Coulomb Excitation of Medium-Weight Even-Even Nuclei*

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The yields of gamma rays resulting from Coulomb excitation have been measured for twenty-nine eveneven medium weight nuclei ($92 \le A \le 130$). The projectiles used for effecting Coulomb excitation were variable-energy protons (1.5 to 3.3 Mev) and doubly-ionized helium ions (8 to 10 Mev). In the most favorable cases the absolute γ -ray yields are determined to an accuracy of $\pm 4\%$. The reduced electric quadrupole transition probabilities, B(E2), are determined to an accuracy of $\pm 7\%$ in favorable cases. For the least favorable case the accuracy is $\pm 20\%$. The observed values for B(E2) are larger than the single particle estimate by factors ranging from 6 to 64. The transition rates in the even-even tin isotopes are larger than the single particle estimate by a factor of 13. A strong correlation is found between the B(E2) and the inverse of the γ -ray energy for transitions with moderately large values for B(E2). The transitions are interpreted in terms of the collective vibrational model. The mass parameters obtained show remarkably little variation and are approximately 10 times the irrotational flow estimate.

I. INTRODUCTION

E ARLY work with the Coulomb excitation process showed that low-lying states of medium weight nuclei (A~100) are strongly excited.¹⁻³ The E2 γ -ray transition rates extracted from the Coulomb excitation cross sections are as much as 50 times larger than the Weisskopf single-particle estimate. These fast transition rates suggest that a collective motion of the nucleons is playing an important role; however, examples of welldeveloped rotational spectra with their remarkable regularities are not found. Certain regularities, which are less striking than those for rotational spectra, have been discerned in the low-lying states of eveneven nuclei of medium weight.⁴ The ratio of the positions of the first and second excited states, the spin sequence of the states and the relative transition rates suggest "near harmonic" spectra.⁵ Two types of collective motion which generate such spectra have been proposed.^{4,6} Recently, the detection of the Coulomb excitation of the second 2⁺ state in a number of medium-weight nuclei has provided additional information which supports the collective-model interpretation of these states.7

In addition to this evidence for the importance of collective motion, it is also apparent that shell structure plays a significant role in the excited states of mediumweight nuclei. This is seen in the rather strong and fairly systematic variation in the energy of the first excited state of even-A nuclei with changes in proton or neutron number. Since E2 transition rates are an unusually sensitive indication of the presence of collec-

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tive motion, extensive and accurate information on these transition rates should provide a better understanding of the interplay of the collective-model and shell-model aspects of these states. From the experimental viewpoint, the prospect of making accurate determinations of Coulomb excitation cross sections by γ -ray yield measurements is favorable for mediumweight nuclei. The γ rays have convenient energies: internal conversion is small; and, most important of all, there exists a fairly complete set of isotopically enriched samples with good enrichment factors from which targets may be prepared. In this paper we wish to report the results of Coulomb excitation measurements on 29 even-A medium-weight nuclei.

II. EXPERIMENTAL PROCEDURE

The projectiles used for effecting Coulomb excitation were variable-energy protons and singly- and doublyionized helium ions accelerated by the 5.5-Mv ORNL electrostatic generator. The energy spread of the incident particles was approximately 0.1% and the absolute energies were determined to 0.2%. The ion beam current integrator used to determine the number of particles striking the target had an accuracy of better than 0.5% for currents equal to or larger than 0.01 microampere.

Under optimum conditions the ratio He⁺⁺/He⁺ from the radio-frequency ion source was less than 0.01 and for this reason the beam current of He++ was limited to a few hundredths of a microampere. Since He⁺⁺ ions and molecular hydrogen ions have approximately equal e/m values, measures were taken to eliminate hydrogen from the ion source. Even so, a small amount of molecular hydrogen equal to about $\frac{1}{3}$ of the He⁺⁺ current was observed. However, because the e/m values are slightly different we were able to resolve the two beams with the analyzing magnet.

Targets were mounted at 45° with respect to the incident beam on a target support which was a stainless steel tube with 0.005-inch wall thickness. The γ rays resulting from Coulomb excitation were detected with

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¹G. M. Temmer and N. P. Heydenburg, Phys. Rev. 98, 1308 (1955).

² P. H. Stelson and F. K. McGowan, Phys. Rev. 99, 112 (1955). ³ Mark, McClelland, and Goodman, Phys. Rev. 98, 1245 (1955).

⁴G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955). ⁵ G. Scharff-Goldhaber, Phys. Rev. 103, 837 (1956)

⁶ L. Wilets and M. Jeans, Phys. Rev. 102, 788 (1956). ⁷ P. H. Stelson and F. K. McGowan, Bull. Am. Phys. Soc.

Ser. II, 2, 267 (1957).

thallium activated NaI crystals mounted on DuMont photomultiplier tubes. Two sizes of cylindrical crystals with dimensions $1\frac{1}{2}$ in. diameter—1 in. length and 3 in. diameter—3 in. length were used. Pulse-height spectra were measured with a sliding 20-channel analyzer of ORNL design.⁸ The detector was placed at 235° with respect to the incident beam and, usually, with a distance of 10 cm from the target to the front face of the crystal. The prolific x-ray yields produced by proton bombardment were attenuated by placing a series of thin metallic shields in front of the detector.

The energy response of the scintillation spectrometer was calibrated by the use of radioactive sources, e.g., Be⁷, Cs¹³⁷, Hg²⁰³, which emit γ rays with accurately known energies. To avoid inaccuracy resulting from slight variations in the gain of spectrometer with changes in counting rate, the crystal was simultaneously irradiated by two sources of calibration γ rays and the γ rays from Coulomb excitation.

Metallic targets of normal and enriched isotopic abundance were prepared either by electrodeposition onto 5-mil nickel backings or by sintering metallic powders into thin foils.⁹ The sintered foils were mounted on nickel target backing by forming a sandwich of the nickel backing, the foil and a nickel collar; the collar was then spot-welded to the backing. The targets were 75 to 150 mg/cm² in thickness by 0.5 inch in diameter and were, therefore, thick to the impinging projectiles.

In more detail, the various targets were prepared as follows. Enriched targets of ruthenium, palladium, and tin were made directly from the metallic powders obtained from the Stable Isotopes Division by sintering at room temperature under a pressure of 25 tons/in.² The enriched molybdenum isotopes were furnished as oxides. These oxides were reduced to metallic powders by reaction with hydrogen gas at elevated temperatures and then sintered into foils. The enriched cadmium targets were prepared by electrodeposition from cadmium cyanide baths to 95% depletion using a platinum anode. An especially pure target of normal tin was made by electrodeposition from a sodium stannate bath. The target of normal tellurium was made by first depositing a thin layer of lead from a lead fluorobate bath and then depositing the tellurium from a solution of tellurium dioxide in a mixture of hydrofluoric and sulfuric acids. A target of normal zirconium was prepared by spot-welding a 2-mil zirconium (hafnium-free) foil to a nickel backing.

1. Gamma-Ray Spectra

The γ -ray spectra for about 30 different targets of normal and enriched isotopic abundance were measured for a number of incident projectile energies.

In some cases both protons and α particles were used. Representative spectra for these targets are shown in Figs. 1 to 24. The isotopic abundance of the isotope of interest is listed in column 4 of Table II. The spectra for targets enriched in the odd-*A* isotopes are needed for the complete interpretation of the even-*A* spectra. These were measured but are not shown here; they are discussed in another paper.¹⁰

In addition to pulses from γ rays resulting from Coulomb excitation, the spectra contain pulses from (a) the local background, (b) the γ rays from proton bremsstrahlung, and (c) the γ rays from nuclear reactions produced by bombardment of minute amounts of light-element target impurities. Information was previously presented to show that the proton bremsstrahlung is a slowly varying function of the Z of the target material.² A tin target was used to obtain the bremsstrahlung spectrum plus local background for the analysis of tellurium, cadmium, palladium, and ruthenium spectra. Since no Coulomb excitation was observed for the enriched Mo⁹² target, this spectrum was taken as the proton bremsstrahlung plus local background for the analysis of the other molybdenum spectra. An examination of the representative spectra shows that in favorable cases, e.g., Ru¹⁰⁴ spectrum, the bremsstrahlung plus local background is less than 1% of the peak height of the Coulomb excited γ ray whereas in the least favorable cases, e.g., Te¹³⁰ excitation in a normal Te target, the bremsstrahlung plus local background is comparable to the height of the Coulomb excited γ -ray peak. Fortunately, in most spectra the bremsstrahlung intensity is so low that the possible uncertainty in the intensity gives a negligible error in the γ -ray yield.

As is well known, gamma rays from nuclear reactions produced by bombardment of low-Z target impurities cause troublesome interference in Coulomb excitation experiments. For example, the 440-kev γ ray from $Na^{23}(p,p')Na^{23}$ is seen in many of the spectra. To identify these spurious γ rays, the γ -ray spectra of most of the light elements ($Z \leq 30$) were measured for a series of proton energies ranging from 1.5- to 4.0-Mey. The presence of target impurities, in addition to producing well-resolved γ -ray peaks, in many cases also raised the continuum level appreciably above the bremsstrahlung level (partly the result of pulses from Compton-recoil electron distributions resulting from higher energy γ rays incident on the detector). In fact, for α -particle spectra, the somewhat irregular continuum from target impurities (see tin spectra) has prevented the definite identification of bremsstrahlung produced by α particles. It was found that preparation of targets by electrodeposition eliminated impurities to a considerable extent. For example, the Cd¹¹⁴ spectrum has a continuum which is accounted for by the proton bremsstrahlung plus local background.

⁸Kelley, Bell, and Goss, Oak Ridge National Laboratory Physics Division Quarterly Progress Report ORNL-1278, 1951 (unpublished).

⁽unpublished). ⁹ The isotopically enriched samples were obtained from the Stable Isotopes Division of Oak Ridge National Laboratory.

 $^{^{10}\,\}mathrm{F.}$ K. McGowan and P. H. Stelson, Phys. Rev. 109, 901 (1958).

The number of emitted γ rays per microcoulomb of incident particles was obtained from the pulse-height spectrum by the use of the formula

$$s = \frac{N(E_0)}{n(E_0)} \times \frac{1}{\Delta E} \times \frac{1}{\epsilon} \times \frac{1}{R_{\gamma}} \times \frac{1}{A_{\gamma}}, \qquad (1)$$

where $N(E_0)$ is the counting rate in counts per microcoulomb at the peak of the full-energy peak; $n(E_0) = 1/(a_\gamma \pi^{\frac{1}{2}})$ and a_γ is the half-width of the fullenergy peak at 1/e amplitude; $\Delta E =$ window width of the pulse-height analyzer; ϵ = detection efficiency; R_{γ} is the ratio of the area of the full-energy peak to the total area of the pulse-height spectrum for the γ ray; and A_{γ} is the correction for absorption of the γ ray by material between the source and crystal.

We now consider the accuracy with which the γ -ray yields are determined by discussing in turn the errors in the quantities on the right-hand side of Eq. (1). The errors given are regarded as standard deviations.

 $N(E_0)$.—In favorable cases where the excitation is strong and the isotopic enrichment is high, e.g., Cd¹¹⁴ or Ru^{104} , $N(E_0)$ has an error of 1.5%. This results from the statistical and reading error (1%), the channel width error (1%) and the current integrator error (<0.5%). On the other hand, in the least favorable cases the error in $N(E_0)$ may be as much as 15% and is then the principal error in $\mathcal{I}(\gamma' s/\mu coul)$. These large errors are caused by the combination of relatively weak excitations with strong interference from γ rays excited in the target impurities or in the other isotopes of the target.

