charge. A qualitative explanation of this characteristic follows: assume that the breakdown voltage is suddenly applied. The current will not increase until a chance carrier enters the junction and triggers the microplasma. In general there will be a lag between the current and the voltage, which will be a function of such factors as the operating point on the instantaneous characteristic, the temperature, and the degree of illumination. Since the measurement involves the average inductance, the significant variable will be the average lag of the current with voltage. As the pulse duration increases, the inductive current will increase; but when the pulse duration is sufficient, a voltage increase will not trigger any new pulses and the lag between voltage and current will disappear. Thus the inductive current would be expected to go through a maximum and then go to zero when the microplasma becomes stable, as is shown in Fig. 18. The same characteristic would be expected to go through several maxima for the uniform junction, as is shown in Fig. 16.

CONCLUSION

Although the detailed mechanism of localized breakdown in silicon is not fully understood, the terminal characteristics of a single microplasma suffice to give a qualitative explanation for some of the terminal characteristics of *p-n* junctions in this region.

ACKNOWLEDGMENTS

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Birefringence Caused by Edge Dislocations in Silicon

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A calculation is given of the intensity distribution in a beam of plane polarized or circularly polarized infrared light after transmission through a crystal of silicon containing a single-edge dislocation or a simple tilt boundary. It is apparent that care must be taken to differentiate between an edge dislocation and the region around the extremum of an inclusion, since both give similar intensity contours, the only difference being one of absolute magnitude.

1. INTRODUCTION

Bond and Andrus[1] (referred to as B.A. in the sequel) have recently successfully utilized photoelastic methods to investigate the stress distribution in the immediate neighborhood of an edge dislocation in silicon. The purpose of this paper is to calculate the expected intensity distribution near a single-edge dislocation and a simple tilt boundary in silicon when plane-polarized or circularly polarized infrared light is used. We are then in a position to criticize certain of the theoretical results and experimental conclusions quoted by B.A. In Sec. 2, we consider the single-edge dislocation. The intensity distribution around an edge dislocation has been given by B.A. where it appears, however, to have suffered an anomalous reflection about the slip-plane (compare Fig. 2 with Fig. 1 of their paper). The simple tilt boundary is discussed in Sec. 3, and, as might be expected, the intensity near such a boundary falls off exponentially with distance from the boundary; by a suitable choice of the type and density of the discrete edge dislocations within the tilt boundary, the results of this last section can, of course, be used to estimate the intensity distribution near a simple twin interface.[2] In both the latter sections, the analysis is carried out using the *isotropic* elastic approximation. Finally, in Sec. 4, a brief discussion of the various numerical magnitudes is given.

[1] W. L. Bond and J. Andrus, Phys. Rev. **101**, 1211 (1956).

2. SINGLE-EDGE DISLOCATION

We erect a system of orthogonal Cartesian coordinates x_i $(i=1, 2, 3)$ in the crystal, such that x_1 is parallel to the slip direction, and x_2 is normal to the glide plane. The strains around a positive edge dislocation, of strength b, situated at the origin are then

$$e_{11}=-\frac{Ax_2}{r^4}[x_1^2(3-2\nu)+x_2^2(1-2\nu)],$$
$$e_{22}=\frac{Ax_2}{r^4}[x_1^2(1+2\nu)-x_2^2(1-2\nu)], \qquad (1)$$
$$e_{12}=\frac{2Ax_1(x_1^2-x_2^2)}{r^4},$$

[2] R. Bullough, Proc. Roy. Soc. (London) **A241**, 568 (1957).

where $r^2=x_1^2+x_2^2$, $A=b/[4\pi(1-\nu)]$ and ν is Poisson's ratio. These strains are well defined at all points in the body, apart from the singular points along the x_3 axis and here a cylinder of material, of small radius r_0, must be removed. The principal strains e_{11}', e_{22}', and the angle $\theta(x_i)$ between the principal axes x_i' and the fixed axes x_i can be found from (1):

$$e_{11}'=\frac{A}{r^2}[x_1-(1-2\nu)x_2],$$

$$e_{22}'=-\frac{A}{r^2}[x_1+(1-2\nu)x_2], \qquad (2)$$

$$\theta=\tfrac{1}{2}\tan^{-1}\left(\frac{x_2^2-x_1^2}{2x_1x_2}\right).$$

To be specific, θ is defined as the angle between the slip direction (the x_1 axis), and the x_1' principal axis.

