K-Meson Dispersion Relations. II. Applications

P. T. MATTHEWS AND ABDUS SALAM
Imperial College, London, England
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The present experimental data on K^+ and K^- scattering on protons are used in the dispersion relations of the previous paper. The Λ and Σ parities are assumed to be both positive and the K^+-p potential is taken to be repulsive. The dispersion relations then indicate that an attractive K^--p potential implies pseudoscalar K mesons, and a repulsive K^--p potential implies scalar K mesons. In both cases the coupling constants are of the order of unity. If data on K-n scattering were available it would not be necessary to assume the relative parity for Λ and Σ hyperons, but this could also be determined from the dispersion relations.

1. INTRODUCTION

We consider here what physical information may be obtained from the dispersion relations, whose theoretical aspects have been discussed in the previous paper.[1]

At present only a limited amount of experimental data is available, mostly on the proton interactions, and the only useful equation is the proton equation in the form (I.31). Evaluated at threshold, this relates the s-wave scattering lengths to integrals over total cross sections and to the bound-state terms. We adopt the convention that Λ has positive parity and assume that Σ has the same parity.[2] We also accept the conclusion that the K^+-p potential is repulsive.[3] The equation then indicates that the parity of the K meson is determined by the sign of the K^--p potential, being pseudoscalar for an attractive potential, and scalar for repulsive. Present experiments suggest the former to be the case.[4] Provided the integrand is reasonably smooth, the unphysical continuum makes an almost negligible contribution. The qualitative conclusion about K parity is also rather insensitive to the values of the total cross sections. However, knowledge of these cross sections up to about 1 Bev would give a simple relation between the K-meson coupling constants g_Λ^2 and g_Σ^2. With the present rough data this relation merely shows that these constants are of the order of unity both for the pseudoscalar and for the scalar case.

2. DISPERSION RELATIONS

The two dispersion relations for K mesons and protons, written in the form (I.12), are

$$\begin{aligned}\mathrm{Re}M_p^+(\omega)=&\frac{1}{\pi}\int_K^\infty\left(\frac{k'\sigma_{\mathrm{sc}}^+(\omega')}{\omega'-\omega}+\frac{k'\sigma_{\mathrm{sc}}^-(\omega')}{\omega'+\omega}\right)d\omega' \\ &+\frac{1}{\pi}\int_{K_0}^\infty\frac{|k'|\sigma_{\mathrm{ab}}^-(\omega')}{\omega'+\omega}d\omega' \\ &+\frac{2\pi}{N}\frac{(M_\Lambda^2/2N)+N\pm\Lambda}{(M_\Lambda^2/2N)+\omega}\left(\frac{g_\Lambda^2}{4\pi}\right) \\ &+\frac{2\pi}{N}\frac{(M_\Sigma^2/2N)+N\pm\Sigma}{(M_\Sigma^2/2N)+\omega}\left(\frac{g_\Lambda^2}{4\pi}\right)+C,\end{aligned}\tag{1}$$

$$\begin{aligned}\mathrm{Re}M_p^-(\omega)=&\int_K^\infty\left(\frac{k'\sigma_{\mathrm{sc}}^-(\omega')}{\omega'-\omega}+\frac{k'\sigma_{\mathrm{sc}}^+(\omega')}{\omega'+\omega}\right)d\omega' \\ &+\frac{1}{\pi}\int_{K_0}^\infty\frac{|k'|\sigma_{\mathrm{ab}}^-}{\omega'-\omega}d\omega' \\ &+\frac{2\pi}{N}\frac{(M_\Lambda^2/2N)+N\pm\Lambda}{(M_\Lambda^2/2N)-\omega}\left(\frac{g_\Lambda^2}{4\pi}\right) \\ &+\frac{2\pi}{N}\frac{(M_\Sigma^2/2N)+N\pm\Sigma}{(M_\Sigma^2/2N)-\omega}\left(\frac{g_\Sigma^2}{4\pi}\right)+C.\end{aligned}\tag{2}$$

Here[5] k', ω', and ω are momentum and energies in the laboratory system; $\sigma_{\mathrm{sc}}^\pm$ is the total cross section for scattering K^+ or K^- mesons on protons, including change exchange, but not absorption; σ_{ab}^- is the absorption cross section for K^- on protons; C is a constant which is eliminated when the equations are combined into their usable forms (I.31) or (I.32); K_0 is the lower limit of the unphysical continuum and is approximately $K/2$. The plus (minus) sign must be taken in the "bound state" terms if K and Λ (or K and Σ) have the same (opposite) parity. Further

$$M_Y^2=Y^2-N^2-K^2,\tag{3}$$

where $Y=\Lambda$ or Σ and each symbol denotes the mass of the corresponding particle. (N here denotes nucleon.)