 $n(E_0)$ or $a_{\gamma}(E_0)$.—For strong, clean excitations the $a_{\gamma}(E_0)$ were determined directly from the spectra with an accuracy of 1%. The ratio $a_{\gamma}(E_0)/E_0$ is a measure of the resolution of the spectrometer and should, in principle, depend only on the γ -ray energy. Therefore, in case of weak excitation or complicated spectra, one can achieve better accuracy by determining $a_{\gamma}(E_0)$ indirectly from the observed γ -ray energy and the knowledge of the curve $a_{\gamma}(E_0)/E_0$ vs E_{γ} . By the use of a series of radioactive sources, we determined this curve to an accuracy of 2% for γ rays with energies of 100-kev to 2000-kev. However, it was found that the resolution for the cases of strong γ rays from Coulomb excitation was slightly poorer (3 to 5% increase in a_{γ}) than those deduced from the resolution curve obtained from radioactive sources. Further study of this difference showed that it was the result of the large number of soft x-rays present during Coulomb excitation which, after energy degradation to fluorescent x-rays of the graded shield, random sum with the nuclear γ ray being detected. When $a_{\gamma}(E_0)$ was obtained indirectly by the use of the curve $a_{\gamma}(E_0)/E_0$ vs E_{γ} we assigned an error of 3%.

 ΔE .—The average window width for the 20 channels,

 ΔE , was determined to an accuracy of 0.5%. In addition, there are variations in the individual channel widths of 1% but this is considered as an error in $N(E_0)$.

 ϵ .—Values of the total efficiency have been computed by numerical integration for the crystal shapes used for the case of small sources placed at various distances from the crystal surface along the axis of the cylinder.¹¹ The error in the numerical integration is a few tenths of one percent. The values taken for the cross section (the total cross section minus the coherent scattering cross section) are believed to be known to 1% for the range of γ -ray energies of interest here. The error in the cross section results in a smaller error in ϵ , especially for the $3 \text{ in} \times 3 \text{ in}$. crystals with which most of the measurements were made, because these large crystals are approaching the total absorption condition for γ rays of a few hundred kev energy. For measurements made with the 3 in. \times 3 in. crystals, the uncertainty in the distance from the target to the front face of the crystal introduces a 1% error in ϵ and an additional 1% error arises from the uncertainty in the dimensions of the crystal. The combination of these errors gives the result that ϵ is known to within 1.5 to 2% for the $3 \text{ in.} \times 3 \text{ in.}$ crystal. From similar condiderations, the error in ϵ for the $1\frac{1}{2} \times 1$ in. crystal is 2 to 3%.

 R_{γ} .—The quantity R_{γ} , which is defined as the ratio of the area under the full-energy peak to the total area of the γ -ray spectrum, varies with (a) the γ -ray energy, (b) the shape and size of the crystal, and (c) the position of the source with respect to the crystal. Values for R_{γ} for the 3 in. $\times 3$ in. crystal were obtained by measuring spectra of radioactive sources which emit only one γ ray. These spectra were measured with the scintillation counter suspended in the center of a large room to reduce distortion of the spectra at low pulse heights caused by γ rays from the source which first scatter from surrounding objects and then interact with the crystal. The values obtained for R_{γ} are in good agreement with the results of Lazar, Davis, and Bell,¹² who have made more extensive measurements. We have used their values of R_{γ} for the $1\frac{1}{2} \times 1$ in. crystal. The values for R_{γ} are believed to be accurate to 3% for both the 3×3 in. and $1\frac{1}{2} \times 1$ in. crystals. The way in which formula (1) is written assumes that the fullenergy peak is of Gaussian shape. Although this is a very good approximation, it was found that with our 3×3 in. detector as one went far down the high-energy side of the full-energy peak, there occurred a considerable deviation from the Gaussian curve fitted to the peak. The extent of this deviation varied with the γ -ray energy. Consequently, there was a small difference between the actual area under the peak (used in the definition of R_{γ}) and that given by fitting a Gaussian to the peak. Since Gaussian shapes were assumed in

¹¹ See P. R. Bell, Beta- and Gamma-Ray Spectroscopy, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 132. ¹² Lazar, Davis, and Bell, Nucleonics 14, 52 (1956).

analyzing the Coulomb excitation spectra, a small correction was applied to the values of R_{γ} to account for this difference in areas.

 A_{γ} .—In our arrangement, the Coulomb-excited γ rays which are detected emerge from the front face of the target, i.e., the face which the beam strikes. Absorption occurs in (1) a small increment of the target itself, (2) the 5-mil stainless steel target holder tube, (3) the x-ray graded shield, (4) the 5-mil aluminum crystal housing, and (5) the thin layer of Al₂O₃ (\sim 30 mg/cm²) surrounding the crystal. Although the correction for (1) is appreciable for proton excitation of the lowenergy γ rays in the rare-earth region, it is negligible for the γ rays reported here. Absorptions by (2) to (5) are small and may be calculated with fair accuracy. The total absorption for the lowest energy γ ray (Ru¹⁰⁴, 358 kev) was less than 4%. The error in $\mathcal{I}(\gamma's/\mu \text{coul})$ introduced by the uncertainty in A_{γ} is thought to be less than 0.5%.

Compounding the errors outlined above, one has the has the result that under the most favorable conditions the absolute γ -ray yield is determined to an accuracy of 4%.

One reason for giving a discussion of the individual sources of error in the γ -ray yields is that the relevant error is different for different applications. For example, one important feature of the γ -ray yields is the variation in yield of a given γ ray with changes in projectile energy. In this case only the error in the area of the full-energy peak is of interest.

The γ -ray yields are given in column 3 of Table I for the corresponding projectile energies listed in column 2. The isotopic abundance of the nucleus of interest is given in column 4 of Table II. The errors given are those for the area of the full-energy peak. The absolute error in the gamma-ray yields may be obtained approximately by combining this error with an additional 4% error (resulting mainly from errors in R_{γ} and ϵ).

To obtain the reduced electric quadrupole transition probabilities, $B(E2)_{ex}$, one must convert from the γ -ray yields to the number of nuclear excitations. This is done by taking into account (a) the isotopic abundance, i.e., converting to a 100% basis, (b) the total internalconversion coefficient, α_T , and (c) the angular distribution of the γ rays. The values of α_T were obtained from tables calculated by Rose *et al.*¹³ These values are listed in column 3 of Table II. For the γ rays of interest here, α_T is never more than 2% and since the calculations are fairly accurate, the error in B(E2) caused by the uncertainty in α_T is negligible. In general, the error in the isotopic enrichment is 1% or less.

The γ -ray yields are those for an observation angle of 235°. The angular distribution of the γ rays is of the form $A_0P_0+A_2P_2+A_4P_4$. Since A_2 is always much larger than A_4 , the magnitude of the correction for the

angular distribution was minimized by measuring the γ -ray yields at an angle of 235° because P_2 is zero at this angle. A correction for A_4P_4 , which is now known both theoretically and experimentally,¹⁴ was applied to the γ -ray yields to obtain A_0 . This correction, which depends on E_i , ΔE , Z_2 , Z_1 , and m (see below for the definition of these quantities) varied from 0 to 10%. The quantities $N(\text{excitations}/\mu\text{coul})$, are given in column 4 of Table I.

III. EXTRACTION OF $B(E2)_{ex}$

The formula for the cross section for electric quadrupole excitation is

$$\sigma(E2) = \frac{2\pi^2 m^2 V_f^2}{25 Z_2^2 e^2 \hbar^2} B(E2)_{\text{ex}} g_{E2}(\xi, \eta_i), \qquad (2)$$

where

$$\xi = \frac{Z_1 Z_2 e^2}{\hbar} \left[\frac{1}{V_f} - \frac{1}{V_i} \right], \quad \eta_i = \frac{Z_1 Z_2 e^2}{\hbar V_i}.$$
 (3)

 Z_{1e} and Z_{2e} are the charges of the impinging projectile and the nucleus, respectively. V_i and V_f are the initial and final velocities and *m* is the reduced mass. B(E2) is the reduced transition probability for excitation by E2radiation. The function $g_{E2}(\xi,\eta_i)$ has been accurately evaluated.¹⁵

The B(E2) for decay of the 2⁺ state to the 0⁺ ground state [denoted by $B(E2)_d$] is equal to $\frac{1}{5}$ the B(E2) for excitation [denoted by $B(E2)_{ex}$]. The values given for B(E2) will actually be those for the quantity $B(E2)/e^2$.

To compare thick target γ -ray yields with theory it is necessary to integrate the theoretical cross section. The [excitations/ μ coul], N, of the 2⁺ state is given by

$$N = \operatorname{const} \int_{0}^{E_{i}} \frac{\sigma(E)dE}{S(E)}.$$
 (4)

Substituting from Eq. (2) for the cross section gives the result $B(E2)_{ex} \propto N/Y$, where

$$Y = K^2 \int_0^{E_i} \frac{E_f g_{E2}(\xi, \eta_i) dE}{S(E)}.$$
 (5)

 E_i is the incident energy of the projectile; $E_f = E_i - \Delta E/K$, where ΔE is the energy of the excited state and $K = M_2/(M_1 + M_2)$, M_1 and M_2 being the masses of the projectile and target 'nuclei, respectively; and S(E) is the rate of energy loss of the projectile in the target.

One of the principal sources of error in the absolute values of B(E2) is the uncertainty in the energy loss of the exciting projectiles in the target materials. For example, one sees from formula (5) that if S(E) is taken uniformly 10% too large, then Y will be in error

¹³ Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. 83, 79 (1951), and tables of internal conversion coefficients privately circulated by M. E. Rose.

¹⁴ F. K. McGowan and P. H. Stelson, Phys. Rev. **106**, 522 (1957). ¹⁵ See review article by Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

TABLE I. The nucleus and observed γ -ray energy are given in column 1. Column 3 gives the observed yield of γ rays (at $\theta = 55^{\circ}$) for thick-target bombardment by protons or α particles with energy given in column 2. The errors given are relative errors (see text). The isotopic enrichments of the targets are listed in column 4 of Table II. The yields have not been converted to a 100% isotopic enrichment basis. Column 4 gives the number of excitations per microcoulomb on a 100% isotopic enrichment basis. Corrections for internal conversion (column 3 of Table II) and for the angular distribution were applied to the γ -ray yields to obtain excitations per microcoulomb. Column 5 lists the integral Y (in kev×mg/cm²) which is needed to deduce the $B(E2)_{ex}$. The integral Y is defined by Eq. (5) in the text. Column 6 lists the $B(E2)_{ex}$ in units of cm⁴.

