of the electron-scattering group for their continued help in setting up the experiments, as well as for discussions on experimental and theoretical questions. We also wish to thank Dr. D. G. Ravenhall and Mr. R. Blankenbecler for valuable discussions. One of the authors (H. F. E.) is grateful to the Deutsche Forschungsgemeinschaft for the grant of a stipend which permitted him to stay at Stanford for one year.

PHYSICAL REVIEW VOLUME 110, NUMBER 2 APRIL 15, 1958

Method for Determining the $K_+^0 - K_-^0$ Mass Difference*

MYRON L. GOOD
Radiation Laboratory, University of California, Berkeley, California
(Received October 28, 1957)

The equations of motion for the amplitudes of short-lived (K_+^0) and long-lived (K_-^0) neutral K mesons in an absorber are simplified for the case of dominance of the decay term. For the case of a thick absorber, a simple relation between the intensities of scattered and unscattered regenerated K_+^0, at zero degrees, results; the relation is sensitive to the $K_+^0 - K_-^0$ mass difference.

THE phenomenon of regeneration of the short-lived neutral K meson in a beam of long-lived neutral K's is a crucial test of the particle-mixture hypothesis of Gell-Mann and Pais[1]; what we wish to show here is that it also permits a rather direct determination of the difference in mass of the two particles.

The process has been studied theoretically[2,3]; preliminary experimental verification of the basic ideas involved has been obtained by Lederman *et al.*[4] and by Fowler, Lander, and Powell,[5] and others.

The theory of the process is independent of the questions of whether or not charge conjugation (C), parity (P), or time reversal (T) are valid symmetry operations.[6,3]

For the short-lived and long-lived particles, respectively, we adopt, following Lee and Yang, the names K_+^0 and K_-^0. Otherwise the notation used is that of reference 3.

First, we observe that, in most, if not all, circumstances K_+^0 decay predominates over absorption processes, so that it is a good approximation to set

$$\beta ck\left|\frac{n-n'}{2}\right| \ll \frac{1}{2\gamma\tau_+}.$$

(This is precisely what makes the regeneration so small.[2]) With this approximation, the solutions for the amplitudes (α_+,α_-) of K_+^0, K_-^0 in the absorber simplify considerably:

$$\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix} \simeq \left[-\begin{pmatrix}R\\ R^2\end{pmatrix} e^{-(i\omega_+ + 1/2\gamma\tau_+)t} + \begin{pmatrix}R\\ 1\end{pmatrix} e^{-i\omega_- t}\right] e^{\frac{1}{2}i\beta ck(n+n')t}, \quad (1)$$

where

$$R \simeq \frac{\beta\gamma ck(n-n')}{2[(1/2\tau_+)+i(\omega_+^0-\omega_-^0)]}, \quad \text{and} \quad |R| \ll 1.$$

The initial conditions have been taken as $\alpha_+(0)=0$, $\alpha_-(0)=1$, and $1/\tau_-$ has been neglected in comparison with $1/\tau_+$. If we now confine ourselves to thicknesses (L) large compared with the K_+^0 decay distance, we can drop the first term, and we have, for the K_+^0 intensity emerging from the absorber,

$$|\alpha_+(L)|^2 \simeq |R|^2 e^{-\frac{1}{2}N(\sigma+\sigma')L}. \quad (2)$$

This refers to the unscattered regenerated K_+^0. The K_+^0 intensity regenerated by scattering through an angle ϕ at depth x is, in the same spirit (evaluated at $\phi=0$),

$$\left|\left(\frac{dI_s}{dx}\right)dxd\Omega\right|_{\phi=0} \simeq \frac{k^4}{16\pi^2 N}|n-n'|^2 dxd\Omega\, e^{-\frac{1}{2}N(\sigma+\sigma')x}. \quad (3)$$

To evaluate the scattered K_+^0 intensity emerging from the absorber at $\phi=0$, we must multiply Eq. (3) by the probability of escape of the regenerated K_+^0 without decay or further scattering, and must integrate with respect to x. (This gives the contribution of single scattering. The particles scattered as K_-^0 can of course, rescatter into K_+^0, and so on. Thus double and higher-order scatters can contribute also. For the time being, we consider only single scattering.) The scattered K_+^0, being incoherent with the incident beam, decays with essentially the exponent of the first term of Eq. (1),

* This work was done under the auspices of the U. S. Atomic Energy Commission.
[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] K. Case, Phys. Rev. **103**, 1449 (1956).
[3] M. L. Good, Phys. Rev. **106**, 591 (1957).
[4] Lande, Lederman, and Chinowsky, Phys. Rev. **105**, 1925 (1957).
[5] Fowler, Lander, and Powell, Bull. Am. Phys. Soc. Ser. II, **2**, 236 (1957).
[6] Lee, Yang, and Oehme, Phys. Rev. **106**, 340 (1957).