Plane-Polarized Light

If the crystal plate is set up between a polarizer and analyzer which are crossed, so that in the absence of the doubly-refracting plate no light is transmitted by the system, then the intensity of light transmitted T may easily be shown to be

$$T=a^2\sin^2(2\gamma)\sin^2(\delta/2). \qquad (3)$$

In the above expression, δ is the phase difference in the two principal directions at any point of the crystal plate, a is the amplitude of the incident plane-polarized light, and γ is the angle between the incident plane-polarized light vector (polarizer) and a principal direction. We define β as the angle between the polarizer and the slip direction; then

$$2\gamma=2\theta+2\beta. \qquad (4)$$

The phase angle δ is proportional to the thickness of the plate t, the birefringence Δn, and inversely proportional to the wavelength of the incident light λ. We have, therefore,

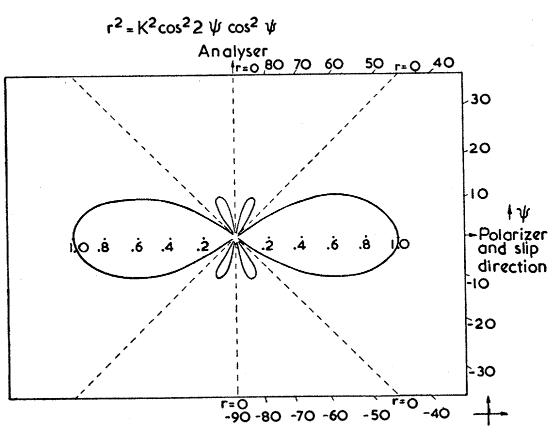

FIG. 1. Intensity plot when the slip direction is parallel to the polarizer.

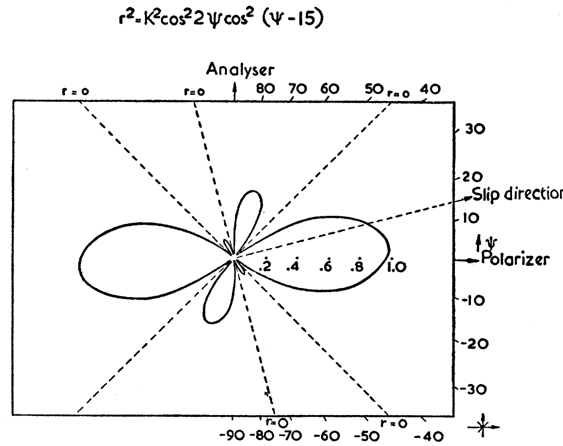

FIG. 2. Intensity plot when the slip direction makes an angle of 15° with the polarizer. $K=1$.

$$\delta=(2\pi t/\lambda)\Delta n, \qquad (5)$$

where the birefringence produced by the principal strains (2) is

$$\Delta n=C\{e_{11}'-e_{22}'\}=2CAx_1/r^2, \qquad (6)$$

and C is the *mean* strain-optic coefficient.[3] Since the birefringence is always small (except at the core of the dislocation and here the elastic approximation breaks down anyway), we make the approximation:

$$\sin(\delta/2)\approx\delta/2. \qquad (7)$$

Then from (3), using (4), (5), (6), and (7), we obtain

$$T=\frac{4a^2A^2\pi^2t^2C^2}{\lambda^2}\frac{x_1^2}{r^4}\sin^2(2\theta+2\beta). \qquad (8)$$

It is convenient to introduce polar coordinates (r,ψ), defined by $\cos(\psi-\beta)=x_1/r$, $\sin(\psi-\beta)=x_2/r$, and thus, from (2),

$$2\theta=2\psi-2\beta-\tfrac{1}{2}\pi. \qquad (9)$$

With this choice of coordinates, $\psi=0$ is the polarizer direction and

$$T(r,\psi)=(B^2/r^2)\cos^2(2\psi)\cos^2(\psi-\beta), \qquad (10)$$

where $B=2aA\pi tC/\lambda$.

The contours of constant intensities are given, therefore, by the polar equation:

$$r^2=K^2\cos^2(2\psi)\cos^2(\psi-\beta), \qquad (11)$$

where $K^2=B^2/T$. The results are plotted for $K=1$ in Fig. 1 ($\beta=0$), Fig. 2 ($\beta=15°$), Fig. 3 ($\beta=45°$), and Fig. 4 ($\beta=90°$). A plot for $\beta=15°$ has been given by

[3] Since the strains (2) have been deduced using the isotropic (elastic) approximation, it would clearly be inconsistent not to use a mean isotropic value for C. Strictly, for a cubic crystal, C will be a function of θ.

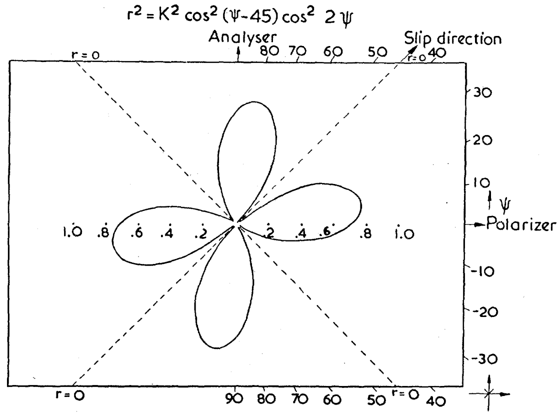

FIG. 3. Intensity plot when the slip direction makes an angle of 45° with the polarizer. $K=1$.