There are two other relations for neutrons and K

[1] P. T. Matthews and Abdus Salam, Phys. Rev. **110**, 565 (1958), preceding paper to be referred to as I.

[2] This assumption would not be necessary if data were available on the K-neutron interaction. See Sec. 6.

[3] Many different groups agree on this. The most thorough calculation is by Igo, Ravenhall, Tiemann, Lanutti, Goldhaber, Goldhaber, and Thaler, *Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957* (Suppl. Nuovo cimento, to be published). See also G. Costa and G. Patergnani, Nuovo cimento **5**, 448 (1957). R. Sternheimer, Phys. Rev. **106**, 1027 (1957).

[4] Allex, Biswas, Ceccarelli, and Crussard, Nuovo cimento **6**, 511 (1957). The conclusion is also supported by the calculations of Igo *et al.*, *Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957* (Suppl. Nuovo cimento, to be published).

[5] $\hbar=c=1$. Scattering lengths are expressed in terms of $1/K\approx0.4\times10^{-13}$ cm and cross sections in terms of $1/K^2\approx1.6$ mb.

mesons which are exactly similar except that protons are replaced by neutrons throughout, the term in g_Λ is put equal to zero, and g_Σ^2 is replaced by $2g_\Sigma^2$. These equations may alternatively be expressed in terms of isotopic spin and strangeness by means of the relations:

$$M_p^+ = M_1^+, \qquad M_n^- = M_1^-,$$
$$M_p^- = \tfrac{1}{2}(M_0^- + M_1^-), \quad M_n^+ = \tfrac{1}{2}(M_1^+ - M_0^+), \tag{4}$$

where, on the right-hand side, the upper and lower suffixes denote strangeness and isotopic spin respectively. (Expressed in these terms, the relations also refer to the K^0 and $\bar{K}^0$ processes, though these do not appear to have much practical value at the moment.)

3. PHYSICAL INTERPRETATION

The only possible equation which can be used with the present very poor experimental data is that obtained by taking the difference of the two relations (1) and (2), (I.32), at threshold. Before considering any experimental numbers a few preliminary relations are introduced.

For M_p^+ near threshold there are no inelastic processes and thus

$$\mathrm{Re}M_p^+(K) = \pm 4\pi (E_c/N)a, \tag{5}$$

where a defines the s-wave scattering length.

For M_p^-, where there are competing inelastic processes, we have the general relation

$$(4\pi E_c/N)^2 \sigma_{\mathrm{el}}^c(\theta) = [\mathrm{Re}M(\theta)]^2 + [\mathrm{Im}M(\theta)]^2. \tag{6}$$

By use of the optical theorem, (I.13), the real part of the forward scattering amplitude is

$$[\mathrm{Re}M_p^-]^2 = (4\pi E_c/N)^2 \sigma_{\mathrm{el}}^c(\theta=0) - k^2\sigma_T^2 \equiv (4\pi E_c/N)^2 b^2. \tag{7}$$

The final equality defines b.

The bound-state terms are

$$\left(\frac{2\pi}{N}\right)\frac{(M_\Lambda^2/2N)+N\pm\Lambda}{(M_\Lambda^2/2N)\pm K} \approx \left(\frac{2\pi}{13}\right)\frac{[30,\,-2]}{1\pm 7}\left(\frac{7}{K}\right), \tag{8}$$

$$\left(\frac{2\pi}{N}\right)\frac{(M_\Sigma^2/2N)+N\pm\Sigma}{(M_\Sigma^2/2N)\pm K} \approx \left(\frac{2\pi}{13}\right)\frac{[33,\,-2]}{2\pm 7}\left(\frac{7}{K}\right). \tag{9}$$

The alternative values given in square brackets depend on whether the K parity is the same as, or opposite to that of the hyperon involved.