Nucleus $(E_{\gamma} \text{ in kev})$	$E_p \text{ or } E_{\boldsymbol{\alpha}} \ (\mathrm{Mev})$	Yield of γ 's/ μ coul	Excitations/ μ coul	Y	$B(E2)_{ex}$
Mo ⁹⁴ (874)	$3.00 \\ 2.70 \\ 2.40$	$(2.48\pm0.25)\times10^4$ $(8.15\pm1.60)\times10^3$ $(1.60\pm0.26)\times10^3$	$(3.02\pm0.30)\times10^4$ $(9.82\pm1.90)\times10^3$ $(1.91\pm0.31)\times10^3$	1.41×10^{3} 4.10×10^{2} 7.79×10	$(2.54\pm0.25)\times10^{-49}$ $(2.85\pm0.55)\times10^{-49}$ $(2.91\pm0.48)\times10^{-49}$
Mo ⁹⁶ (775)	$3.00 \\ 2.70 \\ 2.40$	$(5.63\pm0.85)\times10^4$ $(2.20\pm0.33)\times10^4$ $(5.16\pm0.50)\times10^3$	$(6.33 \pm 0.95) \times 10^4$ $(2.47 \pm 0.37) \times 10^4$ $(5.73 \pm 0.55) \times 10^3$	2.60×10^{3} 9.40×10^{2} 2.24×10^{2}	$(2.89\pm0.44)\times10^{-49}$ $(3.12\pm0.47)\times10^{-49}$ $(3.04\pm0.29)\times10^{-49}$
Mo ⁹⁸ (780)	$3.00 \\ 2.70 \\ 2.40$	$(5.17\pm0.35)\times10^4$ $(2.01\pm0.35)\times10^4$ $(5.83\pm0.55)\times10^3$	$(5.27\pm0.36)\times10^4$ $(2.04\pm0.35)\times10^4$ $(5.86\pm0.55)\times10^3$	2.51×10^{3} 9.00×10^{2} 2.12×10^{2}	$(2.49\pm0.17)\times10^{-49}$ $(2.69\pm0.46)\times10^{-49}$ $(3.28\pm0.30)\times10^{-49}$
Mo ¹⁰⁰ (530)	$3.00 \\ 2.70 \\ 2.40$	$(5.19\pm0.50)\times10^4$ $(2.69\pm0.20)\times10^4$ $(1.13\pm0.10)\times10^4$	$(5.29\pm0.51)\times10^{5}$ $(2.72\pm0.20)\times10^{5}$ $(1.14\pm0.10)\times10^{5}$	1.06×10^4 5.29×10^3 2.12×10^3	$(5.93 \pm 0.60) \times 10^{-49}$ $(6.10 \pm 0.48) \times 10^{-49}$ $(6.39 \pm 0.61) \times 10^{-49}$
Ru ⁹⁶ (840)	3.00	$(2.80\pm0.4) \times 10^4$	$(2.80\pm0.4)\times10^{4}$	1.52×10 ³	$(2.54 \pm 0.35) \times 10^{-49}$
Ru ⁹⁸ (654)	$3.00 \\ 2.70 \\ 2.40 \\ 2.10$	$(1.04\pm0.03) imes10^{5}$ $(4.61\pm0.18) imes10^{4}$ $(1.54\pm0.08) imes10^{4}$ $(3.32\pm0.27) imes10^{3}$	$(1.55\pm0.05) imes10^5$ $(6.83\pm0.27) imes10^4$ $(2.26\pm0.12) imes10^4$ $(4.81\pm0.39) imes10^3$	4.9×10^{3} 2.03×10^{3} 6.18×10^{2} 1.24×10^{2}	$\substack{(4.37\pm0.14)\times10^{-49}\\(4.65\pm0.18)\times10^{-49}\\(5.05\pm0.27)\times10^{-49}\\(5.36\pm0.44)\times10^{-49}}$
Ru ¹⁰⁰ (540)	3.00 2.70 2.40 2.10	$(3.34\pm0.09) imes10^{5}\ (1.73\pm0.05) imes10^{5}\ (7.05\pm0.21) imes10^{4}\ (1.84\pm0.15) imes10^{4}$	$(3.69\pm0.10) imes10^5\ (1.90\pm0.06) imes10^5\ (7.67\pm0.23) imes10^4\ (1.98\pm0.16) imes10^4$	9.40×10^{3} 4.54×10^{3} 1.76×10^{3} 4.81×10^{2}	$\begin{array}{c} (5.43 {\pm} 0.15) {\times} 10^{-49} \\ (5.78 {\pm} 0.18) {\times} 10^{-49} \\ (6.02 {\pm} 0.18) {\times} 10^{-49} \\ (5.69 {\pm} 0.14) {\times} 10^{-49} \end{array}$
Ru ¹⁰² (475)	3.00 2.70 2.40 2.10 1.80 1.50	$(6.85 \pm 0.17) \times 10^{5}$ $(3.37 \pm 0.09) \times 10^{5}$ $(1.49 \pm 0.04) \times 10^{5}$ $(4.96 \pm 0.13) \times 10^{4}$ $(1.03 \pm 0.03) \times 10^{4}$ $(8.4 \pm 1.3) \times 10^{2}$	$\begin{array}{c} (7.19 {\pm} 0.18) {\times} 10^5 \\ (3.52 {\pm} 0.10) {\times} 10^5 \\ (1.54 {\pm} 0.04) {\times} 10^5 \\ (5.09 {\pm} 0.13) {\times} 10^4 \\ (1.04 {\pm} 0.03) {\times} 10^4 \\ (8.35 {\pm} 1.3) {\times} 10^2 \end{array}$	$\begin{array}{c} 1.31 \times 10^4 \\ 6.83 \times 10^3 \\ 2.97 \times 10^3 \\ 9.63 \times 10^2 \\ 1.91 \times 10^2 \\ 1.50 \times 10 \end{array}$	$(7.59\pm0.19)\times10^{-49}$ $(7.12\pm0.20)\times10^{-49}$ $(7.17\pm0.19)\times10^{-49}$ $(7.30\pm0.19)\times10^{-49}$ $(7.53\pm0.22)\times10^{-49}$ $(7.69\pm1.2)\times10^{-49}$
Ru ¹⁰⁴ (358)	3.00 2.70 2.40 2.10 1.80 1.50	$(1.49\pm0.04) imes10^{6}\ (8.80\pm0.22) imes10^{5}\ (4.39\pm0.11) imes10^{5}\ (1.94\pm0.05) imes10^{5}\ (6.11\pm0.16) imes10^{4}\ (1.01\pm0.03) imes10^{4}$	$\begin{array}{c} (1.58 {\pm} 0.04) {\times} 10^6 \\ (9.31 {\pm} 0.23) {\times} 10^5 \\ (4.61 {\pm} 0.11) {\times} 10^5 \\ (2.02 {\pm} 0.05) {\times} 10^5 \\ (6.30 {\pm} 0.16) {\times} 10^4 \\ (1.03 {\pm} 0.03) {\times} 10^4 \end{array}$	2.31×10^4 1.37×10^4 7.14×10^3 3.04×10^3 9.19×10^2 1.54×10^2	$\begin{array}{c}(9.45{\pm}0.24){\times}10^{-49}\\(9.39{\pm}0.23){\times}10^{-49}\\(8.92{\pm}0.21){\times}10^{-49}\\(9.18{\pm}0.23){\times}10^{-49}\\(9.47{\pm}0.24){\times}10^{-49}\\(9.24{\pm}0.29){\times}10^{-49}\end{array}$
$Pd^{104}(555)$	$3.30 \\ 2.40 \\ 2.10$	$(3.21\pm0.10)\times10^{5}$ $(2.80\pm0.10)\times10^{4}$ $(7.45\pm0.30)\times10^{3}$	$(5.08 \pm 0.16) \times 10^{5}$ $(4.35 \pm 0.17) \times 10^{4}$ $(1.15 \pm 0.05) \times 10^{4}$	1.43×10^4 1.32×10^3 3.28×10^2	$(5.63\pm0.18)\times10^{-49}$ $(5.22\pm0.16)\times10^{-49}$ $(5.56\pm0.17)\times10^{-49}$
Pd ¹⁰⁶ (513)	3.30 3.00 2.70 2.40 2.10	$(6.21 \pm 0.15) imes 10^5$ $(3.34 \pm 0.10) imes 10^5$ $(1.66 \pm 0.05) imes 10^5$ $(6.73 \pm 0.16) imes 10^4$ $(1.98 \pm 0.05) imes 10^4$	$\begin{array}{c} (7.47{\pm}0.18){\times}10^5 \\ (4.00{\pm}0.12){\times}10^5 \\ (1.98{\pm}0.06){\times}10^5 \\ (7.95{\pm}0.19){\times}10^4 \\ (2.32{\pm}0.06){\times}10^4 \end{array}$	$\begin{array}{c} 1.81 \times 10^{4} \\ 1.03 \times 10^{4} \\ 5.03 \times 10^{3} \\ 1.91 \times 10^{3} \\ 5.34 \times 10^{2} \end{array}$	$\begin{array}{c} (6.54{\pm}0.16){\times}10^{-49} \\ (6.16{\pm}0.19){\times}10^{-49} \\ (6.24{\pm}0.20){\times}10^{-49} \\ (6.60{\pm}0.16){\times}10^{-49} \\ (6.89{\pm}0.18){\times}10^{-49} \end{array}$
Pd ¹⁰⁸ (433)	3.30 3.00 2.70 2.40 2.10	$\begin{array}{c} (1.12 \pm 0.03) \times 10^6 \\ (6.66 \pm 0.18) \times 10^5 \\ (3.58 \pm 0.08) \times 10^5 \\ (1.59 \pm 0.04) \times 10^5 \\ (7.53 \pm 0.24) \times 10^4 \end{array}$	$\begin{array}{c} (1.19{\pm}0.03){\times}10^6\\ (7.02{\pm}0.19){\times}10^5\\ (3.76{\pm}0.08){\times}10^5\\ (1.66{\pm}0.04){\times}10^5\\ (7.80{\pm}0.25){\times}10^4 \end{array}$	2.50×10^4 1.50×10^4 8.00×10^3 3.62×10^3 1.24×10^3	$\begin{array}{c} (7.53 {\pm} 0.20) {\times} 10^{-49} \\ (7.42 {\pm} 0.20) {\times} 10^{-9} \\ (7.45 {\pm} 0.16) {\times} 10^{-49} \\ (7.27 {\pm} 0.17) {\times} 10^{-49} \\ (9.95 {\pm} 0.35) {\times} 10^{-49} \end{array}$
Pd ¹¹⁰ (374)	3.30 3.00 2.70 2.40 2.10	$\begin{array}{c} (1.65{\pm}0.04){\times}10^6\\ (1.01{\pm}0.02){\times}10^6\\ (5.62{\pm}0.12){\times}10^5\\ (2.87{\pm}0.10){\times}10^5\\ (1.11{\pm}0.03){\times}10^5 \end{array}$	$\begin{array}{c} (1.82{\pm}0.04){\times}10^6\\ (1.11{\pm}0.02){\times}10^6\\ (6.15{\pm}0.13){\times}10^5\\ (3.12{\pm}0.11){\times}10^5\\ (1.20{\pm}0.03){\times}10^5 \end{array}$	3.22×10^4 2.03×10^4 1.15×10^4 5.93×10^3 2.37×10^3	$\begin{array}{c}(8.95{\pm}0.20){\times}10^{-49}\\(8.67{\pm}0.16){\times}10^{-49}\\(8.72{\pm}0.18){\times}10^{-49}\\(8.34{\pm}0.29){\times}10^{-49}\\(8.03{\pm}0.20){\times}10^{-49}\end{array}$
$Cd^{106}(630)$	3.30 3.00 2.70 2.40 2.10	$\begin{array}{c} (8.96{\pm}0.45){\times}10^4 \\ (3.95{\pm}0.30){\times}10^4 \\ (1.49{\pm}0.15){\times}10^4 \\ (5.43{\pm}0.80){\times}10^3 \\ (6.25{\pm}1.50){\times}10^2 \end{array}$	$(2.67 \pm 0.13) imes 10^{5}$ $(1.17 \pm 0.09) imes 10^{5}$ $(4.38 \pm 0.44) imes 10^{4}$ $(1.58 \pm 0.23) imes 10^{4}$ $(1.81 \pm 0.45) imes 10^{3}$	9.45×10^{3} 4.64×10^{3} 1.88×10^{3} 5.46×10^{2} 1.02×10^{2}	$\begin{array}{c} (5.14 \pm 0.25) \times 10^{-49} \\ (4.58 \pm 0.36) \times 10^{-49} \\ (4.24 \pm 0.43) \times 10^{-49} \\ (5.26 \pm 0.77) \times 10^{-49} \\ (3.23 \pm 0.80) \times 10^{-49} \end{array}$