so that we have

$$|I_s d\Omega|_{\phi=0} \simeq d\Omega \int_0^L \frac{dI_s}{dx}(x) e^{-(L-x)/\beta\gamma c\tau_+} e^{-\frac{1}{2}N(\sigma+\sigma')(L-x)} dx.$$

The absorption terms combine, and we have simply

$$|I_s d\Omega|_{\phi=0} \simeq \frac{k^4}{16\pi^2 N} |n-n'|^2 e^{-\frac{1}{2}N(\sigma+\sigma')L} \beta\gamma c\tau_+ d\Omega. \quad (4)$$

Putting $\beta\gamma = \hbar k/Mc$, we obtain

$$|I_s d\Omega|_{\phi=0} \simeq \frac{k^5 \hbar \tau_+}{16\pi^2 NM} |n-n'|^2 e^{-\frac{1}{2}(\sigma+\sigma')L} d\Omega, \quad (5)$$

where M is the mass of the K^0. In the same terms, the unscattered regenerated intensity is

$$|\alpha_+(L)|^2 \simeq \left(\frac{\hbar^2 k^4 \tau_+^2}{M^2}\right) \frac{|n-n'|^2 e^{-\frac{1}{2}(\sigma+\sigma')L}}{\left|1+i\frac{\omega_+{}^0-\omega_-{}^0}{1/2\tau_+}\right|^2},$$

and the ratio of unscattered to scattered $K_+{}^0$ intensity, at $\phi = 0$ and at momentum $\hbar k$, is

$$\frac{|\alpha_+(L)|^2}{I_s} \simeq \left(\frac{16\pi^2 \hbar \tau_+ N}{Mk}\right) \left[\left|1+i\frac{\omega_+{}^0-\omega_-{}^0}{1/2\tau_+}\right|\right]^{-2}. \quad (6)$$

The factors depending on the cross sections cancel, and the result is sensitive to the mass difference. The cancellation reflects the fact that the two groups of particles represent different aspects of the same phenomenon. However, in the unscattered group the regenerated wave is fed by the incident wave over a time $\approx \tau_+$, and is sensitive to the difference in natural frequency of the two waves; whereas in the scattered group the regeneration takes place in a single event, and does not depend on the mass difference.

The rather similar result obtained in an earlier communication[3] was for absorber thicknesses small compared with the distance the particle travels in one period of the mass-difference oscillation, and hence did not depend on the mass difference.

The terms neglected are of order $|R|^2$ throughout, therefore the fractional error in Eq. (6) should be of the same order as the fraction of $K_-{}^0$ regenerated (for reasonable guesses, the latter is 1% or less).[2] It would seem, then, that a measurement of $|\alpha_+|^2/I_s$ behind a fairly thick absorber (several decay lengths), coupled with a knowledge of the momentum of each event, could be used to determine the mass difference.

Some qualifications need to be made, however; for one thing, one should, in principle, use an absorber that is an isotopically pure element of spin zero (or possibly $\frac{1}{2}$).[3] In practice, the scattering is probably well described by the optical model, and, if so, the nucleus is characterized, for our purposes, entirely by its size. Thus any element would do, but compounds would still be unsatisfactory, in general.

The derivation concerned only single scattering; in a thick absorber, higher-order scattering will take place. These will, in general, smear out the angular distribution for both $|\alpha_+|^2$ and I_s, and thus lower the apparent ratio of $|\alpha_+|^2$ to I_s. (The main point probably is that if an unscattered peak shows up at all, then the mass difference is not large compared with $1/2\tau_+$.)

First-order corrections can be made for higher-order scattering in any particular geometry. The effect of higher-order scattering vanishes, for instance, as the geometrical width of the absorber is made small in comparison with its thickness, since then the scattered particles leave, via the sides, without rescattering.

Finally, only elastic scatterings were considered. Inelastic ones could probably be ruled out on the basis of angular distribution, if not by other means.

As a numerical example, Eq. (6) says that for 100-Mev $K_-{}^0$'s in Pb, with a 1° angular resolution, and for mass difference zero, one has

$$|\alpha_+|^2/(I_s \delta\Omega) \simeq 40.$$

The mass-difference measurement here proposed differs from that pointed out by Treiman and Sachs,[7] in that no parameters of the weak interactions other than the mass difference, are involved.

[7] S. Treiman and R. Sachs, Phys. Rev. **103**, 1545 (1956).