4. EVALUATION

The difference between Eqs. (2) and (1) at threshold may now be written

$$(\pm b) - (\pm a) = \frac{1}{6\pi^2}\left[\int_K^\infty k'[\sigma_{\mathrm{sc}}^-(\omega') - \sigma_{\mathrm{sc}}^+(\omega')]\left(\frac{1}{\omega'-K} - \frac{1}{\omega'+K}\right)d\omega' + \int_{K_0}^\infty |k'|\sigma_{\mathrm{ab}}^-(\omega')\left(\frac{1}{\omega'-K} - \frac{1}{\omega'+K}\right)d\omega'\right] + [\mathrm{BS}]. \tag{10}$$

Here the + or − signs on the left-hand side depend on whether the potential is attractive or repulsive, respectively†; σ_{sc} is the total elastic scattering, including charge exchange, and [BS] denotes the "bound-state" terms.

We estimate the first integral by taking $\sigma_{\mathrm{sc}}^\pm$ to be constant in the energy range $K-2K$ and neglecting the rest of the integral. This is certainly a rather rough approximation. Its implications are considered below.

The second integral includes the unphysical region. It has been shown in I that the integrand may become negative in this region. However, the value of $k\sigma_{\mathrm{ab}}^-$ at the physical threshold can be obtained by experiment, and is expected to be slowly varying in this neighborhood. Also $|k|\sigma_{(\mathrm{ab})}^-$ must go to zero at the lower limit. The indications from the four-field model perturbation-theory calculation of I. Sec. 5 are that in the scalar case this expression passes through zero at an energy of about $\frac{3}{4}K$, but that for pseudoscalar K mesons, it remains positive. This behavior appears rather natural, when combined with the known signs of the contributions from the bound state in the two cases. We estimate this integral by taking $|k'|\sigma_{\mathrm{ab}}^-$ constant in the region $\frac{3}{4}K-2K$. This probably overestimates the contribution from the unphysical region, which in any case turns out to be negligible.

With these approximations Eq. (10) becomes

$$(\pm b) - (\pm a) - [\sigma_{\mathrm{sc}}^- - \sigma_{\mathrm{sc}}^+]/25 - [|k|\sigma_{\mathrm{ab}}^-]/120 = (g_\Lambda^2/4\pi)[-\tfrac{3}{2},\,\tfrac{1}{10}] + (g_\Sigma^2/4\pi)[-2,\,\tfrac{1}{8}]. \tag{11}$$

The alternative values in the bound-state terms are explained after Eq. (9). As stated in the Introduction, we adopt the convention that Λ has positive parity[6] and assume that Σ also has positive parity in accordance with the ideas of global symmetry.[7] It is then to be noticed that the bound-state term is negative for scalar K mesons, and positive for pseudoscalar K mesons. It is this fortunate change of sign which makes the relation so sensitive to the K-meson parity.

† We follow Bethe and de Hoffmann [*Mesons and Fields II* (Row, Peterson and Company, 1955)] in taking the sign of the scattering length to be the same as that of the phase shift. This is opposite to the usual convention in nuclear theory.

[6] P. T. Matthews, Nuovo cimento **6**, 642 (1957).

[7] J. Schwinger, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957). M. Gell-Mann, Phys. Rev. **106**, 1296 (1957). J. Tiomno, Nuovo cimento **6**, 69 (1957).

The total cross section fK^+ or mesons on protons has been measured in nuclear emulsion,[8] by bubble chambers[9] and by counters.[10] All techniques agree in giving a fairly constant cross section in the 0–200 Mev region of 15.2±2.4 mb. This is

$$\sigma_{sc}{}^{+}\approx 10/K^2, \tag{12}$$

and thus

$$4\pi a^2=\sigma_{sc}{}^{+},\quad |a|\approx(6/7K). \tag{13}$$

The evidence on K^- interactions is much less reliable. From nuclear emulsion[10] there is data in the 0–80 Mev range, and bubble chamber data[11] below 30 Mev. This indicates that the elastic scattering in this region is

$$\sigma_{el}{}^{-}\approx 48 \text{ mb}\approx 30/K^2. \tag{14}$$

The charge-exchange scattering is estimated[10] to be somewhat less, giving for the total elastic scattering cross section