Nucleus $(E_{\gamma} \text{ in kev})$	$E_p ext{ or } E_{\alpha}$ (Mev)	Yield of γ's/μcoul	Excitations/ μ coul	Y	$B(E2)_{ex}$
Cd ¹⁰⁸ (630)	3.30 3.00 2.70 2.40	$(5.43\pm1.00) imes10^4$ $(1.97\pm0.35) imes10^4$ $(6.11\pm1.20) imes10^3$ $(2.17\pm0.50) imes10^3$	$(3.75\pm0.69) imes10^{5}\ (1.35\pm0.24) imes10^{5}\ (4.16\pm0.82) imes10^{4}\ (1.45\pm0.34) imes10^{4}$	9.45×10^{3} 4.64×10^{3} 1.88×10^{3} 5.46×10^{2}	$\begin{array}{c} (7.21 \pm 1.3) \times 10^{-49} \\ (5.29 \pm 0.96) \times 10^{-49} \\ (4.02 \pm 0.80) \times 10^{-49} \\ (4.83 \pm 1.14) \times 10^{-49} \end{array}$
Cd ¹¹⁰ (656)	3.30 3.00 2.70 2.40 2.10	$(1.65\pm0.04) imes 10^5$ $(7.52\pm0.19) imes 10^4$ $(2.99\pm0.15) imes 10^4$ $(9.20\pm0.50) imes 10^3$ $(1.46\pm0.08) imes 10^3$	$\begin{array}{c} (2.30{\pm}0.06){\times}10^5 \\ (1.04{\pm}0.03){\times}10^5 \\ (4.12{\pm}0.21){\times}10^4 \\ (1.26{\pm}0.07){\times}10^4 \\ (1.98{\pm}0.09){\times}10^3 \end{array}$	$\begin{array}{c} 8.14 \times 10^{3} \\ 3.92 \times 10^{3} \\ 1.54 \times 10^{3} \\ 4.19 \times 10^{2} \\ 7.26 \times 10 \end{array}$	$\begin{array}{c} (5.14 \pm 0.13) \times 10^{-49} \\ (4.82 \pm 0.14) \times 10^{-49} \\ (4.86 \pm 0.25) \times 10^{-49} \\ (5.47 \pm 0.30) \times 10^{-49} \\ (5.03 \pm 0.23) \times 10^{-49} \end{array}$
Cd ¹¹² (610)	3.303.002.702.402.10	$(2.68\pm0.08) imes 10^5$ $(1.28\pm0.03) imes 10^5$ $(5.73\pm0.14) imes 10^4$ $(1.79\pm0.06) imes 10^4$ $(3.43\pm0.16) imes 10^3$	$\begin{array}{c} (3.14 \pm 0.09) \times 10^5 \\ (1.49 \pm 0.04) \times 10^5 \\ (6.65 \pm 0.16) \times 10^4 \\ (2.06 \pm 0.07) \times 10^4 \\ (3.92 \pm 0.18) \times 10^3 \end{array}$	1.05×10^4 5.27×10^3 2.18×10^3 6.71×10^2 1.33×10^2	$\begin{array}{c} (5.44 {\pm} 0.16) {\times} 10^{-49} \\ (5.14 {\pm} 0.14) {\times} 10^{-49} \\ (5.55 {\pm} 0.13) {\times} 10^{-49} \\ (5.58 {\pm} 0.19) {\times} 10^{-49} \\ (5.36 {\pm} 0.25) {\times} 10^{-49} \end{array}$
Cd ¹¹⁴ (555)	3.30 3.00 2.70 2.40 2.10 9.00 ^a	$\begin{array}{c} (4.30{\pm}0.10){\times}10^5\\ (2.14{\pm}0.05){\times}10^5\\ (1.02{\pm}0.02){\times}10^5\\ (3.61{\pm}0.09){\times}10^4\\ (8.71{\pm}0.27){\times}10^3\\ (2.03{\pm}0.05){\times}10^6\end{array}$	$\begin{array}{c} (4.50\pm0.10)\times10^5\\ (2.23\pm0.05)\times10^5\\ (1.05\pm0.02)\times10^5\\ (3.70\pm0.09)\times10^4\\ (8.86\pm0.28)\times10^3\\ (2.13\pm0.05)\times10^6\end{array}$	$\begin{array}{c} 1.38 \times 10^{4} \\ 7.30 \times 10^{3} \\ 3.30 \times 10^{3} \\ 1.16 \times 10^{3} \\ 2.67 \times 10^{2} \\ 3.46 \times 10^{4} \end{array}$	$\begin{array}{c} (6.11 \pm 0.14) \times 10^{-49} \\ (5.55 \pm 0.12) \times 10^{-49} \\ (5.78 \pm 0.11) \times 10^{-49} \\ (5.80 \pm 0.14) \times 10^{-49} \\ (6.03 \pm 0.19) \times 10^{-49} \\ (5.60 \pm 0.14) \times 10^{-49} \end{array}$
Cd ¹¹⁶ (517)	3.30 3.00 2.70 2.40 2.10	$(3.85\pm0.09) \times 10^{5}$ $(2.16\pm0.05) \times 10^{5}$ $(1.03\pm0.03) \times 10^{4}$ $(4.00\pm0.10) \times 10^{4}$ $(1.14\pm0.03) \times 10^{4}$	$(5.44\pm0.13) imes 10^{5}$ $(3.05\pm0.07) imes 10^{5}$ $(1.37\pm0.04) imes 10^{5}$ $(5.45\pm0.14) imes 10^{4}$ $(1.54\pm0.04) imes 10^{4}$	$\begin{array}{c} 1.66 \times 10^{4} \\ 9.16 \times 10^{3} \\ 4.33 \times 10^{3} \\ 1.63 \times 10^{3} \\ 4.33 \times 10^{2} \end{array}$	$\begin{array}{c} (5.96 \pm 0.14) \times 10^{-49} \\ (6.05 \pm 0.14) \times 10^{-49} \\ (5.75 \pm 0.17) \times 10^{-49} \\ (6.08 \pm 0.16) \times 10^{-49} \\ (6.47 \pm 0.17) \times 10^{-49} \end{array}$
Te ¹²⁶ (673)	3.30 3.00 10.00ª 9.00ª	$(2.59\pm0.25)\times10^4$ $(1.18\pm0.12)\times10^4$ $(3.26\pm0.11)\times10^5$ $(1.76\pm0.06)\times10^5$	$(1.40\pm0.13) imes10^5\ (6.11\pm0.62) imes10^4\ (1.73\pm0.06) imes10^6\ (9.33\pm0.33) imes10^5$	6.45×10^{3} 2.89×10^{3} 3.83×10^{4} 2.07×10^{4}	$\begin{array}{c} (5.25 \pm 0.50) \times 10^{-49} \\ (5.12 \pm 0.50) \times 10^{-49} \\ (5.47 \pm 0.20) \times 10^{-49} \\ (5.45 \pm 0.20) \times 10^{-49} \end{array}$
Te ¹²⁸ (750)	3.30 3.00 10.00 ^a 9.00 ^a	$(2.14\pm0.20)\times10^4$ $(9.22\pm1.00)\times10^3$ $(3.25\pm0.11)\times10^5$ $(1.71\pm0.06)\times10^5$	$(6.51\pm0.65) imes10^4 \ (2.79\pm0.30) imes10^4 \ (1.01\pm0.04) imes10^6 \ (5.32\pm0.19) imes10^5$	4.08×10^{3} 1.62×10^{3} 2.95×10^{4} 1.50×10^{4}	$\begin{array}{c} (3.87 {\pm} 0.38) {\times} 10^{-49} \\ (4.17 {\pm} 0.43) {\times} 10^{-49} \\ (4.14 {\pm} 0.14) {\times} 10^{-49} \\ (4.29 {\pm} 0.15) {\times} 10^{-49} \end{array}$
Te ¹³⁰ (850)	3.30 3.00 10.00ª 9.00ª	$(9.40\pm0.80) imes10^3$ $(4.04\pm0.60) imes10^3$ $(2.02\pm0.09) imes10^5$ $(9.97\pm0.45) imes10^4$	$(2.63\pm0.22)\times10^4$ $(1.72\pm0.13)\times10^4$ $(5.77\pm0.26)\times10^5$ $(2.85\pm0.13)\times10^5$	2.10×10^{3} 7.31×10^{2} 2.10×10^{4} 9.95×10^{3}	$\begin{array}{c} (3.03 \pm 0.30) \times 10^{-49} \\ (3.71 \pm 0.70) \times 10^{-49} \\ (3.33 \pm 0.15) \times 10^{-49} \\ (3.47 \pm 0.15) \times 10^{-49} \end{array}$
Sn ¹¹⁶ (1268)	9.13ª 10.13ª	$(4.02\pm0.37) imes10^4\ (1.03\pm0.09) imes10^5$	$(4.23 \pm 0.39) \times 10^4$ $(1.08 \pm 0.09) \times 10^5$	2.03×10^{3} 5.72×10^{3}	$(2.17\pm0.20) \times 10^{-49}$ $(1.97\pm0.17) \times 10^{-49}$
Sn ¹¹⁸ (1219)	8.12ª 9.13ª 10.13ª	$(1.44{\pm}0.21){ imes}10^4$ $(5.64{\pm}0.41){ imes}10^4$ $(1.45{\pm}0.07){ imes}10^5$	$(1.48\pm0.21)\times10^4$ $(5.80\pm0.42)\times10^4$ $(1.50\pm0.07)\times10^5$	6.95×10^{2} 2.56×10^{3} 6.88×10^{3}	$(2.21\pm0.32)\times10^{-49}$ $(2.35\pm0.17)\times10^{-49}$ $(2.28\pm0.12)\times10^{-49}$
Sn ¹²⁰ (1155)	8.12ª 9.13ª 10.13ª	$(2.31\pm0.27)\times10^4$ $(7.52\pm0.36)\times10^4$ $(1.75\pm0.07)\times10^5$	$(2.29\pm0.27)\times10^4$ $(7.50\pm0.36)\times10^4$ $(1.75\pm0.07)\times10^4$	9.85×10^{2} 3.39×10^{3} 8.72×10^{3}	$(2.43\pm0.28)\times10^{-49}$ $(2.30\pm0.11)\times10^{-49}$ $(2.09\pm0.08)\times10^{-49}$
Sn ¹²² (1130)	8.12ª 9.13ª 10.13ª	$(3.36\pm0.45) imes10^4$ $(9.78\pm0.93) imes10^4$ $(1.97\pm0.16) imes10^5$	$(3.60\pm0.47)\times10^4$ $(1.05\pm0.99)\times10^5$ $(2.12\pm0.16)\times10^5$	1.13×10^{3} 3.80×10^{3} 9.56×10^{3}	$(3.31\pm0.43)\times10^{-49}$ $(2.88\pm0.27)\times10^{-49}$ $(2.33\pm0.17)\times10^{-49}$
Sn ¹²⁴ (1128)	8.12ª 9.13ª 10.13ª	$(2.61\pm0.34)\times10^4$ $(6.87\pm0.47)\times10^4$ $(1.81\pm0.12)\times10^5$	$(2.73\pm0.35)\times10^4$ $(7.20\pm0.50)\times10^4$ $(1.92\pm0.12)\times10^5$	1.15×10^{3} 3.85×10^{3} 9.66×10^{3}	$(2.48\pm0.31)\times10^{-49}$ $(1.95\pm0.13)\times10^{-49}$ $(2.06\pm0.13)\times10^{-49}$
Zr ^{92, 94} (920)	9.00ª	$(7.17 \pm 0.72) \times 10^4$	$(2.05\pm0.21)\times10^{5}$	1.33×104	$(0.79 \pm 0.08) \times 10^{-49}$

TABLE I.—Continued.

^a Doubly charged α particles.

by 10%. The existing information on S(E) for mediumweight elements for protons and α particles of several Mev energy has an estimated accuracy of 10%. Since the yield of γ rays, \mathfrak{I} , which is the other main source of

error in obtaining B(E2), can in some cases be measured to 4%, the error in S(E) is seen to be quite important. In the limit of very thin targets the uncertainty is S(E)is eliminated. However, it has been impractical to con-

TABLE II. Column 2 lists the observed γ -ray energy from Coulomb excitation of the target nucleus in **co**lumn 1. The assigned error is considered to be a standard deviation. Columns 3 and 4 list the total internal conversion coefficient and the isotopic enrichment of the target. Column 5 lists our best value for the $B(E2)_{ex}$ in units cm⁴ (converted to a 100% isotopic abundance basis). The columns headed R.E. and A.E. give, respectively, the relative percentage error (see text) and the absolute percentage error (standard deviations) for the $B(E2)_{ex}$. The column headed $B(E2)_{a/B}(E2)_{ap}$ contains the ratio of the observed decay rate to that for a single-particle transition. The quantities β , C_2 , and $B_2/(B_2)_{irrot}$ which are listed in the last three columns, are defined in the text.