B.A. but they appear to have used $2\gamma=2\theta-2\beta$ which, with our definition of θ and β, is certainly incorrect and, in fact, represents an anomalous reflection about the glide plane (or a rotation of 180° about the slip direction). One of the interesting points concerning these contours is that the intensity apparently vanishes at 45° to the polarizer, no matter what angle the latter makes with the slip direction.

Circularly-Polarized Light

In this case, the intensity distribution of transmitted light is simply

$$T=a^2\sin^2(\delta/2)\approx\frac{a^2\delta^2}{4}=\frac{a^2\pi^2t^2}{\lambda^2}[\Delta n]^2. \tag{12}$$

The intensity is therefore proportional to the square of the actual birefringence, and, from (6), we have

$$T(r,\psi)=B^2\cos^2\psi/r^2. \tag{13}$$

The curves of constant intensity are given, therefore, by the polar equation:

$$r^2=K^2\cos^2\psi. \tag{14}$$

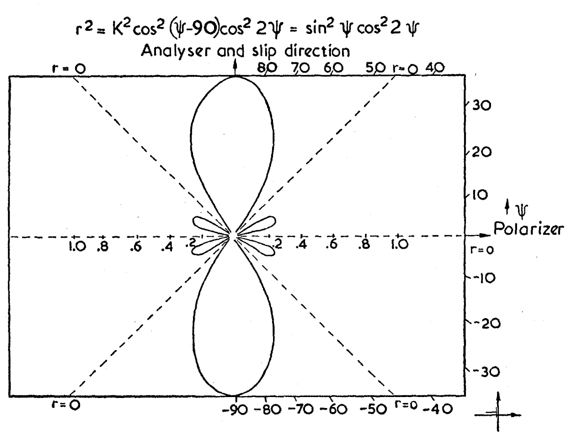

FIG. 4. Intensity plot when the slip direction is perpendicular to the polarizer.

This is plotted in Fig. 5 for $K=1$, and in this case the intensity (birefringence) is zero normal to the glide plane. Equation (14) is, in fact, for each value of K, the polar equation of a pair of circles touching at the origin with their centers on the x_1 axis. Since with circularly-polarized light the intensity is always a minimum in the slip direction (Fig. 5), the method could well be used to identify the slip direction associated with a particular edge dislocation, when, say, the presence of such a dislocation had previously been established by other means.

3. SIMPLE TILT BOUNDARY

This we define as a vertical array of parallel-like positive edge dislocations, equally spaced a distance h apart along the x_2 axis. The required difference of the principal strains and the angle θ are given by

$$\begin{aligned} e_{11}'-e_{22}'&=[(e_{11}-e_{22})^2+e_{12}^2]^{\frac{1}{2}},\\ \tan 2\theta&=e_{12}/(e_{11}-e_{22}), \end{aligned} \tag{15}$$

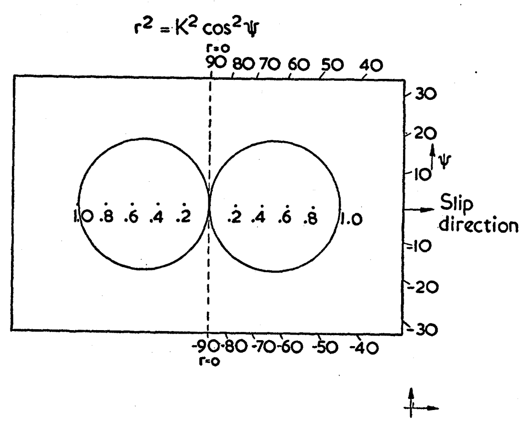

FIG. 5. Intensity plot round a positive edge dislocation when circularly polarized light is used. This is the actual variation of Δn (the birefringence). $K=1$.

where e_{11}, e_{22}, e_{12} are the strains associated with the tilt boundary.