$$\sigma_{sc}{}^{-}\approx 90 \text{ mb}\approx 54/K^2. \tag{15}$$

The experiments on the behavior of $\sigma_{ab}{}^{-}$ are in some confusion at the moment as the bubble chamber data[12] at about 20 Mev do not fit naturally to the emulsion data[11] between 30 and 100 Mev. The experiments have been interpreted[11] as indicating a cross section proportional to $1/(p^c)^2$. Since, by (I.13),

$$\text{Im}M_p{}^{-}\propto p^c\sigma_{sc}{}^{-}+p^c\sigma_{ab}{}^{-}, \tag{16}$$

this implies that $\text{Im}M_p{}^{-}\rightarrow\infty$ at threshold, which is theoretically unacceptable. This is certainly a point which requires much more careful experimental investigation. We assume the theoretically expected $1/p^c$ dependence of the cross section, and consider the two extreme values

$$k\sigma_{ab}{}^{-}\approx 6/K,\ 12/K. \tag{17}$$

In either case the integral over $\sigma_{ab}{}^{-}$ makes a negligible contribution to the dispersion relation.

The bubble chamber[12] and emulsion data[11] are in reasonable agreement on the magnitude of the K^--proton elastic scattering (excluding charge exchange), and we take a value of 48 mb at threshold. Upon using (7), this gives

$$b\approx 1\cdot 6/K,\ 1\cdot 4/K, \tag{18}$$

for the two possibilities considered in (17).

5. PARITIES AND COUPLING CONSTANTS

If the above values are substituted into (11) and if one takes the smaller value in (17), the terms of (11) written in the same order are[5]

$$(\pm\tfrac{3}{2})+(6/7)-(44/25)-(6/120) \\ =(g_\Lambda{}^2/4\pi)[-\tfrac{3}{2},\tfrac{1}{10}]+(g_\Sigma{}^2/4\pi)[-2,\tfrac{1}{8}]. \tag{19}$$

A mean value for b has been taken, since the two cases lead to identical conclusions. The K^+ potential has been taken repulsive.[3] The alternative signs in the first term apply for attractive (+) and repulsive (−) K^--p potentials. In the first case the left-hand side is positive and the equation can be satisfied only by pseudoscalar K mesons, while the coupling constants are given by

$$4/7\approx\tfrac{1}{10}(g_\Lambda{}^2/4\pi)+\tfrac{1}{8}(g_\Sigma{}^2/4\pi). \tag{20}$$

Similarly, if the K^--p potential is repulsive the K meson is scalar with constants satisfying

$$17/7\approx\tfrac{3}{2}(g_\Lambda{}^2/4\pi)+2(g_\Sigma{}^2/4\pi). \tag{21}$$

Assuming $g_\Sigma=g_\Lambda$ we obtain $g^2/4\pi=2.6$ or 0.7 for the pseudoscalar or scalar cases, respectively.

The determination of the sign of the K^--p potential is considerably more difficult than in the K^+-p case owing to the large absorption. A Monte-Carlo calculation has been made by the Göttingen group.[4] They find that the very small amount of inelastic scattering on complex nuclei, in spite of the predominance of this reaction over all others in the K-p interactions, can be explained by assuming a strong attractive potential. This suggestion is also consistent with the observed energy loss as in inelastic scattering. The argument is not conclusive, since other explanations are possible, but if this is accepted, the dispersion relation indicates that the K meson is pseudoscalar.

6. COMMENTS

The sources of error in these relations are (i) the lack of any information on total cross sections above about 200 Mev for K^+ and 80 Mev for K^-, (ii) the curious behavior of $\sigma_{ab}{}^{-}$ near threshold, and (iii) the contribution from the unphysical continuum. Of these the third is negligible unless the extrapolated cross section behaves in a very wild manner (ImM reaching twenty times its threshold value) in the unphysical region. (However, more information on polarization at low energies as discussed in I. Sec. 4, would be very valuable.) The second has been discussed above, in Sec. 3, but certainly calls for more accurate experiments. By far the largest error is the first. Information on the total cross sections up to the highest obtainable

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[8] Hoang, Kaplon, and Cester, Phys. Rev. **107**, 1698 (1957); Widgoff, Pevsner, Fournet-Davis, Ritson, and Schluter, Phys. Rev. **107**, 1430 (1957). These papers contain references to earlier work. See also *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), and *Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957* (Suppl. Nuovo cimento, to be published).