	P		Tastasia	B(Ea)			$B(E2)_d$			C	
Nucleus	(kev)	ατ	abundance	$\times 10^{49}$	R.E.	A.E.	$\overline{B(E2)_{\rm sp}}$	$T_{\frac{1}{2}}(\text{sec})$	β	(Mev)	$B_2/(B_2)_{ m irrot}$
M0 ⁹⁴ M0 ⁹⁶ M0 ⁹⁸ M0 ¹⁰⁰	874 ± 9 775 ± 8 780 ± 8 530 ± 5	$\begin{array}{c} 0.0011 \\ 0.0015 \\ 0.0015 \\ 0.0038 \end{array}$	$\begin{array}{c} 79.1 \ \pm 1.0 \\ 85.94 \pm 1.0 \\ 95.0 \ \pm 1.0 \\ 9.67^{a} \end{array}$	2.70 3.02 2.70 6.14	$\pm 11 \\ \pm 11 \\ \pm 10 \\ \pm 8$	$\pm 13 \\ \pm 13 \\ \pm 12 \\ \pm 10$	21 23 20 48	$\begin{array}{c} 2.0 \times 10^{-12} \\ 3.4 \times 10^{-12} \\ 3.6 \times 10^{-12} \\ 1.1 \times 10^{-11} \end{array}$	$\begin{array}{c} 0.174 \\ 0.181 \\ 0.169 \\ 0.252 \end{array}$	72 59 68 21	12 12 13 8.4
Ru ⁹⁶ Ru ⁹⁸ Ru ¹⁰⁰ Ru ¹⁰² Ru ¹⁰⁴	$840 \pm 10 \\ 654 \pm 6 \\ 540 \pm 5 \\ 475 \pm 5 \\ 358 \pm 3$	$\begin{array}{c} 0.0017 \\ 0.0032 \\ 0.0052 \\ 0.0076 \\ 0.0188 \end{array}$	$\begin{array}{c} 95.51 {\pm} 0.11 \\ 65.1 \ \pm 0.4 \\ 88.9 \ \pm 0.1 \\ 94.24 {\pm} 0.03 \\ 95.1 \ \pm 0.2 \end{array}$	2.54 4.75 5.72 7.33 9.28	$\pm 15 \\ \pm 6 \\ \pm 5 \\ \pm 4 \\ \pm 4$	$\pm 16 \\ \pm 8 \\ \pm 7 \\ \pm 7 \\ \pm 7 \\ \pm 7 \\ \pm 7$	19 35 41 52 64	$\begin{array}{c} 2.7 \times 10^{-12} \\ 5.0 \times 10^{-12} \\ 1.1 \times 10^{-11} \\ 1.6 \times 10^{-11} \\ 5.2 \times 10^{-11} \end{array}$	$\begin{array}{c} 0.160 \\ 0.214 \\ 0.232 \\ 0.259 \\ 0.276 \end{array}$	83 36 25 17.7 10.8	14 9.8 9.6 8.5 8.9
Pd ¹⁰⁴ Pd ¹⁰⁶ Pd ¹⁰⁸ Pd ¹¹⁰	$555\pm 6 \\ 513\pm 5 \\ 433\pm 4 \\ 374\pm 4$	$\begin{array}{c} 0.0045 \\ 0.0056 \\ 0.0087 \\ 0.0137 \end{array}$	62.20 ± 2.43 82.33 ± 0.03 94.19 ± 0.11 91.42 ± 0.32	$5.47 \\ 6.46 \\ 7.42 \\ 8.60$	$\pm 5 \\ \pm 4 \\ \pm 4 \\ \pm 4$	土 7 土 7 土 7 土 7 土	38 43 48 55	$\begin{array}{c} 9.8 \times 10^{-12} \\ 1.2 \times 10^{-11} \\ 2.5 \times 10^{-11} \\ 4.5 \times 10^{-11} \end{array}$	$\begin{array}{c} 0.214 \\ 0.227 \\ 0.241 \\ 0.255 \end{array}$	31 25 18.8 14.4	10.6 9.7 9.9 9.9
$\begin{array}{c} Cd^{106} \\ Cd^{108} \\ Cd^{110} \\ Cd^{112} \\ Cd^{114} \\ Cd^{116} \end{array}$	630 ± 10 630 ± 10 656 ± 6 610 ± 6 555 ± 5 517 ± 5	$\begin{array}{c} 0.0036\\ 0.0036\\ 0.0032\\ 0.0039\\ 0.0050\\ 0.0061 \end{array}$	$\begin{array}{c} 32.9 \ \pm 0.5 \\ 14.2 \ \pm 0.2 \\ 70.0 \ \pm 0.5 \\ 83.5 \ \pm 1.0 \\ 94.2 \ \pm 1.0 \\ 71.2 \ \pm 0.5 \end{array}$	$\begin{array}{r} 4.70 \\ 5.35 \\ 5.04 \\ 5.42 \\ 5.84 \\ 6.00 \end{array}$	$\begin{array}{c} \pm \ 9 \\ \pm 20 \\ \pm \ 6 \\ \pm \ 5 \\ \pm \ 4 \\ \pm \ 4 \end{array}$	$\pm 11 \\ \pm 21 \\ \pm 8 \\ \pm 7 \\ \pm 7 \\ \pm 7 \\ \pm 7 \\ \pm 7$	32 35 32 34 35 36	$\begin{array}{c} 6.0\!\times\!10^{\!-\!12} \\ 5.3\!\times\!10^{\!-\!12} \\ 4.6\!\times\!10^{\!-\!12} \\ 6.2\!\times\!10^{\!-\!12} \\ 9.2\!\times\!10^{\!-\!12} \\ 1.3\!\times\!10^{\!-\!11} \end{array}$	$\begin{array}{c} 0.186\\ 0.195\\ 0.187\\ 0.193\\ 0.197\\ 0.197\end{array}$	46 41 47 42 36 33	12 10 10.4 10.5 10.5 10.8
${f Te^{126}}\ {Te^{128}}\ {Te^{130}}$	673 ± 7 750 ± 10 850 ± 10	$\begin{array}{c} 0.0032 \\ 0.0024 \\ 0.0018 \end{array}$	18.69ª 31.81ª 34.44ª	$5.32 \\ 4.12 \\ 3.40$	$^{\pm 5}_{\pm 6}_{\pm 7}$	$^{\pm}_{\pm} {}^{7}_{8}_{\pm} {}^{9}_{9}$	28 21 17	3.8×10^{-12} 2.9×10^{-12} 1.9×10^{-12}	$0.163 \\ 0.141 \\ 0.127$	64 94 130	11 12 13
Sn ¹¹⁶ Sn ¹¹⁸ Sn ¹²⁰ Sn ¹²² Sn ¹²⁴ Zr ^{92, 94}	$1268 \pm 10 \\ 1219 \pm 10 \\ 1155 \pm 10 \\ 1130 \pm 10 \\ 1128 \pm 10 \\ 920 \pm 10 \\$	0.0008 0.0009 0.0010 0.0010 0.0010 0.0010	$\begin{array}{c} 92.64 \pm 0.07 \\ 94.91 \pm 0.19 \\ 98.14 \pm 0.07 \\ 88.92 \pm 0.06 \\ 90.26 \pm 0.06 \\ 34.5^{a} \end{array}$	2.07 2.28 2.20 2.52 2.13 0.79	$\pm 11 \\ \pm 10 \\ \pm 8 \\ \pm 10 \\ \pm 9 \\ \pm 11$	$\pm 13 \\ \pm 12 \\ \pm 10 \\ \pm 12 \\ \pm 11 \\ \pm 13$	12 13 13 14 12 6.3	$\begin{array}{c} 4.2 \times 10^{-13} \\ 4.6 \times 10^{-13} \\ 6.3 \times 10^{-13} \\ 6.0 \times 10^{-13} \\ 7.9 \times 10^{-13} \\ 5.5 \times 10^{-12} \end{array}$	$\begin{array}{c} 0.111\\ 0.115\\ 0.112\\ 0.119\\ 0.108\\ 0.100 \end{array}$	260 230 230 200 240 230	14 13 14 13 16 23

^a Targets with natural isotopic abundance.

struct thin targets of accurately known thickness from the limited amounts of enriched isotopes available. In order to reduce the error in B(E2) from the uncertainty in S(E) for thick-target experiments, measurements were made with thick and thin targets of silver (Z=47) and gold (Z=79), since for these two elements one can make stable uniform thin targets of accurately known thickness. From a comparison of the values for B(E2) obtained from thin and thick targets for a series of incident energies, one can deduce the error in S(E)and alter S(E) until the yields from thin and thick targets give consistent values for B(E2). The details of these experiments are given in the appendix. It is believed that the error in S(E) for protons in silver has been reduced to 4%. The new values are approximately 10% larger than those previously used.² The stopping power for α particles in silver was obtained using the assumption that $S_{\alpha} = 4S_p$ at an α -particle energy which is 4 times the proton energy. The resulting values for $S(E)_{\alpha}$ are in good agreement with the early measurements of Rosenblum.¹⁶ The stopping power of silver was

used to obtain the stopping power of other medium weight elements on the assumption that S(E) expressed in kev/mg/cm² varies as $Z^{-\frac{1}{2}}$. This procedure ignores "shell effects" which might be as large as several percent.¹⁷

Other sources of error in the values for $B(E2)_{ex}$ result from errors in (a) the incident energy, E_i , (b) the excitation energy of the state, ΔE , and (c) the actual numerical evaluation of the integrals, Y. The integrals are quite sensitive to changes in E_i and ΔE . For example, in the least favorable cases, e.g., α -particle excitation of the tin states, a 1% alteration of E_i changes Y by 5 to 10% and a 1% alteration of ΔE changes Y by 5%. In the most favorable cases, e.g., proton excitation of Ru¹⁰⁴, a 1% alteration of E_i changes Y by 5% and a 1% alteration of ΔE changes Y by 2%. Since we have determined the incident energies to 0.2%, the errors in $B(E2)_{ex}$ resulting from errors in E_i are 1 to 2%. In general the excitation energies have an accuracy of about 1% which means the resulting errors in $B(E2)_{ex}$ range from 2 to 5%. The integrals, Y, were obtained

¹⁶ S. Rosenblum, Ann. phys. 10, 408 (1928).

¹⁷ A. Winther (private communication).

from numerical integrations by the use of Simpson's rule. It is believed that they are correct to within 2%; the uncertainty results in part from the actual numerical integration and in part from reading and interpolation errors in $g_{E2}(\xi, \eta_i)$.

The values for the integrals, Y, in units of kev×mg/cm², are given in column 5 of Table I. The last column contains the values for $B(E2)_{ex}$ deduced from the $N(\text{excitations}/\mu\text{coul})$ and Y. The errors listed for the $B(E2)_{ex}$ in this table reflect only the error in the area of the full-energy γ -ray peak since we are interested here in the constancy of $B(E2)_{ex}$ for different bombarding energies.



FIG. 1. Pulse-height spectrum for 3.0-Mev protons on Mo^{94} target. The curve labeled proton bremsstrahlung also includes local background. The curve labeled 100 (for Mo^{100}) and the peak labeled Mo^{95} indicate the contributions from Coulomb excitation of these isotopes in the enriched Mo^{94} target.



FIG. 2. Pulse-height spectrum for protons on Mo⁹⁶ target.



FIG. 3. Pulse-height spectrum for protons on Mo⁹⁸ target.



normal molybdenum target.

Our best values for the $B(E2)_{ex}$ are given in column 5 of Table II. The columns headed R.E. and A.E. list percentage errors assigned to the $B(E2)_{ex}$. The relative error (R.E.) includes all the errors discussed above in $N(\text{excitations}/\mu\text{coul})$ and in Y which enter into the comparison of the relative transition rates for the different nuclei. The absolute percentage error (A.E.) contains additional sources of error (mainly the error in the absolute stopping power) which are common to all the transitions measured.