It follows from (1) that

$$(e_{11}-e_{22})=-4Ax_1^2\sum_{n=-\infty}^{+\infty}\frac{(x_2-nh)}{[x_1^2+(x_2-nh)^2]^2},$$

$$e_{12}=2Ax_1\sum_{n=-\infty}^{+\infty}\frac{[x_1^2-(x_2-nh)^2]}{[x_1^2+(x_2-nh)^2]^2},$$

and these sums can be obtained by standard methods of contour integration; we obtain

$$\begin{aligned} (e_{11}-e_{22})&=-\frac{4A\pi^2x_1}{h^2}\frac{\sinh X_1\sin X_2}{[\cosh X_1-\cos X_2]^2},\\ e_{12}&=-\frac{4A\pi^2x_1}{h^2}\frac{[1-\cosh X_1\cos X_2]}{[\cosh X_1-\cos X_2]^2}, \end{aligned} \tag{16}$$

where $X_1=2\pi x_1/h$ and $X_2=2\pi x_2/h$. Thus, from (15) and (16),

$$e_{11}'-e_{22}'=\frac{4A\pi^2 x_1}{h^2[\cosh X_1-\cos X_2]}, \qquad (17)$$

$$\tan 2\theta=\frac{1-\cosh X_1\cos X_2}{\sinh X_1\sin X_2}. \qquad (18)$$

Plane-Polarized Light

With the polarizer inclined at an angle β to the glide planes of the dislocations in the boundary, the transmitted intensity is

$$T=\frac{\pi^2B^2}{h^2}\frac{X_1^2}{[\cosh X_1-\cos X_2]^2}\sin^2(2\theta+2\beta), \qquad (19)$$

where θ and B are given by (18) and (10), respectively. Therefore the equation for contours of constant T is given by

$$[\cosh X_1-\cos X_2]^2=(\pi^2K^2/h^2)X_1^2\sin^2(2\theta+2\beta). \qquad (20)$$

Note: if $h\rightarrow\infty$, then θ in the above equations [given by (18)] becomes θ for the single dislocation [given by (2)], and the intensity (19) becomes identical to (8) for the single-edge dislocation. This, of course, provides a check on the veracity of the above expressions.

Circularly-Polarized Light

In this case, the transmitted intensity T is, from (17) and (12),

$$T=\frac{\pi^2B^2}{h^2}\frac{X_1^2}{[\cosh X_1-\cos X_2]^2}, \qquad (21)$$

and therefore the contours of constant intensity are

$$[\cosh X_1-\cos X_2]^2=(\pi^2K^2/h^2)X_1^2. \qquad (22)$$

When $h\rightarrow\infty$, the above expression tends to the single dislocation expression [Eq. (14)].

4. DISCUSSION

The magnitude of the maximum intensity T_m near a single positive edge dislocation when plane-polarized infrared light is used can be calculated from (8). Thus, at a distance r from the dislocation,

$$T_m=a^2 4A^2\pi^2t^2C^2/\lambda^2r^2, \qquad (23)$$

where $t=$ thickness($\sim$2 mm), $A=b/[4\pi(1-\nu)]$ ($\sim 4\times 10^{-9}$ cm), b is the magnitude of the Burgers vector of the dislocation (equal to the width of the "extra half-plane" above the positive edge dislocation), ν is Poisson's ratio, $\lambda=$wavelength of incident light ($=10^{-4}$ cm), $C=$mean strain-optic coefficient, and $a=$amplitude of plane-polarized light. With the above values, we obtain

$$T_m=2.6\times 10^{-9}C^2a^2/r^2. \qquad (24)$$

B.A. state that the intensity 50 microns from the dislocation is about a thousandth of what it would be in the absence of Nicol prisms. According to (24), the *maximum* intensity at a distance $r=5\times 10^{-3}$ cm from the dislocation is

$$T_m=10^{-4}C^2a^2, \qquad (25)$$

and therefore the ratio of this itensity to the intensity of the incident light $T_i=2a^2$ is

$$T_m/T_i=5\times 10^{-5}C^2. \qquad (26)$$

The ratio given by B.A. is thus $20/C^2$ times greater than our estimate. This discrepancy factor varies from $20(C=1)$ [4] to $4(C=2.3)$.[5] The latter figure almost removes any discrepancy in magnitude with B.A. and confirms that their Fig. 2 does, in fact, represent the intensity distribution around an *ordinary* dislocation. However, the large difference in measured C and the consequent larger effect on the value of $20/C^2$ emphasizes the care that must be taken in identifying a photograph, such as Fig. 2 of B.A., as being associated with an *ordinary* dislocation. For, if we assume $C=1$, then the discrepancy disappears if the photograph is not the intensity around an edge dislocation with a single extra half-plane, but is, in fact, the intensity distribution around a macrodislocation with an "extra half-plane" about 4 A wide.[6]

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[4] P. D. Fochs (private communication).

[5] W. L. Bond (private communication).

[6] Similar observations have recently been reported on Rochelle salt [V. L. Indenbom and N. A. Chernycheva, Doklady Akad. Nauk. S.S.S.R. **111**, 596 (1956)] but, in this case, the "extra half-plane" is about 100 A wide, and is perhaps more aptly described as the extremum of an inclusion and not as a macro-dislocation.