[9] Meyer, Perl, and Glaser, Phys. Rev. **107**, 279 (1957).

[10] Kerth, Kyera, and van Rossum, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[11] See reference 3 which includes data reported at the *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957). See also reports in the *Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957* (Suppl. Nuovo cimento, to be published), from groups at Bern, Bristol, Brookhaven, Göttingen, and the University of California Radiation Laboratory, Berkeley.

[12] Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp, Nuovo cimento **5**, 1026 (1957).

energies would thus provide detailed information on the coupling strengths.

However, it is to be noticed that our qualitative conclusion, relating the K-meson parity to the sign of the K^{-}-p potential, is rather insensitive to the precise value of these integrals. It could only be changed if, in fact, the contribution from σ_{sc}^{-} is so large that the integral over total cross sections, in (19), becomes greater in magnitude than the combination of the scattering lengths. We consider this very unlikely as the scattering cross section almost certainly decreases from the very high threshold value, which we have assumed constant throughout the $K-2K$ total energy region. However, if this turns out to be the case, the K meson will be unambiguously scalar, independent of the sign of the K^{-}-p potential.

In the above analysis we have made the assumption that Λ and Σ have the same parity. This will not be necessary when sufficient information is available on the neutron interactions to make use of the neutron equation corresponding to (10). This involves g_Σ^2 only and, given the signs of the K^{+} and K^{-} potentials, will determine g_Σ and the relative $K-\Sigma$ parity. This information may then be used in (10) to determine g_Λ and the relative $K-\Lambda$ parity.

If the cross sections were known to high enough energies, it would also be possible to use equations of the type (I.31) to get information on the effective ranges of the K-meson potentials.

It appears, thus, that the dispersion relations are a very powerful tool for determining parities and the strengths of the K-meson interactions and it is to be hoped that more experiments will be performed soon to try to provide the relevant information, particularly the neutron data.

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Parity of the K Meson from a Dispersion Relation

C. Goebel
University of Rochester, Rochester, New York
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It is remarked that the forward dispersion relations for $K^{\pm}$-nucleon scattering offer a determination of the "strong interaction parity" of the K meson. At present, the sign of the parity, determined in this way, depends on one uncertain experimental datum: the sign of the K^{-}-p scattering length.

ALTHOUGH the absolute parity of the K meson is not observable, its relative parity in strong interactions is well defined. We make the natural convention of defining the Σ and N (nucleon) parities to be the same; thus, to say that the K meson is pseudoscalar is to say that the final orbital state of $N\rightarrow K+\Sigma$ is a p wave, and so on. This is the sense of the "parity of the K meson" we use in this note. Of course, the relative parity of Σ to Λ is well defined and might be negative, although at the moment the evidence favors the same parity for Σ and Λ.

In principle, the parity of the K meson can be determined by observing any process in which it interacts strongly, but the interpretation of the result, unless it depends on a selection rule so that only phenomenology is needed, requires comparison with a field-theoretical calculation. Such calculations in meson theory have so far been shown to agree with experiment only when the situation was in fact controlled by a "threshold theorem." We point out in this note the possibility of determining the parity from meson-nucleon scattering data by using a zero-angle dispersion relation. Dispersion relations, while having a much lower ratio of output to input information than perturbation theory, have a much greater reliability of operation than the latter.

It was recognized independently by Goldberger *et al.*[1] and by the author that if the difference of the π^{+}-p and π^{-}-p forward scattering amplitudes at high energy is less than that of order ω (laboratory energy), which implies that the difference of the total cross sections vanishes at infinite energy, then the dispersion relation at high energy has a limiting form. This form [Eq. (1) below] relates the (p wave) coupling constant, the difference of the (s wave) scattering lengths, and a "dispersion" integral over the difference of the cross sections. This relation is in fact satisfied by the experimental pion scattering data.[2]

We propose to assume that the same holds for the K meson, namely that the K^{+}-p and K^{-}-p cross sections

[1] Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).
[2] Reference 1; for a discussion using the latest data, see J. M. Cassels, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), p. II-4.