IV. REMARKS ON INDIVIDUAL CASES

A. Zirconium (Z=40).—The γ -ray spectrum resulting from 9-Mev α -particle bombardment of a normal zirconium target is shown in Fig. 21. Both Zr⁹² and Zr⁹⁴ are known to have first excited states at 920 key



FIG. 5. Pulse-height spectrum for protons on Ru⁹⁶ target.



FIG. 6. Pulse-height spectrum for protons on Ru⁹⁸ target.

from other types of experiments.¹⁸ Therefore, the 920kev γ ray is interpreted as resulting from Coulomb excitation of the first excited state of both nuclei and the $B(E2)_{\text{ex}}$ is the average for the two transitions.

B. Molybdenum (Z=42).—Figures 1 to 4 show spectra obtained from 3.0-Mev proton bombardment of targets enriched in Mo⁹⁴, Mo⁹⁶, Mo⁹⁸, and normal molybdenum, respectively. The normal molybdenum target was used to obtain the $B(E2)_{ex}$ for Mo¹⁰⁰ because insufficient enriched Mo¹⁰⁰ was available. We had previously reported the energy of the first excited state of Mo¹⁰⁰ as (540 ± 7) kev²; a somewhat more accurate measurement, made in the manner described above, gives (530 ± 5) kev.

A 320-kev γ ray is observed in several of the spectra (it is especially strong in Mo⁹⁶), and a 620-kev γ ray is seen in the Mo⁹⁴ spectrum. We believe that these γ rays do not result from Coulomb excitation of

¹⁸ G. L. Griffith, Phys. Rev. **103**, 643 (1956); Hayward, Hopper, and Ernst, Phys. Rev. **98**, 231(A) (1955).

molybdenum but we are not able to identify the reactions producing these γ rays. We have determined that both γ rays are in coincidence with hard radiation (>1 Mev) which eliminates the possibility that they result from Coulomb excitation of molybdenum.

The positions of the excited states and the $B(E2)_{ex}$ for the four molybdenum isotopes agree well with those obtained by Temmer and Heydenburg who studied the Coulomb excitation of these isotopes by the use of α particles.¹⁹

C. Ruthenium (Z=44).—Figures 5 to 9 show representative spectra for proton excitation of enriched samples of Ru⁹⁶, Ru⁹⁸, Ru¹⁰⁰, Ru¹⁰², and Ru¹⁰⁴, respectively. The γ ray from the weak excitation of the 840-kev



FIG. 7. Pulse-height spectrum for protons on Ru¹⁰⁰ target.



FIG. 8. Pulse-height spectrum for protons on Ru¹⁰² target.

¹⁹ G. T. Temmer and N. P. Heydenburg, Phys. Rev. 104, 967 (1956).

state in Ru⁹⁶ is superimposed on a γ ray of the same energy resulting from the Al $(p,p'\gamma)$ reaction in a 0.15% target impurity of aluminum. The Al $(p,p'\gamma)$ reaction also produces a strong γ ray of 1010 kev. By the measurement of the intensity of these two γ rays from proton bombardment of aluminum, it was possible, by normalizing to the 1010-kev peak, to infer the fraction of the 840-kev peak caused by the aluminum γ ray. This is shown in Fig. 5.

The values of $B(E2)_{ex}$ for the 475-kev state of Ru¹⁰² showed a considerable increase at the lower proton bombarding energies. We have concluded that this was caused by a small amount of lithium in the target (~5 ppm by weight). The high yield of 478-kev γ rays from the Li⁷($p,p'\gamma$) reaction increases much more slowly with increasing proton energy than does the



FIG. 9. Pulse-height spectrum for protons on Ru^{104} target.



FIG. 10. Pulse-height spectrum for protons on Pd¹⁰⁴ target.



FIG. 11. Pulse-height spectrum for protons on Pd¹⁰⁶ target.



FIG. 12. Pulse-height spectrum for protons on Pd¹⁰⁸ target.

yield of the 475-kev γ rays from Coulomb excitation of Ru¹⁰². The γ -ray yields given in Table I have been corrected for the presence of the 478-kev γ rays from Li(p,p'). This correction amounted to 5% at $E_p=3.0$ Mev.

Coulomb excitation of Ru^{96} and Ru^{98} has not previously been reported. No information on the excited states of Ru^{96} is available from radioactive decay studies. The decay of both Tc^{98} and Rh^{98} to Ru^{98} results in the emission of 650-kev γ rays^{20,21} which agrees with the level position of 654 kev found from Coulomb excitation.

Temmer and Heydenburg¹⁹ have reported energies and $B(E2)_{ex}$ for Ru¹⁰⁴, Ru¹⁰², and Ru¹⁰⁰; the latter being

 ²⁰ G. E. Boyd and Q. V. Larson, J. Phys. Chem. 60, 707 (1956).
 ²¹ S. Katcoff and H. Abrash, Phys. Rev. 103, 966 (1956).

assigned on the basis of systematics. The agreement as to the positions of the levels is good. It is difficult to judge how well the values for $B(E2)_{\rm ex}$ compare, since they do not give explicit errors. Our relative values are considerably different from theirs. For example, we find the ratio of the $B(E2)_{\rm ex}$ for Ru¹⁰⁴ to Ru¹⁰⁰ is 1.62 ± 0.11 whereas their value is 3.5.

D. Palladium (Z=46).—Figures 10 to 13 show representative spectra for proton bombardment of enriched targets of Pd¹⁰⁴, Pd¹⁰⁶, Pd¹⁰⁸, and Pd¹¹⁰. The 440-kev γ ray from Na($p, p'\gamma$) is present in the palladium spectra and this γ ray intefers with the determination of the yield of the 433-kev γ ray from Coulomb excitation of Pd¹⁰⁸. We have applied a correction by taking the average value for the intensity of the 440-kev sodium γ ray observed in the other palladium spectra and subtracting this from the yield of the Pd¹⁰⁸ γ ray. A



FIG. 13. Pulse-height spectrum for protons on Pd¹¹⁰ target.



FIG. 14. Pulse-height spectrum for protons on Cd106 target.



FIG. 15. Pulse-height spectrum for protons on Cd¹⁰⁸ target.



FIG. 16. Pulse-height spectrum for protons on Cd¹¹⁰ target.

concentration of approximately 300 ppm by weight of sodium in the target is indicated by the intensity of the 440-kev γ ray. The correction at 3.0-Mev proton energy amounted to 4%. Temmer and Heydenburg have also measured the Coulomb excitation of the four even-even isotopes of palladium.¹⁹ Here, also, there is a considerable divergence in the relative values of the $B(E2)_{\text{ex}}$. For instance, we find the ratio of the $B(E2)_{\text{ex}}$ for Pd¹¹⁰ to Pd¹⁰⁴ is 1.57±0.10 whereas their ratio is 2.26.

E. Cadmium (Z=48).— γ -ray spectra resulting from proton Coulomb excitation of targets enriched in Cd¹⁰⁶, Cd¹⁰⁸, Cd¹¹⁰, Cd¹¹², Cd¹¹⁴, and Cd¹¹⁶ and normal Cd, respectively, are shown in Figs. 14 to 20. The percentage abundance of Cd¹⁰⁶ and Cd¹⁰⁸ in the enriched samples was not very favorable because of the low natural abundance of these isotopes. The situation is made still more difficult by the superposition of the γ ray of the other isotopes on those of Cd¹⁰⁶ and Cd¹⁰⁸. Consequently the errors on the γ -ray yields for Cd¹⁰⁶ and Cd¹⁰⁸ are considerably larger than those for the other isotopes. Coulomb excitation of Cd¹⁰⁶ and Cd¹⁰⁸ has not previously been reported. Nothing is known about excited states of Cd¹⁰⁶ from radioactive decay. The decay of both Ag¹⁰⁸ and In¹⁰⁸ to Cd^{108 22} results in the emission of a γ ray which has an energy in agreement with the 630±10 kev determined from Coulomb excitation.

As indicated in Table I, the yield of the 555-kev γ ray from Cd¹¹⁴ was also measured for 9-Mev α -particle



FIG. 17. Pulse-height spectrum for protons on Cd¹¹² target.



FIG. 18. Pulse-height spectrum for protons on Cd¹¹⁴ target.



FIG. 19. Pulse-height spectrum for protons on Cd¹¹⁶ target.



FIG. 20. Pulse-height spectrum for protons on normal cadmium target.

bombardment to check that the $B(E2)_{ex}$ obtained from proton and α -particle excitation are consistent. The average value of the $B(E2)_{ex}$ obtained with protons and that obtained with α particles differ by 4%. This difference is believed to be reasonable in view of the facts (a) that a 3 in.×3 in. NaI crystal was used for the α -particle work whereas a $1\frac{1}{2}$ in.×1 in. crystal was used in the earlier proton work, and (b) that the relation used to convert from proton stopping power to α -particle stopping power is probably not strictly valid.

Temmer and Heydenburg¹⁹ have reported Coulomb excitation of Cd¹¹⁰, Cd¹¹², Cd¹¹⁴, and Cd¹¹⁶. Our values for the level positions agree, to within the assigned errors, with their values. We find the ratios of the $B(E2)_{\rm ex}$ for Cd¹¹⁶ to Cd¹¹⁶ and Cd¹¹⁶ to Cd¹¹² are 1.19

500

²² W. Mead, University of California Radiation Laboratory Report UCRL-3488, 1956 (unpublished); Perlman, Bernstein, and Schwartz, Phys. Rev. 92, 1236 (1953); M. C. Joslin and B. V. Thosan, Proc. Indian Acad. Sci. 43A, 255 (1956).

 ± 0.08 and 1.11 ± 0.07 , respectively, whereas they obtained the somewhat larger values of 1.51 and 1.35.

F. Tin (Z=50).—Figures 21, 22, and 23 show spectra for 10-Mev α -particle excitation in targets of normal Sn and enriched Sn¹¹⁶, Sn¹¹⁸, Sn¹²⁰, Sn¹²², and Sn¹²⁴. The spectra indicate excitation of numerous other γ rays in addition to those assigned to Coulomb excitation of the 2⁺ states of the even-even tin isotopes. The origins of some of these γ rays have been determined and these are identified in the figures. Most of those which have not been identified are believed to result from reactions



FIG. 21. Pulse-height spectra for α particles on normal zirconium and normal tin targets.



FIG. 22. Pulse-height spectra for α particles on $\mathrm{Sn^{116}}$, $\mathrm{Sn^{118}}$, and $\mathrm{Sn^{120}}$ targets.



501

FIG. 23. Pulse-height spectra for α particles on Sn¹²² and Sn¹²⁴ targets.

produced in small amounts of low-Z target impurities. Alkhazov et al.²³ have studied Coulomb excitation of $\mathrm{Sn^{117}}$ and $\mathrm{Sn^{119}}$ and they find γ rays of 865 and 1030 kev in Sn¹¹⁷ and 907 kev in Sn¹¹⁹. These γ rays could account for the broad peak at 470 pulse-height units observed in the spectrum for normal tin.

The angular distribution of the 1155-kev γ rays excited in the Sn¹²⁰ target was measured to be $1 + (0.29 \pm 0.02) P_2 - (0.04 \pm 0.03) P_4$ for an α -particle energy of 10 Mev. The expected angular distribution for the Coulomb excitation of a 2^+ state under these experimental conditions is $1+0.30P_2-0.09P_4$.

