, since the wave function will not adjust itself very much to accommodate a weak tensor force (compared to the singlet force). Nevertheless, the tensor force would probably still be more effective (relative to the central force) in ${}_{\Lambda}H^3$ than in the heavier fragments. Thus again the apparent spin-dependence of the forces is reduced, but by a small amount. This can be seen by referring to Table II and noting that:

$$\text{for } {}_{\Lambda}H^3, \quad \tfrac{3}{4}\Omega_s+\tfrac{1}{4}\Omega_t'=300,$$
$$\text{for } {}_{\Lambda}He^5, \quad \tfrac{1}{4}\Omega_s+\tfrac{3}{4}\Omega_t=220.$$

If $\Omega_t'>\Omega_t$, the predicted value for Ω_t is still essentially equal to the previously predicted value. Thus we guess that in the singlet-favored case a rather strong tensor potential would be required to provide some of the triplet-state interaction predicted. This point is of some interest when these empirical ΛN potentials are used to test a K-meson theory of hyperon-nucleon forces.[4]

Finally, as a check on the binding energy calculations, it is of interest to note that if one calculates the inhibition, I, of mesonic decay of hyperfragments[13] (due to the Pauli principle), that one also obtains a measure of the Λ wave function. One finds, using $\delta=0.29$ as determined in Sec. I for ${}_{\Lambda}Li^7$, that $I=0.04$. This is in agreement with other quoted results,[5] and roughly agrees with the measured value. The total lifetime of hyperfragments is roughly the same as the free lifetime, so that the inhibition must be approximately equal to the mesonic to nonmesonic ratio. For Li fragments, the mesonic to nonmesonic ratio is approximately 1/15. If the accuracy of these measurements could be improved, an improved version of this method of determining δ (i.e., of obtaining the Λ wave function) would become valuable for determining $\langle\Omega\rangle$. In an improved version it would also be necessary to consider the "stimulated" mesonic decay via virtual Σ hyperons in the nucleus.[14] At present, it is obviously much less sensitive than the binding-energy determination of $\langle\Omega\rangle$.

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[11] A. Feingold, Phys. Rev. **101**, 258 (1956).
[12] For example, R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952).
[13] H. Primakoff, Nuovo cimento **3**, 1394 (1956).
[14] Baldo-Ceolin, Dilworth, Fry, Greening, Huzita, Cimentani, and Sichirallo, Nuovo cimento **7**, 328 (1958).