The locations of the first excited states of Sn¹¹⁶, Sn¹²⁰, and Sn¹²² are known from radioactive-decay studies. The first excited state of Sn¹¹⁸ and Sn¹²⁴ were not previously known. From the decay of In¹¹⁶, Slätis et al.²⁴ determined the energy of the first excited state of Sn¹¹⁶ to be (1274 ± 6) kev and this agrees well with our value of (1268 ± 10) kev. McGinnis²⁵ has determined the energy of the first excited state of Sn¹²⁰ to be 1180 kev from the study of the decay of Sb¹²⁰. From measurements on the decay of Sb¹²² both Farrelly et al.²⁶ and Glaubman²⁷ have determined the energy of the 2⁺ state in Sn^{122} to be (1137±6) kev and (1152±15) kev, respectively, and these values agree with our value of (1130 ± 10) kev.

We have been informed by private communication of work by Alkhazov et al.23 in which Coulomb excitation of the even-even isotopes of tin has been measured.

²³ Private communication from Alkhazov, Lemberg, Andreev, and Erokhina of the Leningrad Physico-Technical Institute, U.S.S.R.

 ²⁴ Slätis, Dutoit, and Siegbahn, Arkiv Fysik 2, 321 (1950).
 ²⁵ C. L. McGinnis, Phys. Rev. 98, 1172(A) (1955).

²⁶ Farrelly, Koerts, Benczer, van Lieshout, and Wu, Phys. Rev. **99**, 1440 (1955). ²⁷ M. J. Glaubman, Phys. Rev. 98, 645 (1955).



FIG. 24. Pulse-height spectrum for protons on normal tellurium target.

Their values for the energies of the 2⁺ states and the $B(E2)_{ex}$ values are in reasonable agreement with our results.

G. Tellurium (Z=52).—The γ -ray pulse-height spectrum observed when a normal tellurium target was bombarded by 3.3-Mev protons is shown in Fig. 24. Targets of enriched isotopic abundance were not used; others have studied the Coulomb excitation of tellurium with enriched targets.¹⁹ The use of a normal tellurium target limited the isotopes in which Coulomb excitation could be studied to Te¹²⁶, Te¹²⁸, and Te¹³⁰. The positions in the pulse-height spectrum of other weak, unresolved γ rays are indicated by arrows. Coulomb excitation of normal tellurium was also measured with 9- and 10-Mev α particles and the resulting γ -ray yields were more precisely determined than those for proton excitation because the background was much lower with α -particle excitation. The values for $B(E2)_{ex}$ obtained with proton and α -particle excitation (see Table I) are in satisfactory agreement.

Information on the location of the 2^+ state in Te¹³⁰ is not available from radioactive decay work. Our value of 850 kev is in good agreement both with other Coulomb excitation results¹⁹ and with the value of 830 ± 20 kev obtained by Sinclair from the (n,n')reaction.28

Benczer et al.29 have established that the 2+ state of Te¹²⁸ is excited in the decay of I¹²⁸. Their value for the energy of 750 ± 7 kev compares well with the Coulomb excitation result of Heydenburg and Temmer $(750 \text{ kev})^{19}$; the (n,n') result of Sinclair $(760\pm 20) \text{ kev}^{28}$; and with our result of (750 ± 10) kev.

Several values are available for the energy of the 2^+ state of Te¹²⁶ but here the agreement is not quite so

good. From the radioactive decay of I¹²⁶, Marty et al. find 670 kev³⁰; Perlman and Welker find 651 ± 10 kev³¹; and Koerts et al. find 650±10 kev.32 From Coulomb excitation, Heydenburg and Temmer obtain 662 kev,¹⁹ and from (n,n') Sinclair finds 680 ± 20 kev.²⁸ Our value is 673 ± 7 kev.

Our values for the $B(E2)_{ex}$ for Te¹²⁶, Te¹²⁸, and Te¹³⁰ are not in good agreement with those obtained by Heydenburg and Temmer. Their absolute values are considerably smaller than ours and the ratios of the $B(E2)_{ex}$ for the different isotopes differ somewhat from those we obtain. For instance, we find that the ratio of the $B(E2)_{ex}$'s for Te¹²⁶ to Te¹³⁰ is 1.56 ± 0.13 whereas they obtain 1.23.

V. COMPOUND NUCLEUS REACTIONS

The above discussion of the errors involved in the determination of the values of $B(E2)_{ex}$ assumed that the observed γ -ray yields resulted solely from Coulomb excitation. In this section we wish to investigate the validity of this assumption by considering (a) the comparison of the experimental results with the theory of Coulomb excitation and (b) the magnitude of the



FIG. 25. Cross sections for Coulomb excitation by protons of the 358-kev state in Ru¹⁰⁴, the 555-kev state in Cd¹¹⁴ and the 874-kev state in Mo⁹⁴. The calculated cross section for compound nucleus formation for ruthenium (Z=44) is shown. The calculated cross sections for excitation of the 358-kev state of Ru¹⁰⁴ and the 874-kev state in Mo⁹⁴ by compound-nucleus inelastic proton scattering are also shown.

²⁸ R. M. Sinclair, Phys. Rev. 102, 461 (1956).

²⁹ Benczer, Farrelly, Koerts, and Wu, Phys. Rev. 101, 1027 (1956).

 ³⁰ Marty, Langevin, and Hubert, Compt. rend. **236**, 1153 (1953).
 ³¹ M. L. Perlman and J. Welker, Phys. Rev. **95**, 133 (1954).
 ³² Koerts, Macklin, Farrelly, van Lieshout, and Wu, Phys. Rev.

^{98, 1230 (1955).}

 γ -rays yields to be expected from the alternative mechanism of compound nucleus inelastic proton or α -particles scattering.

For most of the nuclei studied, the γ -ray yields were observed over a considerable range of projectile energies. In some cases, both protons and α particles were used to excite a given state. When the variations in yield are compared to those predicted by the theory of Coulomb excitation, good agreement is found for most of the nuclei. Significant discrepancies were found in three cases, but these discrepancies are reasonably accounted for by the interference from γ rays produced by reactions in the small amounts of light element impurities in the targets. Angular distributions of the γ rays have been measured for a range of projectile energies for several of the nuclei and to within experimental error these distributions agree with those expected for Coulomb excitation.¹⁴ However, these arguments based on the agreement between experiment and theory of the yields and angular distributions are somewhat weakened by the fact that the most accurate experimental results are for those nuclei which exhibit the strongest excitation whereas possible interference from compound nucleus reactions should be most important for those cases in which the cross sections are the smallest. With reference to Fig. 25, it is seen that for a given projectile energy the observed cross sections may differ by a factor of 100 or more for different nuclei (see Ru¹⁰⁴ and Mo⁹⁴).

Measured values of relevant cross sections for compound nucleus inelastic scattering cross sections, $[\sigma(x,x')]_{comp}$, do not exist. The first step in the theoretical estimate of such cross sections is the estimate of the cross section for formation of the compound nucleus, $\sigma_{\rm comp}$. Recently, some experimental information on $\sigma_{\rm comp}$ for protons on medium weight nuclei has been obtained from the study of (p,n) reactions.^{33,34} These cross sections were found to be fairly well accounted for by straightforward theoretical estimates based on barrier penetrabilities. As a result, for protons on the nuclei in which we are interested, $\sigma_{\rm comp}$ may be estimated to within a factor of 2. The estimated cross section for ruthenium (Z=44) as a function of proton energy is shown in Fig. 25. Estimates of σ_{comp} for α particles on medium weight nuclei are considerably more uncertain than those for protons. The interaction radius is not well known because of the lack of experimental information. Theoretical estimates of σ_{comp} given by Blatt and Weisskopf³⁵ for tin (Z = 50) for two possible interaction radii are shown as a function of α -particle energy in Fig. 26. Probably a factor of 10 uncertainty exists in the estimates of σ_{comp} for α particles.

The values for σ_{comp} may be taken as an upper limit for the excitation of the 2⁺ by $[\sigma(x,x')]_{comp}$. A com-

(John Wiley and Sons, Inc., New York, 1952).



FIG. 26. Cross sections for the Coulomb excitation by α particles of the 1155-kev state in Sn¹²⁰ and the 555-kev state in Cd¹¹⁴. The calculated cross section for compound nucleus formation by α particles on Sn is indicated by the cross-hatched band. The calculated cross section for the excitation of the 1155-kev state in Sn¹²⁰ by compound-nucleus inelastic α -particle scattering at 10-Mev is also shown.

parison of the cross sections for protons presented in Fig. 25 shows that $\sigma_{\rm comp}$ is never more than a few percent of the cross section for Coulomb excitation of Ru¹⁰⁴. On the other hand, $\sigma_{\rm comp}$ is actually larger than the cross section for Coulomb excitation of Mo⁹⁴. As one would expect, the situation is improved for α particles; the relatively small cross section for Coulomb excitation of Sn¹²⁰ is still at least 10 times larger than $\sigma_{\rm comp}$.

To obtain estimates of $[\sigma(x,x')]_{comp}$ it is next necessary to estimate the relative probabilities for the different possible modes of decay for the states of the compound nucleus. If the excitation energy of the compound nucleus is above the threshold for neutron emission, this mode of decay will generally be dominant. However, we omit the consideration of neutron emission, since, if it exists, it serves to reduce still further the expected values of $[\sigma(x,x')]_{comp}$. We restrict the possible modes of decay of the compound states to radiative capture (partial width Γ_{γ}), and to re-emission of charged particles which either leave the residual nucleus in the ground state (partial width Γ_x) or in the first 2⁺ excited state (partial width $\Gamma_{x'}$). According to

 ³³ Johnson, Galonsky, and Ulrich, Phys. Rev. 109, 1232 (1958).
 ³⁴ J. P. Schiffer and L. L. Lee, Jr., Phys. Rev. 107, 640 (1957).
 ³⁵ J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics*

the theory of nuclear reactions, one expects the charged particle widths, Γ_x and $\Gamma_{x'}$, to be equal to $T_l(x \text{ or } x')$ $\times D_J/2\pi$, where T_l is the barrier penetration factor and D_J is the average spacing of states with spin J at the excitation energy of the compound nucleus. Therefore, if one has values for D_J he can obtain Γ_x and $\Gamma_{x'}$ for each type of compound state, i.e., given spin and parity. Newton's formula³⁶ was used to obtain values of D_J .³⁷ The radiation widths, Γ_{γ} , were calculated by the use of Cameron's formula.³⁸ This formula is based on Newton's level spacing formula and assumes dipole emission. It also contains a constant which was adjusted to give a good fit to observed neutron capture radiation widths.

The calculated values for $[\sigma(x,x')]_{comp}$ are shown in Figs. 25 and 26 for 2.7- and 3.0-Mev protons on Ru¹⁰⁴, 3.0-Mev protons on Mo⁹⁴, and for 10-Mev α particles on Sn¹²⁰. It is difficult to judge the reliability of these estimates of $[\sigma(x,x')]_{comp}$ because not enough is known about the general reliability of Newton's formula for the level spacing and of Cameron's formula for the width for radiative capture. The salient result of these estimates is that in each case $[\sigma(x,x')]_{comp}$ is much less than σ_{comp} . The predominant mode of decay for the compound states is by radiative capture. These estimates suggest that interference from $[\sigma(x,x')]_{comp}$ for the proton and α -particle energies employed in the present experiments introduces a negligible error in the determination of Coulomb excitation cross sections. For the case of the very weak Coulomb excitation of Mo⁹⁴, the calculated contribution to the γ -ray yield from $[\sigma(x,x')]_{comp}$ is less than 1%.

VI. DISCUSSION

In column 8 of Table II we list the ratio of the observed $B(E2)_d$ to that expected for a transition between states of the single-particle model. We have taken $B(E2)_{sp}$ to be equal to $(1/4\pi)|\frac{3}{5}R_0^2|^2$ where $R_0=1.20\times10^{-13}A^{\frac{1}{3}}$ cm.³⁵

The even-even tin isotopes present an interesting situation. The protons in these nuclei, having completed the major shell at 50, should make these nuclei quite rigid. The large excitation energy of the 2^+ states is compatible with this view and one might have expected that the transition rates would be those given by $B(E2)_{\rm sp}$. However, the observed rates are approximately 13 times larger. Although the protons form a closed shell, the neutron numbers for the tin isotopes are about midway between the major shells of 50 and 82. Consequently one would expect the neutrons to exert a deforming or softening action on the nuclei and perhaps the enhanced transition rates reflect this action. The energy of the first excited state of even-even

medium weight nuclei shows a strong variation with changes in neutron and proton number. Although the over-all trend is a smooth decrease in excitation energy with removal from closed shells, some interesting exceptions to this behavior have appeared. The anomalous behavior of Mo⁹⁶ and Mo⁹⁸ has been reported¹⁹ and we confirm this result. The cadmium isotopes are also exceptional. The four heaviest isotopes establish a trend with neutron number which, if continued by Cd^{108} and $\mathrm{Cd}^{106},$ would place the excited states above that for Cd¹¹⁰ (656-kev) whereas both isotopes have excited states at 630-kev. The variation in excitation energies with proton number clearly indicates the unusual stability of the 40-proton configuration. The relatively small value for the transition rate of Zr⁹² and and Zr⁹⁴ probably results from the combined effects of neutron numbers near to 50 and the increased stability of the 40-proton configuration.

The values for $B(E2)_{ex}$ also show strong and fairly systematic trends with variations in neutron and proton number. The plot of the observed $B(E2)_{ex}$ vs neutron number is given in Fig. 27. A close correlation is found between the $B(E2)_{ex}$ and the excitation energy of a given state. To exhibit this we have also plotted in Fig. 27 the quantity $3.24/E_{2+}$ (Mev). The constant 3.24 was chosen to give a good fit to those nuclei with large $B(E2)_{ex}$. The anomalous behavior of the excitation energies for the molybdenum isotopes is also reflected in the values for $B(E2)_{ex}$.

In order to more clearly demonstrate the observed dependence of the $B(E2)_{ex}$ on the energy of the excited state, a plot of $\log B(E2)_{ex}$ vs $\log E_{2+}$ is given in Fig. 28. The straight line in the figure has a slope of -1, corresponding to a E^{-1} energy dependence.



FIG. 27. Plot of the experimental values for $B(E2)_{ex}$ vs neutron number. The values for $B(E2)_{ex}$ (referred to the left-hand ordinate) are designated by different symbols for different elements. To demonstrate the close correlation between the $B(E2)_{ex}$ and the energy of the 2⁺ state, we have also plotted the quantity $3.24/E_{2+}$ (Mev) vs neutron number. These values, which are connected by solid lines for a given element, refer to the right-hand ordinate.

⁸⁶ T. D. Newton, Can. J. Phys. 34, 804 (1956).

³⁷ Excitation energies of the compound nucleus were calculated by the use of mass values given by A. H. Wapstra, Physica 21, 385 (1955).

³⁸ A. G. W. Cameron, Can. J. Phys. 35, 666 (1957).

Two types of collective motion have been proposed to account for "near harmonic" spectra; the free oscillation of a nucleus about the spherical shape,^{4,15} and the " γ unstable" motion.⁶ From the $B(E2)_{\rm ex}$ and the E_{2+} one obtains the parameters B_2 and C_2 which are appropriate to the description of quadrupole vibrations about the spherical shape.^{19,15} C_2 represents the effective surface tension and B_2 represents the mass transported by the collective motion. Formulas (6) show the relation between these quantities:

$$C_2(\text{Mev}) = 0.995Z^2 E_{2+}(\text{Mev}) / [B(E2)_d/B(E2)_{sp}],$$
 (6a)

$$B_2/(B_2)_{\rm irrot} = \frac{241(Z^2/A^{5/3})}{E_2 + ({\rm Mev})[B(E2)_d/B(E2)_{\rm sp}]}.$$
 (6B)

The value $(B_2)_{irrot}$ is that expected for the case of irrotational flow. The values for C_2 and $B_2/(B_2)_{irrot}$ are listed in Table II. Of the two experimental quantities, E_{2+} and $B(E2)_d$, which enter into the calculation of the parameter C_2 and $B_2/(B_2)_{irrot}$, the $B(E2)_d$ has the dominant error. Since both parameters vary inversely with $B(E2)_d$, the percentage errors given in Table II as R.E. and A.E. will also be approximately correct for C_2 and $B_2/(B_2)_{irrot}$.

The parameters appropriate to the "shape unstable" model are β and $B_2/(B_2)_{irrot}$, where β is

$$\beta = [2.91 \times 10^{26}] [B(E2)_{\text{ex}}]^{\frac{1}{2}} / ZA^{\frac{1}{3}}.$$
(7)

The values of β are listed in Table II. Since β depends on the square root of $B(E2)_{ex}$, the percentage error



FIG. 28. Plot of $\log B(E2)_{\text{ex}}$ vs $\log E_{2+}$. The straight line has a slope of -1 corresponding to a E^{-1} dependence for the $B(E2)_{\text{ex}}$.

in β is approximately one half the percentage errors R.E. and A.E.

For those nuclei where the values for C_2 and $B_2/(B_2)_{\rm irrot}$ may be compared to the earlier results of Heydenburg and Temmer,¹⁹ the observed differences result from differences in the experimental values for $B(E2)_{\rm ex}$. In spite of these differences, both sets of values for the parameters point to the rapid variation in C_2 and to large values for $B_2/(B_2)_{\rm irrot}$. Our values for $B_2/(B_2)_{\rm irrot}$ display more uniformity; in fact, the constancy of $B_2/(B_2)_{\rm irrot}$ for a large number of the nuclei is a striking feature. It is also interesting to note that some of the values for β in Table II are as large as those for nuclei which exhibit rotational spectra.

APPENDIX. ENERGY LOSS OF PROTONS IN SILVER AND GOLD

The number of γ rays excited in a target by the Coulomb excitation process is proportional to

$$\int_{Ef}^{E_i} \frac{\sigma(E)dE}{S(E)},\tag{1}$$

where

$$E_f = E_i - \int_0^x S dx. \tag{2}$$

 $\sigma(E)$ is the cross section and S(E) is the energy loss. The shape of the cross section curve as a function of energy for Coulomb excitation has been accurately calculated and has been experimentally confirmed. The absolute value of $\sigma(E)$ depends on $B(E2)_{\text{ex}}$. One has that $\sigma(E) = BX(E)$, where X(E) is known. Therefore one has

$$B \propto 1 \bigg/ \int_{E_f}^{E_i} \frac{X(E)dE}{S(E)}$$

Now, for a thick target $(E_f=0)$ one sees that if S(E) is uniformly 10% too large then B will be in error by 10%. On the other hand, if the target is sufficiently thin, the error introduced in the integrand is just counteracted by the change in the limit E_f . So, from thin-target measurements on silver and gold, one can get good values of B, even with an error in the value taken for S(E). One then compares these values with those obtained by bombarding thick targets with protons of various incident energies. From this comparison one can deduce the error in S(E) and alter S(E) until yields from thick and thin targets give consistent values for B. It is not necessary to know the γ -ray detection efficiency to high accuracy since it drops out in the comparison of thick and thin targets. However, it is necessary to know quite accurately the thickness x of the thin target, since this enters into the determination of B.

The thin foils were chosen to be only moderately thin because it is desirable to have sufficient Coulomb excita-

$E_n(Mey)$	79Au $(dE/d\rho x)_p$ kev/(mg/cm ²)	$E_n(Mev)$	$47 \text{Ag} \\ (dE/d_{\rho}x)_{p} \\ E_{-}(\text{Mev}) \text{kev}/(\text{mg/cm}^{2})$		
5.0	27.8	3.3	50.7		
4.8	28.5	3.2	51.6		
4.6	29.2	3.1	52.4		
4.4	30.0	3.0	53.4		
4.2	30.8	2.9	54.3		
4.0	31.6	2.8	55.4		
3.8	32.7	2.7	56.4		
3.6	33.7	2.6	57.5		
3.4	34.9	2.5	58.6		
3.2	36.1	2.4	59.7		
3.0	37.3	2.3	61.0		
2.8	38.7	2.2	62.3		
2.6	40.3	2.1	63.7		
2.4	42.2	2.0	65.2		
2.2	44.4	1.9	66.8		
2.0	46.7	1.8	68.5		
1.8	49.3	1.7	70.5		
1.6	52.3	1.6	72.7		
1.4	55.9	1.5	75.1		
1.2	60.1	1.4	77.8		
1.0	65.1	1.3	80.8		
0.8	71.3	1.2	84.1		
		1.1	87.9		
		1.0	92.5		
		0.9	98.0		
		0.8	104.0		

TABLE III. $dE/d\rho x$ for protons in Au and Ag.

tion yield to make the yield of the proton bremsstrahlung (from foil and foil backing) small in comparison.

Gold and silver foils in 6×6 in. squares were obtained from Baker and Company, Inc. These were successively cut into smaller squares in order to study the macroscopic uniformity. In this way profiles of the thickness of the original large foils were obtained. Small foils for the experiment were chosen from those sections showing the greatest uniformity. It is judged that the thickness of the foils used is known to 0.5% for the gold foils (thickness $\sim 10 \text{ mg/cm}^2$) and to 1.0% for the silver foils (thickness $\sim 6 \text{ mg/cm}^2$). The silver foils were mounted on tin backings of approximately 80 mg/cm² thickness, and the gold foils were mounted on bismuth backings of approximately 120 mg/cm² thickness. Both the tin and bismuth backings were prepared by electrodeposition onto 5-mil nickel foils. Three thin targets each of silver and gold were prepared to check the reproducibility of the measurements.

Since the foils are only moderately thin, one expects that the values obtained for B are not completely independent of the values taken for S(E). We have studied this problem in the following way: We assumed that the shape of S(E) was known. We then changed the absolute values by a certain percentage, for example, 10%, and carried out numerical integration to find the resultant change in the thin-target B value for various incident proton energies and for the foil thicknesses used. The ratio of the percentage change in B to the percentage change in S(E) was 0.35 for the worst case (3-Mev protons on gold foil) and 0.08 for the best case (5-Mev protons on gold foil). This ratio is then useful in the determination of the values of S(E) which will give the same B values for thick and thin targets.

Measurements were made at proton energies of 3.0, 3.5, 4.0, 4.5, and 5 Mev for gold and at 2.4, 2.7, 3.0, and 3.3 Mev for silver. Coulomb excitation of the states at 550 and 277 kev in gold and at 419 and 316 kev in silver was studied.

Two small corrections were applied. First, one must take into account the fact that angular distribution of the gamma rays from Coulomb excitation varies (slowly) with the incident proton energy. The angular distributions are known for silver and gold.³⁹ Appropriate averages over target thickness have been determined and applied to measurements. Second, allowance was made for the fact that the multiple small-angle Rutherford scattering makes the effective path length in the foil somewhat larger than the foil thickness. This increase can be calculated by the use of the theory of Goudsmit and Saunderson⁴⁰ and Williams.⁴¹ For our cases it was found that the largest correction for this effect was 0.5% (3-Mev protons on the gold foil).

Our best values for S(E) for silver and gold are given in Table III. These measurements are of an integral type and are useful in obtaining accurate values of Bfrom Coulomb excitation with thick targets. However, they are not suited for a detailed examination of the shape of S(E). Rather, one must assume that he knows the shape of S(E) and then determine whether the experiments are consistent with the assumed shape. Our shape for S(E) for silver is of the form $S(E) \propto E^{-\frac{1}{2}}$, and that for gold is very nearly of this shape. Theoretical consideration and experimental results⁴² suggest this energy dependence for S(E). If one accepts the fact that the shapes for S(E) are correct (and the experiments are consistent with these shapes), then it is felt that the absolute values of S(E) are determined to within $\pm 4\%$ (standard deviation).

- ⁴⁰ S. Goudsmit and J. L. Sauderson, Phys. Rev. 57, 24 (1940).
 ⁴¹ E. J. Williams, Phys. Rev. 58, 292 (1940).
- ⁴² A. Winther (private communication).

³⁹ F. K. McGowan and P. H. Stelson, Phys. Rev. 99, 127 (1955).